John Adams
Library.

IN THE CUSTODY OF THE
BOSTON PUBLIC LIBRARY.

SHELF No
* ADAMS
* 262.13

John Adams

THE ELEMENTS OF Astronomy, PHYSICAL and GEOMETRICAL.

By DAVID GREGORY M. D. *Savilian* Professor of ASTRONOMY at *Oxford*, and Fellow of the Royal-Society.

Done into *English*, with Additions and Corrections.

To which is annex'd,
Dr. HALLEY'S Synopsis of the Astronomy of Comets.

In Two Volumes.

Vol. I.

LONDON:
Printed for J. NICHOLSON at the *King's-Arms* in *Little-Britain*, and sold by J. MORPHEW near *Stationers-Hall*. M DCC XV.

Price of the Two Vols. 12s. Bound.

XX
ADAMS
Nov. 13

THE AUTHOR'S PREFACE.

MY design in publiſhing this Book, was, that the Celeſtial Phyſics, which the moſt ſagacious *Kepler* had got the ſcent of, but the Prince of Geometers Sir *Iſaac Newton*, brought to ſuch a pitch as ſurpriſes all the World, might, by my care and pains in illuſtrating, become eaſier to ſuch as are deſirous of being acquainted with Philoſophy and Aſtronomy. The Title informs you ſufficiently that the Arithmetical or Calculatory part of Aſtronomy is here omitted, tho' that, perhaps, may be publiſhed hereafter in its proper place. As for the Phyſics, it is all taken out of the abovemention'd Authors; but is here intermix'd with Aſtronomy, in ſuch places as ſeem'd proper and convenient; the Geometry to be met with in it, I have either borrow'd elſewhere, and quoted the place where 'tis to be found,

or deliver'd it Lemmatrically. Whatever is done in each Section, you have it expreſs'd either in the Title or Preface thereof, in ſuch a manner, as that thoſe who are leſs vers'd in the more abſtruſe parts of Geometry, or leſs concern'd about the Phyſical parts, may paſs over, and only read the Aſtronomy ſeparately and diſtinct from them.

The Celeſtial Phyſics, or Phyſical Aſtronomy, is not only the firſt in dignity of all inquiries into Nature whatever, but the firſt in order, becauſe it is the eaſieſt. For the Sun and Planets are ſeparated from one another by ſo immenſe a diſtance, as renders them incapable of exerting moſt of thoſe forces whereby all Bodies act upon one another; ſo that they have no other force left them whereby they can affect one another, but the ſingle force of univerſal Gravity: Whereas in the production of ſeveral Phænomena, that are obſerv'd upon our Earth, innumerable other forces are exerted, ſuch as are very hard to be diſtinguiſh'd from one another; which notwithſtanding, if not accurately done, in vain do we attempt Nature, and make any inquiry into it. Upon this account it is, that every Problem in the Terreſtrial Phyſics is very operoſe and perplex'd, on the contrary, in the Celeſtial Phyſics, much more eaſy and ſimple; tho' even the latter has its difficulties, ariſing from the different diſtances and magnitudes of the Celeſtial Bodies. For the Fix'd Stars are ſo vaſtly diſtant aſunder, that they have no mutual action upon

each

each other, obſervable by us who are the Inhabitants of the Earth. The Primary Planets are remov'd ſo far from each other, that, tho' they have ſome ſmall power and effect upon one another, yet we cannot be ſenſible of it, till after many Years obſervation. The Secondary Planets are not at ſo great a diſtance from their Primaries, or from the Sun, but that they may be conſiderably affected by the powers of both, (if regard be had to the Quantity of Matter that is in theſe latter,) and this is the ſpring of thoſe manifold inequalities found in them, ſuch as, for inſtance, manifeſtly ſhews itſelf in our Moon; which yet is nothing at all, if compar'd with the inequalities found among Terreſtrial Bodies, which are acted upon by an innumerable variety of other forces, preſſing every way upon them. So that thoſe perſons ſeem to apply their thoughts but to a very indifferent purpoſe in the ſtudy of Nature, that overlook this part of Aſtronomy, from whence the principal and moſt ſimple Laws of Nature are to be learn'd.

That none may think the Phyſics deliver'd in the following Work intirely new and unknown in Aſtronomy, I ſhall take the liberty to ſhew that it was both known and diligently cultivated by the moſt ancient Philoſophers. And I ſhall dwell a little longer upon this argument, becauſe there is no need of ſpending a Preface, either upon the order of the parts of this Work, which may be ſeen in the Index, or upon the uſefulneſs, dignity, hiſtory and pro-

grefs of Aſtronomy, or even of the true Syſtem of the World, approv'd of by *Pythagoras*, and others among the Ancients; theſe things being all of them treated of at large by the common Writers of Aſtronomy. What I ſhall now therefore make out is, that we do ſtill tread in the ſteps of the Ancients in this Phyſical Aſtronomy; inaſmuch as they knew that the Celeſtial Bodies gravitated towards each other, and were retain'd in their Orbits by the force of Gravity; and were alſo apprized of the Law of this Gravity.

For if we look back to the firſt Riſe of Aſtronomy, and take a view of it in its Infancy, as it were, we ſhall find nothing better approv'd of, nothing more univerſally entertained among the ſeveral Sects of Philoſophers, than this notion of the Gravity of the Celeſtial Bodies. That ſaying is well known, ſo often uſed by [a] *Anaxagoras*, and his Scholars, [b] *Achelaus* and [c] *Euripides*. Namely, "That "the Sun and Stars were *fiery or red-hot Stones* "and *Golden Clods*." Of the ſame mind alſo were [d] *Democritus*, *Metrodorus*, and [e] *Diogenes*, By

a *He affirm'd the Sun to be a Maſs of Red-hot Iron.* Diog. Laert. in Anaxag. *That the ſubſtance of the Sun was Stone, that of the Moon, Earth.* Plat. in Apol. Socr.

b *That the Stars were Plates of Red hot Iron.* Stob. Ecl. Phyſ. cap. 25.

c *They ſay that* Euripides *uſed to call the Sun a Clod of Gold, or Golden Maſs.* Diog. Laer. in Anaxag.

d Anaxagoras, Democritus *and* Metrodorus *affirm'd, that the Sun was a Maſs of Iron or Stone red hot.* Plut. de Placit. Phil. lib. 2. c. 20.

e Diogenes *thought that the ſubſtance of the Stars was not unlike that of a Pumice Stone.* Ibid, c. 13.

By these expressions they meant no more, than that they were heavy, dense and fix'd Bodies, (such as Stones are) so as to bear a considerable degree of heat: And that this was really their meaning will evidently appear, if we do but enquire more narrowly into the first Authors of this Opinion. For, as we are told by [f] *Democritus*, these notions about the Sun and Moon are not to be ascrib'd to *Anaxagoras* as their original, for he had really borrow'd them of the Ancients. Nor is it a difficult matter to find out, who they were that he borrow'd them of, or from whom they were handed down to him. He had them from his Master [g] *Anaximenes*, whose Opinion we know was, that the Stars were of a fiery nature and substance, that there were also mingled with them certain Earthly Bodies, which were carried round about them, tho' not visible to us: By which words he plainly means, Planets of a terrestrial nature, performing their revolutions in the System of every Fix'd Star. These notions *Anaximenes* received from *Anaximander*, *Anaximander* from [h] *Thales* himself, who was the Head and Founder of the *Jonic* Philosophy;

---

f Favorinus, *in his various History, relates, that* Democritus *used to say of* Anaxagoras, *that the Opinions which he taught concerning the Sun and Moon, were not his own, but far more ancient than* Anaxagoras'*s time; and that he had stollen them.* Laert. in Democrit.

g Anaximenes *said, the nature of the Stars were fiery, and that there were certain terrestrial Bodies that are invisible, carried together about them.* Stob. Ecl. Phys. c. 25.

h Thales *was of Opinion that the Stars consisted of an Earthly substance, which was continually red-hot.* Stob. Ecl. Phys. c. 25. And so Plutarch. De Plac. Phylos. lib. 2. c. 13.

ſophy; and ſpread this opinion of the Gravity of the Fix'd Stars among his Sect. Nor did this Doctrine concerning the Stars ſtop here, but afterwards it diffus'd it ſelf thro' the *Italic* Philoſophy, the [i] followers of which taught, that each Star was a World in the infinite Æthereal Space, containing Earth, Air and Æther; and that the [k] Moon, not only was like our Earth, but inhabited by Animals of a larger ſize, and furniſh'd with Plants of a more beautiful appearance.

Nor were they ſo abſurd in their conceptions about Gravity, as to think that it was done by the virtue of any point within the Earth, or of a Center, to which all heavy Bodies placed any where tended; but they thought it was done by the [l] power of the whole Matter in the Terreſtrial Globe attracting all things to it ſelf: And as the power of the Loadſtone is compos'd of the powers of the ſeveral

---

i *The* Pythagoreans *affirm'd, that every Star is a World in the infinite Æthereal Space, wherein are contain'd Earth, Air, and Æther,* Plut. de Plac. Philoſ. Lib. 2. c. 13.

k *The* Pythagoreans *aſſerted, that the Moon ſeem'd to be of a ſimilar nature with the Earth, it is inhabited as our Earth is, by Animals, tho' of a larger ſize than ours, and fill'd with the ſame Plants, tho' much more beautiful than ours.* Plut. de Plac. Phil. Lib. 2. c. 30.

l *And yet if every heavy Body inclines towards the ſame place, and does with every one of its parts tend to its middle or center, the Earth certainly will not appropriate to itſelf theſe heavy Bodies, which are its parts, becauſe it is the Center of the Univerſe, but rather becauſe it is the whole, of which they are the parts.* Plut. de facie in Orbe Lunæ.

As for that which is incorporeal, 'tis not probable, nor will they themſelves allow it to be poſſeſs'd with ſo great a power as to draw all things towards it, and retain them about it. Ibid.

ſeveral parts combin'd together, ſo they believ'd that the Gravity towards the whole Earth, reſulted from the Gravity towards each ſingle part of it. Beſides, they believ'd there was a [m] Gravity towards the Moon and Sun, acting in the ſame manner as it does towards the Earth; and that each [n] Planet, like a Stone, whirl'd in a fling, was kept in its Orbit by the ſame principle, and for the ſame reaſon revolving always about us. From ſome things mention'd by [o] *Diogenes Laertius*, concerning *Plato*, which alſo are obſcurely hinted at in his [p] *Timæus*, I am apt to believe with [q] *Galileo*, that the divine Philoſopher ſuppos'd the Mundane Bodies, when they were firſt formed, were moved with a Rectilinear motion (by the means of Gravity,) but after that they

---

m *And this meeting together of Bodies here, and their coalition likewiſe with the Earth's Body, ſhew us the manner how it is probable that the parts, which are aſſembled at the Moon's Body, continue alſo there.* I

n *But the Moon is help'd, and preſerv'd from falling down, by her very Motion and that impetuoſity of her Revolution; as Stones and other weighty Bodies put in Slings and ſwung round, are kept from dropping out by the ſwiftneſs of their Motion, and their being mov'd circularly.* ——— *Wherefore the Moon does not move downwards, as her own weight wou'd naturally carry her, her tendency that way being ſtopt by the violence of her circular revolution.* Ibid.

o *Theſe at firſt were mov'd in a confus'd and irregular manner, but when they were duely adjuſted and rightly ſettled, then the World was eſtabliſh'd by God in juſt order and proportion.* Diog. Laert. in Plat.

p *He gave it a Motion altogether agreeable to its nature as a Body (that is a direct one,)* and a little further. *Therefore he afterwards made it continue its courſe in a Circle.* Plat. in Timeus.

q In his Coſmical Syſtem.

they had arrived to ſome determined places, they began to revolve by degrees in a Curve, the Rectilinear Motion being chang'd into a Curvilinear one. 'Tis from this Doctrine of Gravity, that all Bodies gravitate mutually to one another, 'tis by this that [r] *Lucretius*, taught by *Epicurus* and *Democritus*, labours to prove, that the Univerſe has no Center or loweſt Place, but that there is an infinity of Worlds like ours in the immenſe Space. His Argument runs thus; If the nature of things were bounded any where, then the outmoſt Bodies, ſince they have no other beyond them, towards which they may be made to tend by the force of Gravity, wou'd not ſtand in an Equilibrio, but make towards the inner and lower Bodies, being neceſſarily inclin'd that way by their Gravity, and therefore having made towards one another, during an infinite ſpace of time, would have long ago met, and lye in the middle of the whole, as in the loweſt place. 'Tis evident therefore from hence that *Lucretius*, and thoſe whom he followed, believ'd that all Bodies did Gravitate towards the Matter placed around them, and that every ſingle Body was carried by the more prevailing Gravity, towards that region where there was moſt Matter.

As

r *Suppoſe they all had Bounds, ſuppoſe an End;*
Then Bodies which by Nature muſt deſcend,
And from Eternity purſu'd the Race,
Had long e'er this time reach'd the Loweſt Place. Lucr. l. 1 v. 986.

As it is manifeſt that the Ancients were apprized of, and had diſcover'd the Gravity of all Bodies towards one another, ſo alſo they were not unacquainted with the Law and Proportion which the action of Gravity obſerv'd according to the different Maſſes and Diſtances. For that Gravity is proportional to the Quantity of Matter in the heavy Body, [f] *Lucretius* does ſufficiently declare, [t] as alſo that what we call light Bodies, don't aſcend of their own accord, but by the action of a force underneath them, impelling them upwards, juſt as a piece of Wood is in Water; [u] and further, that all Bodies, as well the heavy as the light, do deſcend *in vacuo*, with an equal celerity. It will be plain likewiſe, from what I ſhall preſently obſerve, that the famous Theorem about the proportion whereby Gravity decreaſes in receding from the Sun, was not unknown at leaſt to *Pythagoras*. This indeed ſeems to be that which he and his followers would ſignify to us by the Harmony of the Spheres: That is, they feign'd *Apollo* playing upon an Harp of ſeven Strings, by which Symbol, as it is abundantly evident from [x] *Pliny*, *Macrobius* and *Cenſorinus*, they meant the Sun in Conjunction with the ſeven Planets, for

---

f *Beſides; why have not Bodies equal Weight*
With thoſe, whoſe Figure is but juſt as great? Lucr. l. 1. v. 415.

t *And this I think a proper place to prove,*
That nothing of it ſelf can upwards move. Lucr. l. 2. v. 178.

u *Therefore thro' Void, unequal Weights muſt be,*
Like Swift in Motion, all of like degree. Lucr. l. 2. v. 228.

x Plin. lib. 2. c. 22. Macrob. lib. 1. c. 19. Cenſorin. c. 11.

for they made him the leader of that Septenary Chorus, and Moderator of Nature; and thought that by his Attractive force he acted upon the Planets (and called it Jupiter's Prison, becauſe it is by this Force that he retains and keeps them in their Orbits, from flying off in Right Lines) in the Harmonical ratio of their Diſtances. For the forces, whereby equal Tenſions act upon Strings of different lengths (being equal in other reſpects) are reciprocally as the Squares of the lengths of the Strings.

[y] For *Pythagoras* as he was paſſing by a Smith's Shop, took occaſion to obſerve, that the Sounds the Hammers made, were more acute or grave in proportion to the weights of the Hammers; afterwards ſtretching Sheeps Guts, and faſtning various Weights to them, he learn'd that here likewiſe the Sounds were proportional to the Weights. Having ſatisfy'd himſelf of this, he inveſtigated the Numbers, according to which Conſonant Sounds were generated. Whether the whole of this Story be true, or but a Fable, 'tis certain *Pythagoras* found out the true ratio between the ſound of Strings and the Weights faſten'd to them. The ſame Tenſion acts upon a String as ſhort again, four times more powerfully: For it produces an Octave, and an Octave is ſounded by a force that is four times greater; for if a String, put upon the ſtretch by a given Weight,

y Macrobius lib. 2. inſomn. Scip. cap. 1,

Weight, generates a given Tone, the ſame String ſtretch'd by a Weight four times greater, will ſound an Octave. Thus likewiſe the ſame Tenſion upon a ſubſeſquialteran Chord, acts in a double ſeſquiquartan ratio: For it generates a Fifth or Diapente, and a String that ſounds a given Note, with a given Weight ought to be ſtretch'd by a Weight that is a double ſeſquiquartan to ſound a Fifth. And univerſally, the Weights which generate all Tones in Strings, are reciprocally as the Squares of the lengths of Strings of equal Tenſion, producing the ſame ſound in any Muſical Inſtrument. *Pythagoras* afterwards applied the proportion he had thus found by experiments, to the Heavens, and from thence learn'd the Harmony of the Spheres. And, by comparing theſe Weights with the Weights of the Planets, and the intervals of the Tones, produced by the Weights, with the interval of the Spheres; and laſtly, the lengths of Strings with the Diſtances of the Planets from the Center of the Orbs; he underſtood, as it were by the Harmony of the Heavens, that the Gravity of the Planets towards the Sun (according to whoſe meaſures the Planets move) were reciprocally as the Squares of their Diſtances from the Sun.

We have thus far been ſhewing what was the Opinion of the Ancients concerning Gravity; and it is evident they were perſwaded that Gravity was not an affection of Terreſtrial Bodies only, but of the Celeſtial alſo,

that

that all Bodies gravitate towards one another; and that the Planets are retain'd in their Orbits by the force of Gravity, and laſtly, that the Gravity of the Planets towards the Sun are reciprocally as the Squares of their Diſtances from it. What the induſtry and ſkill of the Moderns have added to theſe inventions of the Ancients, the following Pages do declare at large.

THE

THE

PUBLISHER

TO THE

READER.

THE *Science of* ASTRONOMY, *which is as much esteem'd and admir'd for its great and manifold uses for the Service of Mankind, as it is delightful and entertaining to the more curious and contemplative, has in all ages been cultivated and improv'd, by Men the most eminent for their parts and learning; and is now brought, as it were, to the utmost degree of perfection, and that chiefly by the superior Genius and Industry of those of our own Nation. But since nothing considerable therein, has been as yet writ in our own Language, I thought I could not oblige my Country-Men more than in publishing an* English *Edition of the most valuable and finish'd piece of* ASTRONOMY *now extant. It is generally*

reckon'd

reckon'd to be a Book that contains not only all the Discoveries and Philosophical Sentiments of the great Kepler, *and the various Hypotheses of the most noted Astronomers before and since his Time; but is chiefly valued by the best Judges, for the large and instructive Comments deliver'd in it, on the Writings of the illustrious Sir* Isaac Newton, *as well as on the several Astronomical Dissertations of the sagacious Dr.* Halley, *which the Reader will find here every where interspers'd.*

And in order to render this work as compleat as possible, I shall, in a very little time, present you with another Volume, containing correct Astronomical Tables, for the ready computing of the Planets Places, Eclipses, &c. *all done by a Person of known ability, from the true Theory of Gravity, deliver'd in this Book: For it was by no means judged proper that I should annex to so intire a piece as this is, any imperfect Tables, drawn from a different Principle from what is here established, such it seems all those as yet published are.*

THE

THE
ELEMENTS
OF
Aſtronomy,
Phyſical and Geometrical.

The First Book.
Of the Syſtem of the World.

Section I.

Concerning the Order, Diſtances, and Periods of the Primary Planets revolving about the Sun, and the principal Phænomena thence ariſing.

Proposition I.

TO *give a general account of the Order, and Periods of the Primary Planets revolving about the Sun, and their Diſtances from it; as alſo what we are to think of the Comets and Fixt Stars.*

The Sun is to be look'd upon as immovable, and placed in the midſt of that immenſe Space,

in which the Planets perform their Revolutions. And there are ſix opake Spherical Bodies that revolve about it, as their Center, from Weſt to Eaſt, [*Fig.* 1.] from *A*, along *B*, *C*, and *D*, in the following Order: Mercury neareſt the Sun, compleating its Revolution in about three Months; next to Mercury, Venus in about ſeven Months and an half; then the Earth in a Year; Mars in about two Years; Jupiter in twelve; and, laſt of all, Saturn, which is outermoſt, in thirty. Their Diſtances from the Sun are nearly the ſame as they are repreſented in the Scheme: namely, ſuppoſing the diſtance of the Earth from the Sun to be divided into ten equal Parts, of theſe the diſtance of Mercury will be about four, of Venus ſeven, of Mars fifteen, of Jupiter fifty two, and that of Saturn ninety five.

'Tis to be obſerv'd, that all their Orbits are not in the ſame Plane, but variouſly inclin'd to one another; ſo that ſuppoſing the Plane of the Earth's Orbit to coincide with the Plane of this Scheme, one half of the Plane of any other Planet's Orbit will be above, and the other half below it; ſo that the Planes interſect in a Line that paſſes thro' the Sun.

Beſides the abovemention'd Planets moving round the Sun, there are likewiſe Comets; and theſe, if they are laſting in their Nature and Motions, move in very excentric Orbits, and conſequently ſhew themſelves only, when they are in thoſe Parts of their Orbits that are neareſt to us and the Sun: Some of them move, like the Planets, from Weſt to Eaſt, others from Eaſt to Weſt; ſome again from North to South, and laſtly, others from South to North. Their Orbits are various and different in Magnitude, Situation, and Inclination both to one another, and to

the

Plate 1 Book 1.

Fig: 1.
p: 2

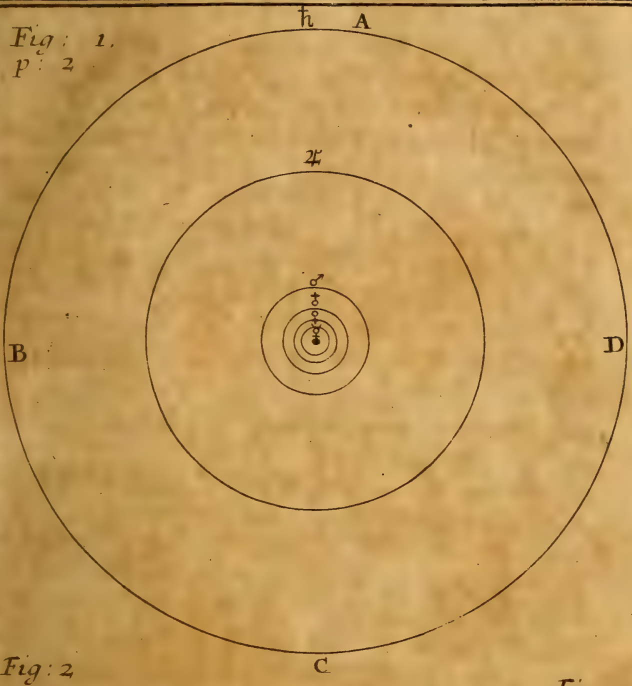


Fig: 2
p: 4

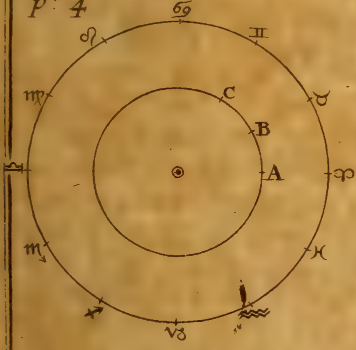
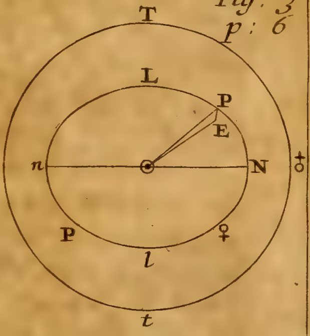


Fig: 3
p: 6



the Orbits of the Planets. Their Periods are not as yet known from Obſervation, nor indeed is it fully certain, that they move in Lines that return into themſelves. But all theſe things, perhaps, will be determined by proper Obſervations made in future Ages.

The reſt of the Mundane Space is to be conceived as divided into Spaces juſt like that we have been deſcribing, each having one of thoſe Stars in its Center, which are called *Fixt Stars*, performing the office of a Sun to it, and having, it may be, Planets and Comets of its own revolving about it.

It is enough at firſt to have a general conception of theſe things as they are here deſcribed: For tho' the Periods of the Planets, and their Diſtances from the Sun, as here laid down, are not exactly true, yet they are neareſt the Truth in round Numbers. Again, tho' the Paths they deſcribe are not perfect Circles, concentric to the Sun, and the Motion of the ſame Planet not perfectly equable; yet the difference is ſo ſmall, that they need not, at preſent, be taken otherwiſe; till by Obſervations and Methods of uſing them, hereafter to be ſhewn in their proper places, all theſe things ſhall be preciſely and exactly ſettled, and theſe niceties examined; or, at leaſt, till the ſame are purpoſely handled, in order to inquire into their Phyſical Cauſes.

PROPOSITION II.

TO deſcribe the Phænomena that ariſe from the ſituation of the Sun and motion of the Earth, as related above.

Firſt, If the Obſerver be ſuppos'd to be plac'd in the Sun, 'tis evident the Earth will ſeem to him to move from Weſt to Eaſt perpetually, as

really it does. Then ſince, beſides the Earth, our Obſerver ſees the Fixt Stars plac'd round him, as it were (the nature of the Eye requiring it) in a concave Sphere, that has the Eye for its Center; 'tis likewiſe evident, that he will obſerve the Earth moving, as it were, among the Fixt Stars, and approaching nearer and nearer the more Eaſtern ones, till, in a Year's ſpace, having compleated its Revolution, it returns to the ſame place among them again. And becauſe the Earth always goes the ſame track over again, the Obſerver will take eſpecial notice of the Stars the Earth paſſes over; alſo the Plane of the Earth's Orbit, and the Circle in the Sphere of the Fixt Stars, called the *Ecliptic*, made by that Plane; and this will be a great Circle, becauſe it paſſes thro' the Sun, or Eye, which is the Center of that Concave Sphere that terminates the Sight. But if the Obſerver, for the advantage of making Obſervations, imagines this Ecliptic divided into twelve equal Parts, or *Signs*, calling them by the name of any neighbouring Conſtellation, or Figure thoſe Stars ſeem to make; in this caſe, I ſay, [*Fig.* 2.] the Earth will ſeem to move from ♈ to ♉, and from thence to ♊, and ſo on; from Weſt to Eaſt, thro' all the Signs, till it return to ♈ again.

Secondly, If you imagine the Obſerver to be removed from the Sun to our Earth, if the Earth be at *A*, where it is ſeen from the Sun among the Fixt Stars, at ♈; the Sun will appear, when ſeen from the Earth, in the oppoſite Sign ♎, among the Fixt Stars; the Earth being then the Center of the Sphere of the Fixt Stars: For the place of the Eye is the Centre of a Sphere, on whoſe ſurface the Stars are conceiv'd

ceiv'd to be plac'd. If the Earth be moved from *A* thro' *B*, to *C*, in its Orbit, or if look'd upon from the Sun, from ♈ thro' ♉, to ♊, *in consequentia*, or according to the order of the Signs; the Sun, to an Obſerver on our Earth, that thinks the place he ſtands upon immovable, will appear to move among the Fixt Stars, according to the order of the Signs alſo, from ♎, thro' ♏, to ♐, *&c.* in the ſame Plane, during the ſame time, and towards the ſame Region of the Heavens, as the Earth ſeen from the Sun does, but in the oppoſite Points of the Ecliptic.

SCHOLIUM.

The like Phænomena happen in reſpect of the Sun and any other Planet; nay indeed, the very ſame, excepting that the time of that Planets revolution about the Sun, or the Sun's apparent revolution about the Planet, when view'd from that Planet, is various, according to the different Period of each, mention'd in the foregoing Propoſition; and that the Plane of the Orbit of that Planet produced, will cut other Stars than thoſe which the Plane of the Earth's Orbit does, when produced; and conſequently, that the Path of the Sun among the Fixt Stars, ſeen from any other Planet, is different from its Path, when ſeen from the Earth, that is, from the Ecliptic.

PROPOSITION III.

TO deſcribe the Phænomena of the Planets ſeen from the Sun, ariſing from their Motion in Orbits, whoſe Planes are inclin'd to the Planes of the Ecliptic.

Since the Orbits of the Earth and Planets are ſo ſituated, as that their Planes are inclin'd to each other, and interſect each other, (as was

ſhown in general in *Prop.* 1.) in right Lines paſſing thro' the Sun; the Inclination of the Plane of the Orbit of each, to the Plane of the Ecliptic, or Earth's Orbit, is to be taken into conſideration, in explaining the Phænomena of the Planets viewed from the Sun. For the Plane of the Ecliptic is taken by Aſtronomers as the Standard to which the Planes of the other Orbits are judged to incline; and that with very good reaſon, ſince it is that in which the Earth (the habitation of the Aſtronomer) moves round the Sun, or in which the Sun ſeems to move round the Earth: And an Obſerver plac'd in any other Planet, would make the Plane of that Planet's Orbit, the Standard of all the reſt, and conſider them as inclin'd to it.

The right Line which paſſes thro' the Sun, and is the common Section of the Plane of the Orbit, with the Plane of the Ecliptic, is call'd the *Line of the Nodes* of that Planet, and the Points themſelves, wherein the Orbit of the Planet cuts the Ecliptic, are call'd the *Nodes*. Thus, [*Fig.* 3.] let ☉*T* ♄ *t* be the Plane of the Orbit of the Earth produced indefinitely, *NPn* the Orbit of any Planet, interſecting the Plane of the former Orbit or Ecliptic in *N* and *n*, which are the Nodes of that Planet; ſo as that one part *NPn* of that Orbit be ſuppos'd above the Plane of this Scheme, and the other, *npN*, below it, (which makes it look like an Ellipſe:) The right Line *Nn* joining the Nodes, being the common Section of the Plane of the Orbit of the Planet with the Plane of the Ecliptic, is the Line of the Nodes.

'Tis evident then, that if a Planet be ſeen from the Sun, when it is in one of the Nodes, as *N*, it will appear to be in the Plane of the Ecliptic. But when gone forwards as far as P, it will

ſeem

ſeem to deviate from the Ecliptic: and (by *Def.* 5. *Elem.* XI.) the Inclination of the right Line ☉*P*, to the Ecliptic, and conſequently of the Planet at *P* ſeen from the Sun, call'd the Planets *Helio-centric Latitude*, is meaſur'd by the Angle *P*☉*E*, where *PE* is ſuppos'd to be a perpendicular let fal from *P* to the Plane of the Ecliptic. This Heliocentric Latitude is continually upon the increaſe, till the Planet is got as far as *L*, its *Limit*; where it is equal to the Inclination of the Plane of the Planet's Orbit, to the Plane of the Ecliptic. But during the Planet's paſſage from thence to the other Node *n*, it is decreaſing, till at laſt it vaniſhes at the Node. Having paſs'd the Node *n*, it begins again, changing its name, becauſe 'tis towards the contrary parts of the Ecliptic, and grows bigger and bigger, till the Planet has arriv'd at the other Limit *l*, from whence again it grows leſs and leſs, till it vaniſhes at the other Node *N*.

The *Orbs* of the Planets (that is, the Planes of their Orbits) are inclin'd to the Ecliptic in the following manner: The Orb of Saturn makes an Angle of 2 $\frac{1}{2}$ Degrees; Jupiter, 1 $\frac{1}{3}$ Degree; Mars, a little leſs than 2 Degrees; Venus, ſomething above 3 $\frac{1}{3}$ Deg. Mercury almoſt 7 Degrees. An account of the Poſition of the Line of the Nodes of each Planet ſhall be given in a more proper place. We have, in theſe two laſt Propoſitions, conſider'd the Planets as they appear to one plac'd in the Sun: becauſe it was neceſſary to the underſtanding their Motion when view'd from the Earth.

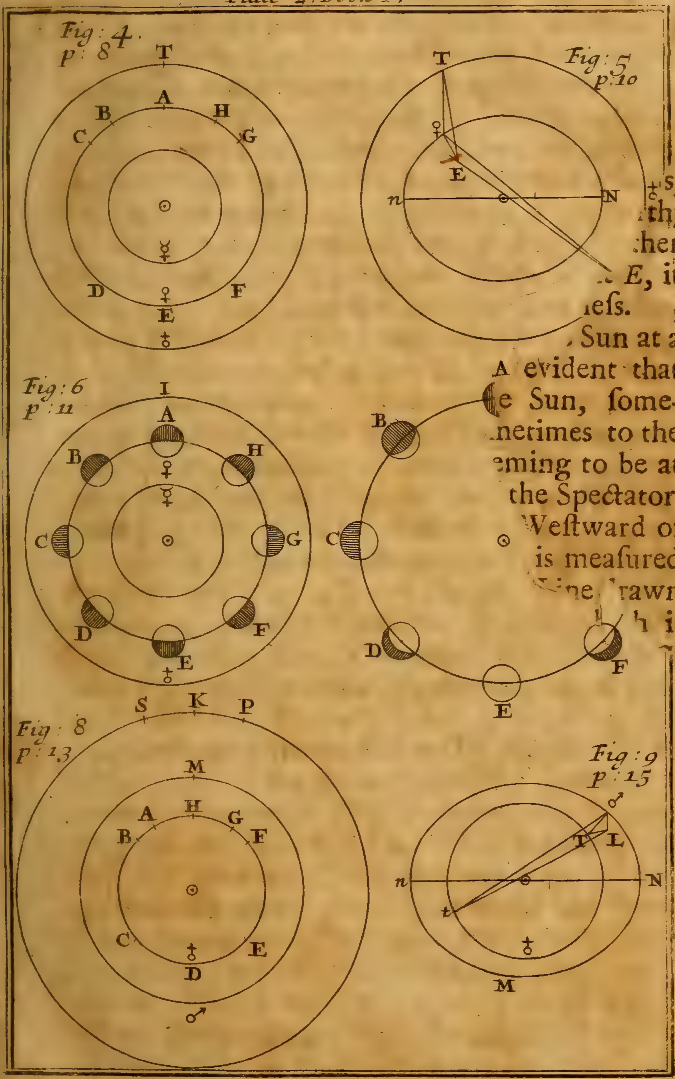
PROPOSITION IV.

TO *describe the Phænomena arising from the Motion of the Earth, and inferior Planets, Venus and Mercury, when view'd from the Earth.*

Since Venus and Mercury revolve about the Sun, in lesser Orbits than the Earth, as you see in the Scheme, [*Fig.* 4.] where *T* represents the Earth, carried in its Orbit *T* ♄ from West to East; and *ACEG*, the Orbit describ'd by Venus in a less space of Time the same way; 'tis evident, that when Venus is in *D E F*, that part of its Orbit that is farthest off the Earth, it will appear to us on the Earth to move *in consequentia*, or according to the order of the Signs, and is then said to be *Direct*. When it is in *G*, moving from thence to *H*, it will appear to move as swift as the Sun, because then its Motion tends directly towards the Earth, and it does not seem to move at all, but as its Orbit is carried along by the Sun, whose Motion is towards the East: Venus moves now therefore slower than before, but is still Direct. When it is got beyond *H* in its Motion, thro' *A* to *B*, it passes between the Earth and Sun, because it is nearer to us than the Sun, and moves swifter than the Earth, (the cause of which we shall hereafter assign,) consequently will seem to us to change its Place among the Fixt Stars, and move *in antecedentia*, or contrary to the order of the Signs, and then it is said to be *Retrograde*, tho' really Direct still, if view'd from the Sun. Between Direct and Retrograde, for instance, about *H*, it will appear *Stationary*, the right Lines that join the Earth and Venus, continuing for some sensible time parallel. Thus likewise, after its Retrogradation, before it becomes Direct again, it will appear Stati-

onary

Plate 2. Book 1.



s,
th,
her
E, it
efs.
Sun at a
evident that
e Sun, fome-
netimes to the
eming to be at
the Spectator.
Weftward of
is meafured

in
de
its
appear
or acco
then ſaid to
ving from th
as ſwift as t
tends directly
not ſeem to
ried along
the Eaſt:
thar bef
be

onary a second time, and is Stationary in order to its being Direct, as it was before in order to its being Retrograde at *H*. In all this affair, regard is to be had to the Motion of the Earth, for Venus is Direct, Stationary or Retrograde, according as it is posited in such parts of its Orbit as have the same relation to the Earth in its Motion as the Points aforementioned have. From what has been said, 'tis evident that Venus, when Retrograde, as at *A*, is nearer to the Earth, and consequently appears bigger than at other times; or the contrary when Direct, as at *E*, it is farther off, and consequently appears less.

And because Venus moves round the Sun at a less distance than the Earth; 'tis evident that it will seem always to attend the Sun, sometimes to go to the West of it, sometimes to the East: all the Heavenly Bodies seeming to be at an equal distance from the Eye of the Spectator. This digression to the Eastward or Westward of the Sun is call'd the *Elongation*, and is measured by the Angle contain'd under a right Line drawn from the Eye to the Sun and Venus, which is never greater than the Angle ⊙ *TC* or ⊙ *TG*, if the Lines *TC* or *TG*, when drawn, are Tangents to the Orbit *A D F*. Consequently the Elongation of Venus will never be above half a Quadrant from the Sun, as is evident from the Semidiameters laid down in *Prop.* i. And when it has arrived to its farthest Elongation it will return to the Sun, and pass as far beyond on the other side as if its Motion were Oscillatory.

Mercury has all the same Phænomena; but its Directions, Stations, and Retrogradations happen oftner, because it finishes its Course in a shorter time, and consequently overtakes the Earth,

Earth oftner than Venus. And ſince Mercury's Orbit is leſs than that of Venus, its greateſt Elongations muſt be alſo leſs, and it muſt be a nearer and more conſtant attendant of the Sun, being, by *Prop.* i. never a whole Sign diſtant from it, and conſequently ſeldom to be ſeen by us.

PROPOSITION V.

TO *deſcribe the Phænomena of the Latitude of the inferior Planets ſeen from the Earth.*

Let T ♄ t be the Earth's Orbit, whoſe Plane is the ſame with that of the Ecliptic; [*Fig.* 5.] and let N ♀ n be the Orbit of an inferior Planet, for inſtance Venus, whoſe Plane is inclin'd to that of the Ecliptic, and therefore will look like an Ellipſe, whoſe greater Axe is the interſection of the Planes, or Line of the Nodes $N n$. And while Venus is in ♀, let the Earth be in T; in which ſuppoſition Venus will be neareſt the Earth, and Retrograde, by *Prop.* iv.

'Tis evident from *Def.* 5. *Elem.* xi. that the Inclination of the right Line ♀T to the Plane of the Ecliptic, or the Latitude of Venus in ♀, ſeen from the Earth (which is hereupon called the *Geocentric Latitude*) is meaſured by the Angle ♀$T E$, the right Line ♀E, being made perpendicular to the Plane of the Ecliptic. If Venus be ſuppos'd to continue in ♀, and the Earth to be at t, in which ſuppoſition Venus is Direct, and furtheſt off the Earth; the Geocentric Latitude of Venus will be the Angle ♀$t E$, leſs than ♀TE, almoſt in the ratio of T♀ to t♀, tho' the Heliocentric Latitude of Venus be in both caſes the ſame.

And all that has been now ſaid is true of Mercury as well as of Venus; conſequently all other

other circumſtances being alike, the Latitude of the inferior Planets is greater, when they are Retrograde and neareſt the Earth; leſs, when Direct and fartheſt off.

Moreover if any inferior Planet be moſt Retrograde and neareſt the Earth, and at the ſame time in or near a Node, it will be found directly between the Obſerver and the Sun: If it be at a conſiderable diſtance from a Node, it will paſs the Sun to the Northward or Southward of him. In like manner, if it be moſt Direct and fartheſt off the Earth, and be alſo near, or in a Node, it will be cover'd by the Sun; if it be otherwiſe ſituated, it will paſs on one ſide of the Sun.

PROPOSITION VI.

TO *explain the Phænomena of the Inferior Planets, ariſing from their being opake Spherical Bodies enlighten'd by the Sun.*

Since all the Planets, as well as the Earth, are opake Spherical Bodies, reflecting the Rays of Light that fall upon them every way, 'tis evident that that half of each Planet which is expoſed or turn'd to the Sun, is enlighten'd by it, while the other half, which is turn'd from the Sun, is in darkneſs. And ſince that half only which is towards the Earth is ſeen by the Obſerver, if we conſider which Face of Venus is enlighten'd in this or that ſituation in reſpect of the Sun, together with the Face that is viſible at the Earth, [*Fig.* 6.] in the part *T* of its Orbit *T* ♄, it will be manifeſt, that when Venus is in *A*, that is, moſt Retrograde and neareſt to us, it is not viſible at all, having its dark Face turn'd to us: And if Venus were at the ſame time in the Plane of the Ecliptic, that is, in one of its Nodes, it would appear like a Spot in the Sun,

be-

being ſituated directly between the Earth and Sun, by the foregoing Propoſition.

When Venus is got into the ſituation *B*, that is, begins to be Retrograde, if view'd from the Earth, then ſome part of the enlighten'd half is turn'd towards the Earth, tho' the greater remaining part belongs to the obſcure half. And becauſe Venus is of a ſpherical Figure, and appears like a Plane, its enlighten'd part will appear Horned, and thoſe Horns turned towards the Weſt, or from the Sun.

When Venus is at C, half the enlighten'd part will be ſeen by us, and then it is ſaid to be half Full; at *D* it is ſaid to be Gibbous, above half the enlighten'd part being viſible; and at *E*, where it is fartheſt off the Earth, and moſt Direct in its Motion, it appears Full, all the enlighten'd Hemiſphere being turned towards us.

Venus will have the ſame variety of Phaſes in its paſſage thro' *F*, *G*, and *H*, that is, it will be Gibbous at *F*, half Full at G, and Horned at *H*, with its Horns turned from the Sun, that is, in the preſent caſe, looking towards the Eaſt, exactly contrary to what they were when it was at *B*.

For the better underſtanding of this matter, we have delineated the ſeveral Phaſes of Venus, as they appear to us, in the Scheme [*Fig.* 7.]; where the ſame Letters are uſed that were in the Diagram [*Fig.* 6.] repreſenting its Phaſis when it was in that part of its Orbit, marked by the ſame Letter. Thus, in this Scheme, [*Fig.* 7.] at the Point *A*, it is drawn all obſcure, becauſe that is its condition in reſpect of the Earth at *T*, when Venus is at *A*, in the former Diagram. At *B* it is repreſented Horned,

be-

becauſe it appears ſo, when it is at *B*, in the former Scheme, and look'd upon from the Earth at *T*; and ſo on in the other caſes of *C*, *D*, *&c.*

The like Phænomena will appear in Mercury, regard being had to its Orbit and Revolution.

SCHOLIUM.

As the Phænomena deſcribed in the three laſt Propoſitions, manifeſtly follow upon the Situation and Motion of the Earth, Venus and Mercury, laid down in *Prop.* I. So it follows converſely, that the Obſervation of theſe Phænomena in them, eſtabliſhes and confirms that Order and Situation, namely, that Venus and Mercury revolve about the Sun, in Orbits, that are included within the Earth's Orbit.

PROPOSITION VII.

TO deſcribe the Phænomena ariſing from the Motion of the Earth, and of the ſuperior Planets, Mars, Jupiter, and Saturn. [*Fig.* 8.]

Let *M* ♂ be the Orbit of any one of the ſuperior Planets; for inſtance Mars, *A C* ♄ *G* the Orbit of the Earth, nearer to the Sun. 'Tis evident, firſt, that this Planet will not always attend the Sun, but ſometimes be diametrically oppoſite to it: For the Earth finiſhing its Revolution ſooner than any of the ſuperior Planets, will ſometimes be exactly between the Sun and that Planet; thus, when Mars is in *M*, the Earth may be at *H*; and, to ſpeak univerſally, the Angle at the Earth, made by lines drawn thence to the Sun and that Planet, may be equal to any given one.

Let us ſuppoſe Mars to be in *M*, and the Earth at the ſame time in *A*; Mars in this caſe will appear Stationary, in order to its being Direct, becauſe the right Lines that join the Earth and Planet

Planet at that moment, will continue parallel for some sensible time; during the whole of which, notwithstanding, Mars will seem to go forwards, as usual, if viewed from the Sun.

While the Earth moves along thro' *B*, *C*, *D*, *E*, *F* and *G*, Mars likewise will seem to go forwards among the Fixed Stars, upon a double account; first, because it really does move about the Sun, *in consequentia*; and then again, because the Earth in the opposite Semicircle is carried the same way, and about the same Center: And, consequently, Mars in this case being most remote from the Earth, and in conjunction with the Sun, view'd from the Earth, will seem to move faster than ordinary, *in consequentia*, and become Direct. But when the Earth is arriv'd to the Point *G*, in respect of Mars at *M* (which some time or other will happen, tho' Mars be carried in the mean while about the Sun, namely, when the Earth has almost overtaken Mars) Mars will again become Stationary, in order to its being Retrograde, as it will be soon after. For when the Earth in its Motion from *G*, thro' *H* to *A*, has pass'd Mars, and that Planet is seen in opposition to the Sun, and biggest, because nearest the Earth, which is lower and swifter, will make Mars appear to move *in antecedentia* from *S* thro' *K* to *P*; whereas in the mean while view'd from the Sun, it seem'd to move as always before, *in consequentia*.

The like Phænomena will happen to Jupiter and Saturn, excepting that Saturn's Retrogradations are more frequent than Jupiter's, and Jupiter's than Mars's: because the Earth oftner overtakes Saturn than Jupiter, and Jupiter oftner than Mars, and passes between them and the Sun.

PRO-

PROPOSITION VIII.

TO describe the Phænomena of the Latitude of the Superior Planets ſeen from the Earth.

Let the Earth's Orbit be *T t* ♄ ; [*Fig.* 9.] that of any ſuperior Planet as Mars ♂ *M*, whoſe Plane is inclined to that of the Ecliptic, and cuts it in the Line of the Nodes *N* ☉ *n* : Let the ſituation of Mars and the Earth, to the Sun, be ſuch, as that Mars being in ♂, the Earth may be in *T*, almoſt between Mars and the Sun; in which caſe Mars is both neareſt the Earth, and conſequently biggeſt and moſt Retrograde, as was ſhown in the foregoing *Propoſ.* Its Geocentric Latitude will be meaſured by the Angle ♂ *TE*, ♂ *E* being ſuppos'd perpendicular to the Plane of the Ecliptic. But if Mars continuing in the ſame ſituation in its own Orbit, and conſequently having the ſame Heliocentric Latitude, the Earth being ſuppoſed be in *t*, the Sun being between it and Mars; in which Caſe, by the foregoing, Mars will be fartheſt off, and conſequently leaſt, and moſt Direct in its Motion; Its Geocentric Latitude meaſured by the Angle ♂ *t E*, is always leſs than the Angle ♂ *TE*, in the ratio of the Diſtances of the Earth from Mars, that is, of the right Lines *T* ♂ , *t* ♂ . Thus in whatever ſituation Mars and the Earth be placed, in reſpect of the Sun, its Geocentric Latitude will be changed, ſo as, *cæteris paribus*, it will be leſs as Mars is nearer to a Conjunction with the Sun, and its ſwifteſt direct Motion; and greater, as it is nearer its Retrogradation and Oppoſition to the Sun.

'Tis evident from what has been ſaid, that none of the Superior Planets can ever be ſeen from the Earth to cover the Sun; tho' any of them may be covered by the Sun, when it is Direct and pretty near a Node.

PRO-

PROPOSITION IX.

TO *describe the Phænomena of the Superior Planets, arising from their being Opake Bodies, and enlightned by the Sun.*

Saturn and Jupiter being Opake Bodies, and illuminated by the Sun, that half of each Planet which is turn'd towards the Sun, (that is, the illuminated half) is likewise turn'd towards the Earth, which is never far off from the Sun, or Center of Saturn and Jupiter's Orbit: For, by *Prop.* i. Jupiter's distance from the Sun is above five times, and Saturn's almost ten times greater than the Earth's distance from it.

In Mars indeed it is something different: For the distance of Mars from the Sun being but half as much more as the Earth's Distance from it; its inlighten'd Hemisphere, towards the Sun, is not always, as to Sense, turn'd towards the Earth. [*Fig.* 10.] Let *T* be the Earth's place in its Orbit *T*♄, 'tis evident that Mars being at *A* or *B*, in Conjunction or Opposition to the Sun, has the same Face towards the Earth, as it has towards the Sun, that is, its enlightened one, and consequently appears Full; but in the situation of the Points *D* or *C* (when the Angle ☉*CT*, or ☉*DT* is greatest, or when ☉*TC*, or ☉*TD* is almost a right Angle) neither is the whole enlighten'd Face seen, nor is that Face that is seen entirely illuminated, but it appears Gibbous, the light being deficient a little towards those parts that are turn'd from the Sun.

SCHOLIUM.

As the Phænomena described in these three last Propositions follow from the Order and Motion of the Earth, Mars, Jupiter, and Saturn, laid

laid down in *Prop.* 1. So by the converſe of them, the Obſervation of theſe Phænomena ſettles and eſtabliſhes that Order.

PROPOSITION X.

TO deſcribe the Phænomena of the Motion of Comets, ſeen from the Earth.

Becauſe different Comets have different Orbits, which can't be determin'd till ſome Obſervations have been made about them: 'Tis evident that during their deſcent, near their Perihelium, or while they are in that part of their Orbit, which is within the Region of the Planets, they have the ſame Phænomena as the Planet next them has, regard being had to the Velocity and Inclination of the Orbit of the Comet.

Conſequently a Comet's apparent Path among the Fixt Stars, will be nearly a Great Circle; excepting that its deviation from it, on this or that ſide of it, will be like that of a Planet, according as the Motion of the Earth is, which carries along with it the Spectator. For were the Comet view'd from the Sun, it would deſcribe a Great Circle among the Fixt Stars accurately. But its deviation from one, on the account of the Motion of the Earth round the Sun, will not be very ſenſible in a ſmall ſpace of time.

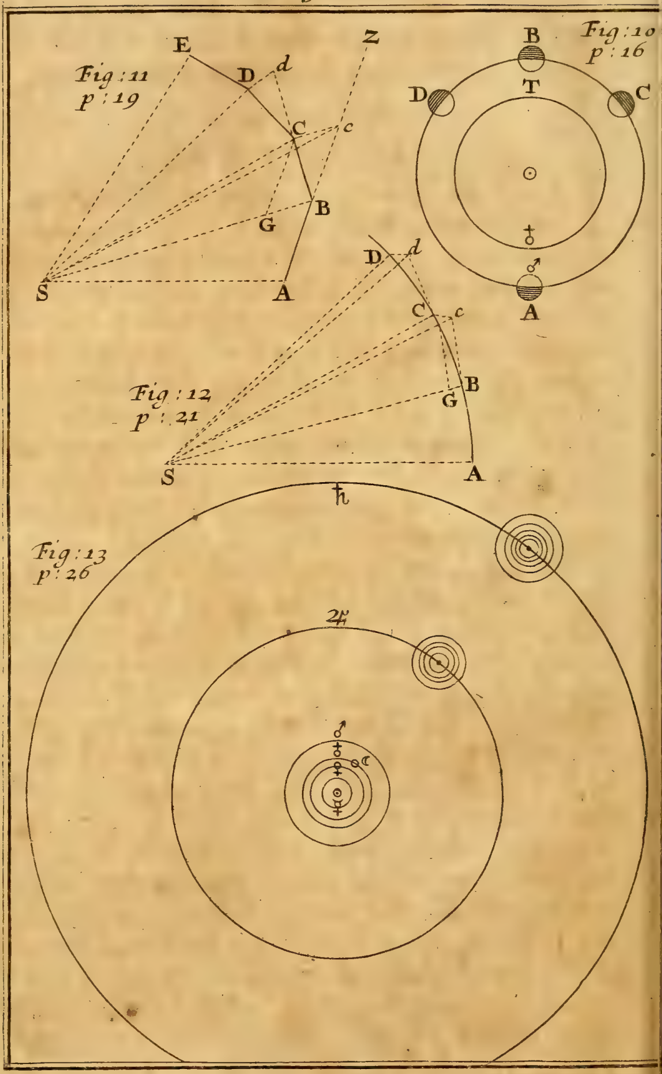
If the Earth happen to be between the Sun and a Comet, that moves *in conſequentia*, the Comet will be Retrograde; for the Earth moves ſo much the ſwifter, as the right lines that connect the Earth and Comet converge towards the Region beyond the Comet: But if the Earth moves ſlower, then the Comet's Motion, by taking away the Earth's Motion, likewiſe becomes ſlower: But if the Sun be between the

Earth and the Comet, the Comet's Motion will appear ſwifter than really it is. On the contrary, the Comets that move *in antecedentia* are ſwifter than they ſhould be, when the Earth is between them and the Sun; and ſlower, or perhaps moving according to the natural Order of the Signs in appearance, when the Earth is ſituated on the contrary ſide. All this happens from the Motion of the Earth, and its various Poſition, as it does in the Planets; that are, according as the Motion of the Earth falls in with, or is contrary to their Motion, ſometimes Retrograde, ſometimes Slower, ſometimes Swifter than they ſhould be, as has been ſhewn in *Prop.* iv and vii. Thoſe Phænomena will be moſt ſenſible, and eaſily taken notice of, a little before the diſappearance of the Comet; its apparent Motion being at that time ſlower, and the Earth's Motion proportionably greater, with which the Spectator is carried.

The Latitude of the Comet likewiſe, *cæteris paribus*, is varied by the different Situation of the Earth, it being greater in Oppoſition; but in Conjunction greater or leſs, according as the Comet is between the Earth and Sun, or the Sun between the Comet and the Earth, as was demonſtrated of the Planets in *Prop.* v and viii.

SEC-

Plate 3. Book 1.



SECTION II.
Of the Direction of the Forces, which retain the Primary Planets in their Orbits.

PROPOSITION XI.

IF a Body be moved according to the Direction of any given right line as A Z, [Fig. 11.] *and at the ſame time be urged by a Centripetal Force tending towards a certain given immoveable Point as* S, *ſituated without the aforeſaid right line; the Line deſcrib'd by the Body will be a Curve, and Concave towards* S, *lying all of it in the ſame immoveable Plane paſſing thro' the right line* A Z *and the Point* S: *And the Areas contain'd under any portions of the Curve and Right lines drawn to the Center* S, *are to one another, as the Times wherein thoſe portions of the Curve were deſcribed.*

Let the Time be imagin'd to be divided into equal parts; in the firſt of which let the Body deſcribe the line *A B*, a part of *A Z*, by its *Vis inſita* alone, by which it tends to move according to the Direction of the Line *A Z*; in the ſecond part of Time it will deſcribe the Line *B c*, equal to the former *A B*, if nothing hinder it: For by the firſt and chief Law of Motion, all Bodies once put into Motion, and not meeting with any Impediment, will continue to move uniformly on, in the ſame right line they were firſt moved in. But when the Body is arrived at the Point *B*, let us ſuppoſe a Centripetal Force tending towards the Point *S*, to act upon it by a ſingle impulſe, ſo as that, had it been impell'd only by that Impulſe, it would in the ſecond part of Time deſcribe the line *B G*. 'Tis evident

that drawing the Line *c C* parallel to *B G*, thro' the Point *c*, and *G C* to *B c*, thro' *G*, the Body acted upon by both Impulſes at once, will arrive, in the ſecond part of time, at the Point *C*, deſcribing the right line *B C*. For as it is well known in Mechanics, a Body deſcribes the Diagonal of a Parallelogram, both the Forces combin'd, in the ſame time as it would do the Sides, with the Forces ſeparated. 'Tis certain likewiſe that the right line *B C* is in the Plane of the Parallelogram *B G C c*, both of whoſe Sides *B G* and *B c* is in the Plane of the Triangle *A S B*, that paſſes thro' the Center *S*, of the Centripetal Forces, and the immoveable right line *A Z*. Beſides, the Triangles *S C B*, *S c B* are equal, becauſe they are upon the ſame Baſe *B S*, and between the Parallels *S B*, *C c*; but *S c B*, *S B A* are equal, becauſe their Baſes are equal, and height the ſame: Conſequently *S B A*, *S C B* alſo are equal. By the ſame method of reaſoning, if in the third particle of Time, a Body deſcribes any other right line as *C D*; it may be proved that the Triangle *S C D* is equal to the Triangle *S B C*, and that the right line *C D* is in the ſame Plane with the right lines *S B*, *B C*; that is, in the ſame with that which is drawn through the right line *A B*, and the Point *S*. And ſo we may go on, as long as the Motion is continued, and in equal times the Area deſcribed by Radii drawn to the immoveable Center of the Forces, will be equally increaſed; and by compounding Ratios, any ſums of Areas, are to one another, as the Times wherein they are deſcrib'd. The Line deſcrib'd by the Body will be in the immoveable Plane, and it will be concave towards *S*, becauſe every part of it that is a right line, as *B C*, declines from the foregoing

A B, towards the Center. If you ſuppoſe the number of the Triangles, *S A B*, *S B C*, *S C D* to be augmented, and their breadth to be diminiſhed *in infinitum*; their Baſes, *A B*, *B C*, *C D*, will form a Curve Line, concave towards the ſame parts, and lying in the ſame Plane; and the Centripetal Force, which before acted, as it were by ſtarts, and at equal intervals of Time, whereby the Body is continually drawn off from the Tangent of that Curve (which the baſes of the Triangles meaſure) now acts conſtantly: And any Areas thus deſcrib'd, *S A B C S*, *S A B C D E S*, will be, as before, proportional to the Times of deſcription. *Q. E. D.*

PROPOSITION XII.

A Body moved in a Curve Line A B C D, [Fig: 12.] *deſcrib'd upon a Plane, and concave towards the ſame parts, and by a Radius drawn to* S, *an immoveable Point ſituated in the ſame Plane towards the Concavity of the Curve, deſcribing Areas proportional to the Times, is urged by a Centripetal Force tending to the Point* S.

Let ſuch a Curve be imagined to be divided into the parts *A B*, *B C*, *C D*, *&c.* differing as little as can be from right lines, ſo as to be deſcribed in equal Particles of Time: Let the Centripetal Force likewiſe be conceiv'd to act only in the points *B*, *C*, *D*, *&c.* by ſtarts, as in the foregoing *Prop.*. Let *A B* be produc'd to *c*, ſo as that *B c* be equal to *A B*; in like manner *B C* to *d*, till *C d* be equal to *B C*; and ſo on. The Triangle *S A B*, will be equal to *S B C*, becauſe, by Hypotheſis, the Areas deſcribed are proportional to the Times; and *S A B* is equal to *S B c*, becauſe *A B* is equal to *B c*: Wherefore *S B C* is equal to *S B c*, and conſequently (by 39

Elem. 1.) *C c* is parallel to *S B*. Moreover, the Body moved along *A B*, in the firſt particle of Time, by its *Vis inſita* alone would deſcribe *B c*, in the ſecond; but it really does deſcribe *B C*, in that ſecond Particle of Time: Wherefore the Force acting in the Point *B*, in conjunction with the *Vis inſita* to carry the Body along *B C*, is directed according to a Right line parallel to *C c*; that is, according to the Right line *B S*. After the ſame manner the Force acting in the point *C*, and in conjunction with the *Vis inſita*, (by which alone the Body would deſcribe the Line *C d* in the third Particle of Time) carrying it in the ſame time along *C D*, is directed according to a Right line parallel to *d D*, that is, according to *C S*: But the Right lines *B S*, *C S*, *&c.* tend towards the Point *S*. Therefore the Centripetal Force, that draws the Body off from the Tangent of the Curve, acts along right lines tending to the immoveable Point *S*. *Q. E. D.*

PROPOSITION XIII.

THE *Forces whereby the Primary Planets, Mercury, Venus, Mars, Jupiter, and Saturn, are drawn off from their Rectilineal Motions, and kept in their Orbits, do not tend towards the Earth, but towards the Sun.*

For every Body that is moved in a Curve Line, is turn'd off from its Rectilineal courſe, that it naturally affects, by ſome Force or other. And it is evident that the Planets move in Curve Lines, becauſe their Orbits return into themſelves. Now this Force exerciſed upon them, does not tend towards the Earth, becauſe the Orbits of two of them, namely, Mercury and Venus, do not ſurround the Earth, as is evident

dent from *Prop.* vi; and consequently they are not concave in all their parts towards the Earth: Wherefore (by *Prop.* xi.) the Forces by which they are kept in their Orbits, do not tend to it. Besides, the Primary Planets, *viz.* Mercury, Venus, Mars, Jupiter and Saturn, in respect of the Earth, do sometimes go forwards from the West towards the East, sometimes backwards from the East towards the West, and sometimes stand still: But the time in which these Motions are perform'd, always flows uniformly; and therefore the Areas describ'd by a Radius drawn to the Earth from any Planet, are not proportional to the Times of description. Consequently, by the foregoing Proposition, each Planet is urged and retained in its own Orbit, by a Force that does not tend towards the Earth.

But (from the *Scholium* to *Prop.* vi.) 'tis certain, that the Orbits of Mercury and Venus surround the Sun; which is evident likewise concerning the Orbits of Mars, Jupiter and Saturn, from the *Scholium* to *Prop.*ix. And all the Planets, in respect of the Sun, are always Direct in their Motion, as is evident from *Prop.* 1; and that almost uniformly, just as the Time flows. 'Tis true indeed, the Planets move a little swifter in some Points of their Orbits, even in respect of the Sun, than in others, as shall be shewn hereafter; but the difference is so small, as it may here very well be neglected. But even at that time the Planets are nearer to the Sun (for their Orbits are not Circles concentric to the Sun) and their Motion is so temper'd, as that the Areas described by Radii drawn to the Sun, are equally augmented, as will appear in the sequel of this Subject. And consequently the Forces, whereby the Planets are drawn off from a Recti-

lineal Motion, and retained in their Orbits, tend towards the Sun.

PROPOSITION XIV.

THE Forces, by which the Comets are kept in their Trajectories (if they be Curvilineal) do not tend towards the Earth, but towards the Sun.

If the Trajectories of the Comets were Right lines, they would not be urged by any Forces at all tending to a point ſituated without thoſe Right lines; for if they were, they would be drawn off from a Rectilineal Motion, and made to deſcribe Curvilineal Orbits, by *Prop.* xi. And if the Trajectories of Comets were Curve lines, yet the Force by which any ſuch Comet is retained in that Curve, is not directed towards the Earth, becauſe the Earth is generally found to be without the Plane of that Trajectory; beſides, in reſpect of the Earth, the Comet is ſometimes Direct and ſometimes Retrograde, and conſequently does not deſcribe Areas proportional to the Times: Yet in reſpect to the Sun, which is placed in the Planes of all their Trajectories, a Comet always moves the ſame way, and the nearer the Sun, the ſwifter; ſo as to encreaſe its Area, which is equally deſcribed by a Radius drawn from the Sun: Wherefore what was aſſerted, is evident from *Prop.* xii.

SECTION

SECTION III.

Of the Order, Diſtances, and Periods of the Secondary Planets revolving about their Primary ones, and their Phænomena; together with the Direction of the Forces, whereby they are kept in their Orbits.

PROPOSITION XV.

TO deſcribe the Order, and Periods of the Secondary Planets, or Satellites, about their Primary Ones, and Diſtances from them.

Of the Six Primary Planets, that revolve about the Sun, there are but three, as we are ſure of by Obſervation, that have Satellites, or others revolving about them, hence called, Secondary Planets. The Earth has one, *viz.* the Moon, compleating its Revolution in $27\frac{1}{3}$ Days, and diſtant about 60 Semidiameters of the Earth from it.

Jupiter has four; the inmoſt of which revolves $1\frac{3}{4}$ of a Day, at the diſtance of $5\frac{2}{3}$ Semidiameters of Jupiter from his Center; the ſecond revolves in $3\frac{3}{5}$ Days, at the diſtance of 9 Semidiameters; the third in $7\frac{1}{6}$ Days, at the diſtance of $14\frac{1}{3}$ Semidiameters; the fourth and outermoſt revolves about Jupiter in the ſpace of $16\frac{3}{4}$ Days, being diſtant from his Center $25\frac{1}{3}$ Semidiameters of Jupiter.

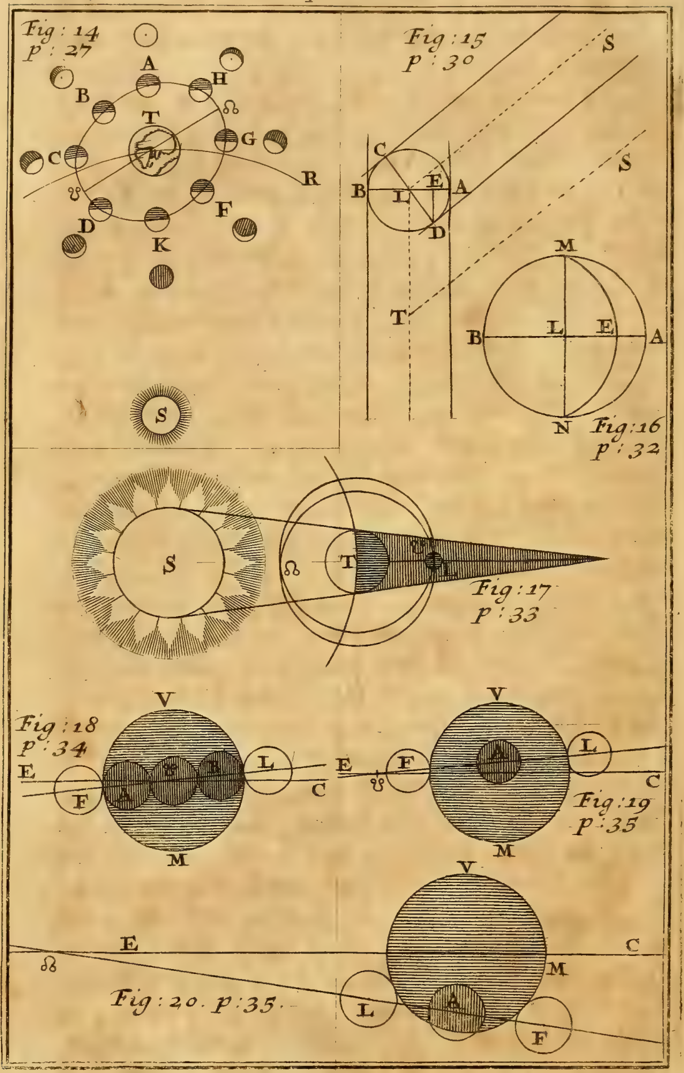
Saturn has five; of which the innermoſt revolves in $1\frac{7}{8}$ of a Day, at the diſtance of $4\frac{3}{8}$ Semidiameters of Saturn, from the Center of Saturn; the ſecond, in the ſpace of $2\frac{3}{4}$ Days, at the

the diſtance of $5\frac{3}{5}$ Semidiameters of Saturn; the third, in $4\frac{3}{5}$ Days almoſt, at the diſtance of 8 Semidiameters; the fourth, in almoſt 16 Days, at the diſtance of 18 Semidiameters; the fifth and outermoſt of all, that have been diſcovered, revolves about Saturn in $79\frac{1}{3}$ Days, at the diſtance of 54 Semidiameters from the Center of Saturn. And the Solar Syſtem, as far as it has been hitherto diſcover'd is of a Figure much like that in the Scheme [*Fig* 13.] But between the two laſt Satellites of Saturn, *Hugenius* ſuſpects there revolves a Sixth, the ſpace being larger than it ſhould be, in proportion to the diſtances of the others, or that there are others beyond the fifth revolving about Saturn, but not ſeen as yet, by reaſon of their Obſcurity. The Planes of the Orbits of the Satellites of the ſame primary Planet, do not coincide, but are variouſly inclin'd to one another, and to the Plane of the Orbit of the Primary one.

Saturn likewiſe is encompaſs'd with a thin plane *Ring*, that does not touch the Body of Saturn at all, but is like an Orbicular Arch, built round about it. The Plane of this Ring is at this time nearly parallel to the Plane of Earth's Equator. And from the various Poſition of it, in reſpect of the Sun illuminating it, it being Opake like a Planet, and in reſpect of the Eye of the Spectator, the various Phaſes of the *Anſæ* of Saturn ariſe, which have ſo long baffled the attempts of Aſtronomers, with their various Shapes. This was firſt diſcover'd by *Hugens*, in his *Syſtema Saturnium*, Printed 1659. The Diameter of the Ring is to the Diameter of Saturn, as 9 to 4; and the breadth of the ſpace between the Ring and the Body of Saturn is equal to the breadth of the Ring it ſelf.

PRO-

Plate 4. Book 1.



PROPOSITION XVI.

TO *describe the Phænomena of the Moon view'd from the Earth, as it is a Spherical Body illuminated by the Sun.*

The Moon, being a secondary Planet, accompanies the Earth *T*, [*Fig.* 14.] during its Revolution about the Sun, in its Orbit *R T*, and moves in its Orbit the same way as the Earth does, from *A* thro' *B*, *C*, *&c.* The Plane of the Moon's Orbit does not coincide with the Plane of the Ecliptic, but is inclin'd to it by an Angle of about five Degrees; and the common intersection of these Planes is the Right Line ☊ ☋; the Points ☊ and ☋, where the Orbit of the Moon intersects the Plane of the Ecliptic are call'd the Nodes of the Moon, as was said before, when we treated of the primary Planets. So that the Orbit of the Moon is to be conceiv'd as being half above and half below the Plane of the Ecliptic, the former ☊ *A* ☋ to the North, the latter ☊ *K* ☋ towards the South, and on that account is represented in the Diagram, in the Figure of an Ellipse. One of the Nodes, *viz.* that where the Moon, having pass'd the Ecliptic, is ascending into the Northern Part, is call'd the *Dragon's Head*, and is mark'd thus ☊; the other, the *Dragon's Tail*, mark'd thus ☋. The Line of the Nodes does not always keep the same position, but is moved with an angular Motion, so as that the Nodes go backwards, *viz.* ☊ thro' *G*, *F*, *&c.* and ☋ thro' *C*, *B*, *&c.* contrary to the Order of the Signs, compleating a Revolution in about 19 Years. From whence 'tis certain, that the Moon can never be found in the Ecliptic above twice in its Monthly Course, namely, when it is in the Node

Node ☊ or ☋; all the reſt of the time it is wide of it; and the Latitude of the Moon's deviation is meaſur'd by the Angle in which the Right line connecting the center of the Moon and Eye is inclin'd to the Plane of the Ecliptic, (by *Def.* 5. *Elem.* xi.) agreeable to what was ſaid above concerning the primary Planets, *Prop.* v. and viii.

Beſides, the Moon, like the other primary Planets, being an Opake, Rough, Spherical Body, reflecting the Sun's Rays falling upon it; 'tis evident, that half of it being turn'd towards the Sun is illuminated and bright, while the other half, that is turn'd from the Sun, continues obſcure and dark. Now only that Hemiſphere of the Moon which is towards the Earth, can be ſeen by an Inhabitant of the Earth viewing it: Conſequently the Phaſes of the Moon will be various, according to the various habitude of the inlighten'd Hemiſphere to that which is turn'd towards the Earth; as was ſhewn in the like caſe concerning the inferior Planets, in *Prop.*vi.

If the Moon be at *A* in the Scheme [*Fig.* 14.] the point juſt oppoſite to the Sun, which we will ſuppoſe to be at ☉, and the Earth in *T*; 'tis evident that the whole enlighten'd face of the Moon is turn'd towards the Earth; this Phaſis or appearance of the Moon is call'd the *Full Moon.* If the Moon be remov'd to *B*, the Sun and Moon continuing as they were, part of the bright or illuminated half will be turn'd from the Earth, and part of the obſcure or darken'd half turn'd towards it; hereupon the Moon will not appear full, but Gibbous, or a little deficient or obſcure in oppoſite to the Sun. The Moon arriving to the ſituation *C*, where the Angle *C T S* is Right, and the Moon in one of

of its *Quadratures*, half the Hemiſphere which is turn'd to the Earth is illuminated, and half dark; on which account it will appear a half Moon, and in the decreaſe. The Moon having got farther, for inſtance, to *D*, but a little of its Face that is turn'd to the Earth is inlighten'd, much the greater part remaining dark: and conſequently the inlighten'd part will appear Horned, by reaſon of the Moon's really ſpherical, but apparently plane Figure; and the Horns will appear turn'd away from the Sun, as ſhall be demonſtrated in the next Propoſition, or in the preſent caſe, turned towards the Weſt. At length when the Moon is come to the point *K*, or into conjunction with the Sun, all the inlighten'd half is turned from the Sun; and conſequently the Moon will be intirely dark, and become a *New Moon*; becauſe it will ſoon appear at *F*, and ſhew its Horns ſtill turn'd from the Sun, and conſequently now towards the Eaſt. Afterwards being upon the increaſe, it will appear a half Moon at *G*, at the end of the firſt Quarter, as it did in *C* at the beginning of the laſt Quarter; then Gibbous at *H*, till it become full again at *A*. For greater evidence and clearneſs of the whole, we have drawn the Appearance of the Moon ſeen by us, correſponding to every diſtinct ſituation, and plac'd it juſt behind it.

Tho' the time of the Moon's Revolution about the Earth be but 27 Days, 7¾ Hours, which is call'd a *Periodic Month*, yet by a Month, or intire *Lunation* is commonly underſtood all that ſpace of time that is ſpent from one New Moon till the next following, which is greater than the abovemention'd Periodic Month; becauſe during the Periodic Month, in which the

Moon

Moon departing from the point *K*, where the New Moon now happens, returns to it again; the Earth itſelf, together with the Moon its Attendant, is carried an intire Sign *in conſequentia*; ſo that the ſaid point of the Orbit of the Moon *K*, is more towards the weſt of the Sun; and conſequently the Moon is not yet arrived to a conjunction with the Sun; but ſtill lacks 2 days and 5 hours to it, or to be become New; that is, to finiſh an intire Lunation, and all the ſeveral Changes of its Appearances; which ſpace of time is call'd a *Synodic Month*, and conſiſts of 29 days, $12\frac{3}{4}$ hours.

A little before and after a New Moon, *viz.* when the Moon is in *D* or *F*, the Rays of the Sun reflected from the Earth, meeting with the Moon, are the cauſe of that faint Light by which the reſt of Moons Diſc beſides its Horns, is then render'd viſible. But when the Moon is got out of the way of the reflexion of the Earth's light, that faint light vaniſhes, being falſly ſuppos'd native.

PROPOSITION XVII.

TO *draw the Phaſis or Appearance of the Moon at any given Time.*

Let a Plane, the ſame with that of the Scheme, for inſtance, paſſing thro' the Centers of the Sun, Earth and Moon, cut the Globe of the Moon, and let its Section be the Circle *A D B C*, [*Fig.* 15.] Let *C D* be a Diameter of the Circle, and *S L*, a right line connecting the Centers of the Sun and Moon, perpendicular to it; and *A B* another Diameter to which the Right line *T L*, connecting the Centers of the Earth and Moon, is perpendicular. From the Point *D*, let *D E* fall a perpendicular to *A B*, meeting it in *E*.

Since

Since a Plane perpendicular to that of this Scheme erected upon the Diameter *C D*, ſeparates the enlighten'd Hemiſphere from the obſcure; and a Plane erected after the ſame manner upon *A B*, ſeparates the viſible from the inviſible Hemiſphere: That part of the Globe of the Moon which is common to both Hemiſpheres, *viz.* that which lies between the Planes erected upon *A L*, *D L*, will be the enlightened part of the viſible Hemiſphere; and that which lies between the Planes erected upon *B L*, *D L*, will be its obſcure or dark Portion. Both theſe Portions of the viſible Hemiſphere, as well the inlightened as the obſcure, are terminated in the oppoſite Points of the Globe of the Moon; namely, thoſe in which a Right line raiſed from *L*, perpendicular to the Plane of the Circle *A C B D*, and produced both ways, meets the Moon's Surface; that line being the common Section of the two Planes erected upon *A B*, and *C D*: The greateſt breadth of theſe parts is in the Circumference *A D B*; the breadth of obſcure being *B D*, of the enlightened *A D*. The Arc *B D* is the meaſure of the Angle *B L D*, which is equal to the Angle *S L T*, contained under the Right lines drawn from the Center of the Moon to the Centers of the Sun and Earth. For if to the equal Angles *S L D*, *T L B*, (becauſe right ones) you add the common Angle *D L T*, the Angles mention'd will be formed, which are conſequently equal. But at any given time, the Angle of the Moon's Diſtance from the Sun, *viz.* *L T S* is known; conſequently *S L T*, its Complement to two right: For the Diſtance of the Moon from the Earth is ſo ſmall, in compariſon with the diſtance of the Earth from the Sun, that the Right lines *T S*, *L S*, drawn from the

the Earth and Sun, may be taken for parallels. Make therefore, that as twice the Radius is to the versed line of the Angle *TLS*, so is *BA* to *BE*. And since the Hemisphere of the Moon seen from the Earth, appears like a Circle, as it happens to any Globe seen at a great Distance, according to the Principles of Optic: Let *AMBN* [*Fig.*16.] be a Circle representing the Disc of the Moon, the same with the former, made by the Section of the Moon with a Plane erected upon *AB*, and consequently described upon the same Center *L*, and with the same Diameter. Because the Moon is at a considerable distance, each point of its Disc will be seen by Right lines parallel to the Right line *LT*. And therefore *BE*, and *AE*, will be the greatest Latitudes of the inlightened and obscure parts in the Disc of the Moon. Draw the Diameter *MN* perpendicular to *AB*; and describe the Semi-Ellipse *MEN*, whose greater Axe is *MN*, and lesser equal to twice *LE*, *MANEM* will be the inlighten'd Part, and *MBNEM* the obscure Part of the Moon's Disc *AMBN*.

For the boundary of illumination on the Spherical Surface of the Moon, by the Sun, which is likewise Spherical, is a Circle; and this Circle seen at a distance obliquely, appears like an Ellipse, whose Semiaxes are *LM*, *LE*, as is evident from Optics: 'Tis seen obliquely by *TL*, because the Right line *SL* perpendicular to its Planes, is inclined to *LT*; excepting when the Moon is Full, where they coincide.

In the Construction of this Problem, we suppos'd half the Globe of the Moon to be inlighten'd, and half likewise to be seen by a Spectator, tho' neither of the Suppositions are rigorously true: For the Sun being bigger than the Moon

Moon, more than half the Moon is inlighten'd; and the Spectator not being at an infinite Distance from the Moon, its intire Hemisphere will not be visible. But the difference in both these cases is so small, that it may very well be neglected: And the Phasis or Appearance of the Moon delineated as above, sufficiently agrees with Observations. That the Scheme may agree with the Heavens, the Line *B A*, to which *M N* connecting the Horns is perpendicular, must be plac'd in such a Situation, as that it may tend directly to the Sun: For the Right line *L A*, seen from *T*, coincides with the line *L S* tending to the Sun.

The Phases of the inferior Planets and Mars, described in *Prop.* VI. and IX. are to be delineated after the same manner; But then if *L* represent a Primary Planet, the Right lines *T S*, *L S* are sensibly inclin'd to one another, and form a Triangle.

PROPOSITION XVIII.

To explain the Phænomena of the Moon arising from the Opacity of the Earth casting a Shadow, that is, an Eclipse of the Moon.

The Earth is an Opake Body, consequently when it is inlighten'd by the Sun, it casts a Shadow towards those parts which are turn'd from the Sun: and this Shadow must always be in the Plane of the Ecliptic; since both Sun and Earth are always there. [*Fig.* 17.] If at any time, the Moon when Full, should happen to be in the Plane of the Ecliptic, or pretty near it, (which it will be, when the Full Moon happens in or near a Node,) 'tis evident the Moon will be immers'd in the Shadow; and consequently deprived of the Sun's Light, by which

it ſhines; that is, it will ſuffer an *Eclipſe* which will be *Total* or *Partial*, according as the whole or only a part of the Moon's Body enters the Shadow.

The Earth's Shadow terminates at laſt in a Point, and does not reach ſo far as Mars: For Mars, tho' in the Plane of the Ecliptic, and oppoſite to the Sun, is not Eclipſed; which it neceſſarily would be, if it were immers'd in the Earth's Shadow: For Mars is an opake Body, as is ſhewn in *Prop.* ix. 'Tis evident at firſt ſight, from this Figure of the Earth, that the Sun is bigger than the Earth: For if the lucid Body be bigger than the opake one, the Shadow will be equally thick and cylindrical; But if the lucid Body be leſs than the opake, the Shadow indeed will be conical, but growing bigger and bigger, and the farther off it is from the opake Body, the thicker it is; and in both caſes will be extended infinitely. As the Sun is bigger than the Earth, ſo the Earth is bigger than the Moon; becauſe the Moon ſometimes is totally Eclipſed by entering into the Earth's Shadow; but the Earth's Shadow is much ſmaller where the Moon enters it, than nearer to the Earth, as is evident from what has been already ſhewn.

Let the Circle *V M* [*Fig.* 18.] repreſent the tranſverſe Section of the Earth's Shadow, where it croſſes the Moon's Orbit. Let *L ☋ F* be the Orbit of the Moon, *C E* the Ecliptic. The duration of ſome Eclipſes is found to be ſo long (for inſtance, four hours,) as to let the Moon go the length of three of its Diameters in the Shadow totally Eclipſed. This happens when the Center of the Moon paſſes thro' the Center of the Earth's Shadow or Circle *V M*: And ſuch an Eclipſe is call'd a *Total* and *Central* Eclipſe.

Some-

Sometimes the Moon is not arrived at, or has already paſſed the Node, when it is entering the Earth's Shadow: Notwithſtanding, if at that time it be near the Node, it will be totally immers'd, and conſequently there will happen a Total, tho' not a Central Eclipſe, nor of ſo long a continuance [*Fig.* 19.] For the way of the Moon thro' the Earth's Shadow, is leſs than the Diameter of the Shadow, becauſe it does not paſs thro' its Center; and conſequently, *cæteris paribus*, will be paſs'd thro' ſooner.

But if the Node be ſo far off from the Shadow, [*Fig.* 20.] as that only part of the Moon is immers'd, as it paſſes in its Orbit, near the Shadow, the Eclipſe will be a partial one, and is ſaid to be of ſo many *Digits* as there are twelfth parts of the Moon's Diameter darken'd, when the Eclipſe is greateſt. For the Diameter of the Moon (like any other Integer) is imagin'd to be divided into twelve Parts or Digits: Which is to be underſtood likewiſe of any other Phaſis of an Eclipſe. In all theſe Caſes, in the beginning of an Eclipſe, the Moon enters the Weſtern part of the Shadow, with the Eaſtern part of her Limb; and in the end of it, ſhe leaves the Eaſtern part of the Shadow with the Weſtern part of her Limb; and all the intermediate time is reckon'd into the Eclipſe; but only ſo much into the total immerſion, as paſs'd while the Moon was totally in darkneſs. The Eaſtern limb of the Moon in its acceſs to the Shadow, does not enter preſently into the thickeſt Darkneſs; but grows darker and darker as it approaches nigher to the Shadow; and this ariſes from the Penumbra, which always accompanies a Shadow which is made by a lucid Body, that is ſeen under a ſenſible Angle. And this Pe-

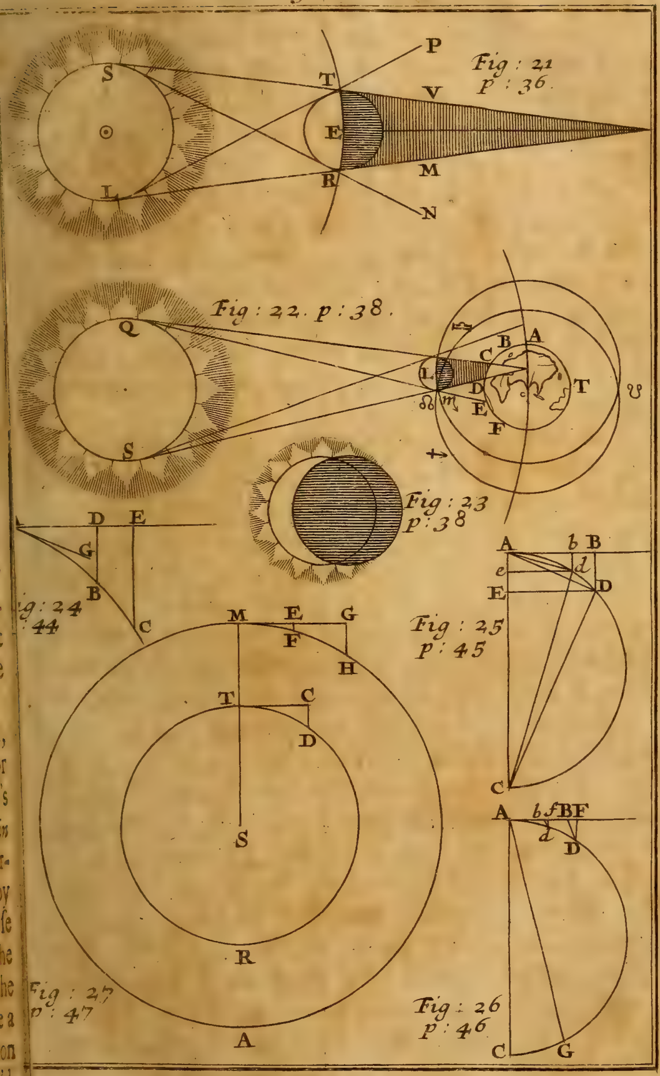
numbra is diffuſed all about the Shadow. Let ☉ be the Sun, [*Fig.* 21.] *E* the Earth, and let Right lines be drawn juſt as in the Scheme: All about the Shadow *TVMR*, where no part of the Sun's light can come, a Penumbra *VTPMRN* is ſpread, ſome part of the Sun's light not been ſtop'd therein: And this Penumbra is darker towards *TV*, *RM* which are the extremities of the perfect Shade; becauſe fewer Rays can arrive thither, the portion of the Sun, from which they are emitted, being ſmaller; but leſs obſcure towards *TP*, *RN*, where more Rays can reach: And beyond which limit all the Rays of the Sun can reach without any hindrance at all, and inlighten according to the degree of their vigour.

The Red colour, that the Moon in the middle of the compleat Shadow is affected withal, and makes her look like a Brick (for in ſome Eclipſes the Moon intirely diſappears,) ſeems to ariſe from the Sun's Rays, either refracted in their paſſage thro' the Earth's Atmoſphere, or reflected by ſome particles of Matter flying about the Earth's Shadow, to the Moon; or from the light of the Stars, or all theſe taken together.

Some kind of Eclipſe or other of the Moon, happens generally twice a Year at leaſt: For there being two Nodes in which the Moon's Orbit croſſes the Ecliptic, and they moving *in antecedentia*, (by *Prop.* xvi.) and the Sun appearing to go thro' the Ecliptic *in conſequentia*, (by *Prop.* ii.) the Sun muſt meet one or other of theſe Nodes twice every Year; and conſequently the Earth's Shadow muſt perforate, as it were, the Moon's Orb in the other Node. If therefore a Full Moon happens juſt at that time, the Moon muſt neceſſarily be totally and centrally Eclips'd,

as

Plate 5 Book 1.



as was demonſtrated above. And tho' a Full Moon does not happen juſt as the Sun and Node meet; yet the inclination of the Orbit of the Moon to the Ecliptic, and the depth of the Earth's Shadow is ſo great, that tho' the Full Moon is diſtant above ten Days from the aforeſaid time, before or after it, and it can be diſtant but fifteen Days, yet the Moon will ſcarce get clear of the Shadow. But if the abovemention'd meeting of the Sun, and Lunar Node, happen on the very Day of the New Moon, or a Day or two before or after, which can happen but ſeldom, the Moon will be far enough off from the Earth's Shadow in the next Full Moon, whether preceding or following, and ſo will eſcape an Eclipſe; conſequently there will be no Eclipſe that half Year.

PROPOSITION XIX.

To explain the Phænomena of the Sun ſeen from the Earth, and ariſing from the Opacity of the Moon, or Eclipſes of the Sun accounted for.

As the Moon upon the interpoſition of the Earth is deprived of the Sun's light, and ſaid to be Eclipſed; ſo, in like manner, if the Earth ſhould be robbed of the Sun's light, by the interpoſition of the Moon, this Phænomena ought to be call'd an Eclipſe of the Earth. But the Obſerver of it being on the Earth, and allowing of no loſs of Light or Eclipſe, nor Motion, nor any other thing that ſeems to argue an imperfection in his place of abode, calls this Phænomenon an Eclipſe of the Sun; for the ſame reaſon as an Inhabitant of the Moon, would ſay the Sun was eclipſed, when the Moon is really entering into the Earth's Shadow.

'Tis evident an Eclipſe of the Sun will happen in every ſuch New Moon, as has the Moon at or near any of the Nodes. For then the ſhadow of the Moon lying directly betwixt the Sun and Earth, reaches to the Earth, and cauſes a total Eclipſe to the Inhabitants of the tract *C D* [*Fig.* 22.] that are immers'd in the thickeſt ſhade. But the Moon's Shadow not being large enough to cover the whole Earth, the circular tract *BC, ED,* which ſurrounds the former *C D,* is cover'd with the Penumbra, and its Inhabitants ſee only a partial Eclipſe of the Sun; which is greater towards *C* and *D,* becauſe a greater portion of the Sun will be cover'd by the Moon; but leſs towards *B* and *E,* the extremities of the Penumbra, and the defect of light is ſcarce ſenſible. At the ſame time, in other places, as *E F,* the bigneſs of the Earth is the reaſon why there is no Eclipſe at all; the Sun inlightning it without any hindrance or impediment.

All the preceding account happens in Nature, juſt as it has been related. But if we look upon the Sun and Moon from our Earth, the Moon will ſeem to cover the Sun more or leſs according as the Spectator is nigher or farther off from the total Shadow [*Fig.* 23.] And becauſe the Moon which covers the Sun can't be ſeen, the Sun will appear to be darken'd in that part which the Moon covers, the reſt only remaining lucid.

Sometimes a central Eclipſe happens not to be total; but a bright Ring ſeems to ſurround the Moon: The reaſon is, at that time the Moon's ſhadow is ſo ſhort that it cannot reach the Earth; either becauſe the Moon is then ſo far off from the Earth; or becauſe the Sun's Rays that graze along the extreme parts of the

Moon are more than ordinarily inflected, and ſo ſhorten the Moon's ſhadow. The bigneſs of a Solar Eclipſe is to be eſtimated juſt as in the Moon, by the digits of the Sun's diameter, that are darken'd by the Moon at the time given.

'Tis evident likewiſe that the Moon moving towards the Eaſt, or from ♎, thro' ♏ to ♐, [*Fig.*22.] the weſtern part *A* of the Earth will be in the ſhadow firſt, which will paſs along the Earth's diſc, like a Spot, thro' *B*, *C*, *D*, *E*, to *F*, where it leaves the Earth. But if the Moon be look'd upon from the Earth, the eaſtern limb of the Moon will firſt cover the weſtern of the Sun, and the weſtern of the Moon will laſt uncover the eaſtern limb of the Sun: And the greateſt darkneſs that happens in a total Eclipſe is ſoon at an end; ſome part of the Sun's disk being preſently uncover'd, almoſt ſo ſoon as the whole was cover'd: and that part, tho' never ſo little, will mightily inlighten the Air.

Tho' the Moon muſt be in a Node the very moment of the New Moon, to cauſe the biggeſt Eclipſe of the Sun that is poſſible, and that the ſhadow of the Moon may go along the middle of the Earth: Yet if ſhe be not far off from thence, the ſhadow of the Moon, or at leaſt part of the Penumbra, will fall upon ſome tract of the Earth, being ſo big, and there cauſe a total, or at leaſt a partial Eclipſe: And in this ſenſe there are more Eclipſes of the Moon than of the Sun. But Eclipſes of the Sun, in any given place, are much fewer than the Eclipſes of the Moon; becauſe the Moon's ſhadow is leſs than the Earth's, and conſequently it does not involve any given place upon the Earth, ſo often as the Earth's ſhadow does ſome part of the Moon.

SCHOLIUM.

The ſame Phænomena will appear in Jupiter and Saturn : For their Satellites or Moons, will be eclipſed, by being immers'd in the ſhadow of their primary Planet ; and thoſe Eclipſes of them are obſerved by us, juſt as the Eclipſes of our Moon may be obſerved from them. In like manner, every Satelles coming between the Sun and its Primary, caſts a ſhadow upon the Primary, which ſeems to move along the Disk of the Primary from Eaſt to Weſt like a Spot. But the Duration, Phaſes, Periods, *&c.* of theſe Phænomena are various, and differing from the like ſeen by us, and ariſing from our Moon, according to the diverſity of the Shadows, Motions and Magnitudes, both of the Primary and Secondary Planet.

PROPOSITION XX.

EAch of the Secondary Planets mention'd in Prop. XV. *is urged by a Force compounded of a Centripetal Force, tending to the center of the Primary, about which it revolves, and of all the Accelerating Force with which the Primary is urged. And therefore the Forces whereby the Satellites are retain'd in their Orbits about the Primary ones, tend towards the centers of their Primary ones reſpectively.*

Becauſe the Satellites of Jupiter and Saturn revolve equably in circular Orbits, concentric with Jupiter and Saturn, they deſcribe Areas about theſe Centers reſpectively, proportional to the Times. In like manner, if the Moon's Orbit differ from a Circle concentric with the Earth, the leſs the Moon appears, (that is, the farther it is from the Earth) the ſlower it moves round the Earth, ſo that ſtill the Area, that the Radius drawn to the center of the Earth de-

ſcribes, is equably augmented. So that univerſally, each of the abovemention'd Satellites deſcribes Areas, by a Radius drawn to the center of its Primary Planet, about that Center, proportional to the Times. If therefore the Syſtem of any Primary Planet, and its revolving Satellites, be ſuppos'd to be urged along parallel Lines, by a Force equal and contrary to that, whereby the Primary Planet in that Point of its Orbit, where it then is, is urged towards the Sun, the Primary Planet will no longer deſcend towards the Sun; and the Secondary will continue to deſcribe the ſame Areas about the Primary as before; that is, proportional to the Times: (For if the Bodies move any how in reſpect of one another, and are urged by equal accelerating Forces along parallel lines; or, which is all one, if the Space in which they perform their Motions be moved uniformly in a right line, they will move all after the ſame manner, as they would do if thoſe Forces were abſent, or the Space were at reſt, in which they are included). So that each Satelles, only urged by the difference of the Forces, will go on to deſcribe, about the center of its Primary, Areas proportional to the Times. Therefore, by *Prop.* xii. the difference of thoſe Forces tends to the Primary Planet as a Center. But before that the whole Syſtem was urged along parallel lines, by a Force equal and contrary to that, whereby the Primary Planet is urged towards the Sun, that is, in its natural State, each Satelles is urged by a Force compounded of the Centripetal Force tending to the center of its Primary, and of all the accelerative Force that the Primary is urged by. Conſequently the 'foreſaid difference of the Forces, is that whereby the

Satelles

Satelles is retained in its Orbit about the Primary, the remaining accelerative Force, whereby the Primary Planet is urged in any Point of its Orbit, conferring nothing to this: But this difference has been ſhewn to tend to the Center; therefore the Forces whereby the Secondary Planets are retained in their Orbits about the Primary, tend towards the Centers of the ſaid Primary reſpectively. Which was to be demonſtrated.

SECTION IV.

Of the Periods of the Primary Planets about the Sun, and Secondary about their reſpective Primary, compar'd together, and of their Diſtances likewiſe compared together, with their mutual relation to one another, the reaſon and cauſes of it.

PROPOSITION XXI.

T*HE Motion of the Secondary Planets revolving about a Primary, is ſuch, as that the Squares of the Periodic Times, are in the ſame ratio as the Cubes of their Diſtances from the Center of the Primary.*

This Theorem is abundantly evident from the Obſervations of Aſtronomers: And by the Squares of the Periodic Times, you muſt underſtand, the Squares of Right lines, or of Numbers, having the ſame ratio as the Periodic Times; in this Senſe alſo are the Squares of the Velocity, and the like.

Thus in the Satellites of Jupiter, where the Periodic Times of the inmoſt, ſecond, third, and

and outmoſt, are reſpectively as 1 $\frac{3}{40}$, 3 $\frac{3}{5}$, 7 $\frac{1}{6}$, and 16 $\frac{3}{4}$ almoſt; and their Diſtances as 5 $\frac{2}{3}$, 9, 14 $\frac{1}{3}$ and 25; the Square of the Periodic Time of the inmoſt, namely 3, is to 13 the Square of the Periodic Time of the ſecond Satelles, as 170, the Cube of the diſtance of the inmoſt from the Center of Jupiter, to 736, the Cube of the diſtance of the ſecond from thence. In like manner 3 is to 51, the Square of the Periodic Time of the third Satelles, as 170 to 2890, the Cube of the diſtance of the third from the Center of Jupiter. And 3 is to 280, the Square of the Periodic Time of the outmoſt Satelles, as 170 to 15800, the Cube of the diſtance of the outmoſt from Jupiter's Center. And therefore, *ex æquo*, the ſame ratio holds between any other two of them compar'd together; as the ſecond with the third or the laſt, or the third with the laſt. This Ratio comes out more exact, if the Diſtances and Periodic Times are taken more accurately.

The ſame will be found to hold in the Satellites of Saturn, if you take the numbers laid down in *Prop.* xv, that their diſtances from Saturn, and their Periods about him may be eſtimated. But the Moon being a ſolitary Satelles, this Propoſition can't be applied to her.

PROPOSITION XXII.

THE Motion of the Primary Planets about the Sun, is ſuch, as that the Squares of their Periodic Times are in the ſame ratio as the Cubes of their Diſtances from the Sun.

Thus, for inſtance in round numbers, the Period of Saturn is 30 Years, Jupiter 12; their Squares are 900 and 144. The diſtance of Saturn from the Sun, is, by Obſervation, to the

diſtance

diſtance of Jupiter from the ſame, as 9 to 5 almoſt; their Cubes are 729 and 125. But 'tis evident thoſe Squares are nearly as theſe Cubes. Alſo the Period of the Earth is a little more than four times greater than the Period of Mercury; conſequently their Squares are as 17 to 1 almoſt: Suppoſing the Diſtance of the Earth from the Sun to be ten parts, the Diſtance of Mercury is, by Obſervation, about 4, or 3, 9; and their Cubes as 1000 to 59: But 17 is to 1 almoſt as 1000 to 59; And ſo on in the other Planets. The more correct the Diſtances and Periods are taken, the nearer you'll come to this Proportion. What has been now ſaid, may ſuffice at preſent: The Propoſition being more exactly to be made out in the third Book.

PROPOSITION XXIII.

THE Spaces that a Body deſcribes, acted upon by any kind of finite Force, (whether it be determinate and unchangeable, or the ſame in continual increaſe, or continual decreaſe) are in the very beginning of the Motion, in the duplicate ratio of the Times.

Let the Times be expreſs'd by the Right lines *AD*, *AE*; [*Fig.* 24.] and the Velocities acquired at the end of thoſe Times, by the Ordinates *D B*, *E C*: And the Spaces deſcribed with theſe Velocities will be as the Areas *A B D*, *A C E*, made up, as it were, of theſe Ordinates; namely, the Space deſcrib'd in every ſmalleſt particle of Time, being as the Velocity and that Particle of Time conjunctly. But, in the beginning of the Motion, the Ordinates *DB*, *EC*, are very near the Point *A*; and in that caſe, the Trilineal Figures *ADB*, *ACE*, are ſimilar rectilineal Triangles, that part of the Curve

A B C,

ABC, which belongs to the infinitely ſmall trilineal Figures, not extending it ſelf beyond the Right line *AG*, which is a Tangent to the Curve in *A*. Now theſe ſimilar rectilineal Triangles, are in the duplicate ratio of their homologous Sides *AD*, *AE*. Conſequently the Spaces deſcrib'd in the beginning of the Motion, are in the duplicate ratio of the Time. Which was to be demonſtrated.

COROLLARY.

From hence we may gather, that the Errors of Bodies deſcribing ſimilar parts of ſimilar Figures in proportional Times, generated byany equal diſturbing Forces ſimilarly applied to the Bodies, and meaſur'd by the diſtances of the Bodies from thoſe places of the ſimilar Figures, to which theſe Bodies would arrive, in the ſame proportional Times, without thoſe Forces, are very near as the Squares of the Times wherein they are generated. For theſe Errors are the Spaces which the Bodies acted upon by the diſturbing Force deſcribe. But if theſe diſturbing Forces are not equal, but in a given ratio ; the Errors are as thoſe Forces and the Squares of the Times conjunctly ; ſuppoſing the Forces ſimilarly applied.

PROPOSITION XXIV.

THe naſcent or evaneſcent ſubtenſe of the Angle of Contact, in any Circle, is in the duplicate ratio of the conterminous Arc.

Let *ADC* [*Fig.*25.] be a Circle, the right line *AB* a Tangent in *A*, conſequently the Angle of Contact *BAD*. I ſay, any Subtenſe that is the neareſt to the Point *A*, for inſtance, *BD* is as the ſquare of the Arc *AD*; that is, the ſubtenſe *BD* is to another ſubtenſe *bd*, in the ſame cir-

circumſtances, as the ſquare of the Arc AD to the ſquare of the Arc Ad.

Draw the Diameter AC, which will be perpendicular to AB, join the Right lines DA, dA, DC, dC, and draw DE, de parallel to AB.

Firſt, let the Subtenſe BD be parallel to AC. Now (by 8 and 17. *Elem.* vi.) $ADq = CA \times AE$, and $Adq = CA \times Ae$. Wherefore $ADq : Adq ::$ $(AE : Ae ::)$ $BD : bd$. But ſince BD, bd are neareſt to the Point A, or in their naſcent condition, the Arcs AD, Ad, and their Subtenſes do not differ from one another; that is, they are equal. Conſequently, in this caſe, BD is to bd, as the Square of the Arc AD, to the Square of the Arc Ad.

Secondly, if the Subtenſe BD of the Angle of Contact be not parallel to AC, but to AG, [*Fig.* 26] the ſame ſtill holds: For, drawing DF, df parallel to AC; then, becauſe DFB dfb are ſimilar Triangles, $BD : bd :: DF : df$; but it has been already ſhewn that DF is to df, as ADq to Adq. Wherefore BD is to bd, as ADq to Adq.

Thirdly, if BD be ſuppos'd to be drawn according to any other certain Law, (for inſtance converging toward the Center;) ſince BD, bd are as near as may be to the Point A, the Angles B, b will be equal; and conſequently, in that caſe, $Bd : bd :: (DF : df ::)$ $ADq : Adq$. Which was to be demonſtrated.

SCHOLIUM.

This is true likewiſe in any other Curve, to which a Circle equi-curve may be drawn; ſuch as all the Conic Sections are. For the Points D, d (being the neareſt that can be to A) muſt be in that other Curve, as well as in the Circle, which is equi-curve to it, by Hypotheſis: Con-

ſequently

ſequently the Properties of *B D*, *b d*, which agree to the Circle, agree likewiſe to this Curve, to which the Circle is equi-curve. And ſince the foregoing Propoſition is true of all Circles, it muſt likewiſe be true of all Curves, that can have equi-curve Circles drawn to them.

PROPOSITION XXV.

Bodies deſcribing different Circles with an equable Motion, are acted upon by Centripetal Forces tending to the Centers of thoſe Circles. And the Forces are to one another as the Squares of the Arcs, deſcribed in the ſame time, applied to the Radii of the Circles. This likewiſe is true of the Centrifugal Forces of the Bodies thus moved.

Becauſe the Bodies, by their *Vis inſita* alone, would deſcribe Tangents, 'tis evident that they are drawn off from their Rectilineal Motion, and retain'd in Circular Orbits by Forces tending to Points within the Circles. But ſince, by ſuppoſition, they are carried in the Circumferences by an equable Motion; the Areas deſcribed by a Radius drawn to the Centers are equally augmented; and therefore are proportional to the time which flows equably; and conſequently theſe Forces, by *Prop.* xii. tend to the ſaid Centers.

Let the Bodies *M* and *T* [*Fig.* 27.] revolving in the Circumferences of the Circles *M A*, *T R*, deſcribe the infinitely ſmall Arcs *M F*, *T D*; From their extremities *F*, *D*, draw the Right lines *F E*, *D C* as far as the Tangents, either parallel to *S M*, *S T*, or diverging from *S*: (For it comes to the ſame, ſince *M F*, *T D* are only naſcent Arcs.) Theſe Right lines are the effects of the accelerating Centripetal Forces; and con-

consequently proportional to them, as their adequate Causes; that is the Centripetal Force of the Body M is to the Centripetal Force of the Body T, as EF to CD. Make the Fig. MGH similar to the Fig. TCD; wherefore GH will be a nascent Right line: And so CD is to GH, as the Arc TD, to the Arc MH. And, by the præceed. GH is to EF as MHq to MFq: And therefore, *ex æquo*, CD is to ET, as ($TD \times MHq$ to $MH \times MFq$; that is, as $TD \times MH$ to MFq, or as) $TD \times MH \times ST$ to $MFq \times ST$: And since TCD, MGH are similar Figures, the Arcs TD, MH are similar; that is, $ST : SM :: TD : MH$. Wherefore $MH \times ST = TD \times SM$. Then by Substitution $CD : EF :: (TDq \times SM : MFq \times ST ::) \frac{TDq}{ST} : \frac{MFq}{SM}$.

But the Centripetal Forces of the Bodies T and M were shewn to be proportional to CD, EF. Therefore the Centripetal Force of the Body revolving in TR, is to the Centripetal Force of the Body revolving in MA, as $\frac{TDq}{ST}$ to $\frac{MFq}{SM}$. And, (because the Motion in both is equable) TD and MF, being describ'd at the same time, have the same ratio with any others describ'd at the same time. Wherefore the Centripetal Force of a Body revolving in TR, is to the Centripetal Force of a Body revolving in MA, as the Square of an Arc described in any time in TR applied to the Radius ST, is to the Square of an Arc described in the same time in MA applied to the Radius SM.

If the abovementioned Bodies be imagined to be tied to S, by two Strings, and kept revolving

ving in the Circles by the help of them; the ſame Force, whereby the String is ſtretched, conſider'd as it is in the String pulling the Body, is call'd the Centripetal Force, but conſidered as it is in the Body ſtretching the String, may be call'd the Centrifugal Force: And conſequently all that has been ſaid concerning the Centripetal Force, is true alſo of the Centrifugal Force. Which was to be demonſtrated.

COROLLARY I.

From hence it follows, that if a Body ſhould be let fall towards the Center, being acted upon by the Centripetal Force, that kept it in the Circle; ſo as that this Force be neither augmented nor diminiſhed in its deſcent, but continue the ſame, and to the firſt impulſe continually ſuperadd a new one equal to it, (which is the *Galilean* Hypotheſis of Gravity:) In the time that the ſaid Body deſcribes an Arc in the Circle, it would, in its deſcent towards the Center, deſcribe a Right-line, equal to that which is produc'd by the Square of that Arc applied to the Diameter of the Circle. For by this *Prop.* 'tis evident it is done in the very beginning of the Fall; and in the preſent Hypotheſis of deſcent, the Spaces run are in the duplicate Ratio of the Times; and ſince the Circle is equably deſcribed, the Squares of the Arcs deſcribed are alſo in the duplicate Ratio of the Times of deſcription: Now the application of theſe Squares, to the Diameter of the Circle, being a conſtant Quantity, does not alter the Ratio. Conſequently it holds perpetually. So that in this Hypotheſis of an uniform Centripetal Force, a Body would in half the Time that it deſcribes an Arc of a Circle in, deſcend the length of a line, equal to that produced by the application

of the Square of half that Arc to the Diameter of the Circle; namely, the ſubquadruple of the Right-line, it would deſcend in double the time, *viz.* in which it deſcribes the intire circular Arc.

COROLLARY 2.

From hence likewiſe it follows, that the centripetal Forces of Bodies, deſcribing different Circles by an equable Motion, are in a ratio compounded of the duplicate ratio of the Velocities directly, and of the ſimple ratio of the Radii inverſely. For the Motion in any Circle being equable, the ratio of the Velocities will be the ſame with that of the Arcs deſcribed in the ſame time; that is, of the Spaces run in the ſame time: But (by this Propoſition) the centripetal Forces are in a ratio compounded of the duplicate ratio of the Arcs deſcribed at the ſame time directly, and of the ſimple ratio of the Radii inverſely: Wherefore the ratio of theſe Forces is compounded of the duplicate ratio of the Velocities directly, and of the ſimple ratio of the Radii inverſely.

PROPOSITION XXVI.

THe ſame things being ſuppoſed, I ſay that the centripetal Forces are reciprocally as the Squares of the Periodic Times applied to the radii of the Circles; or that they are in a ratio compounded of the ſimple ratio of the Radii directly, and of the duplicate ratio of the Periodic Times inverſely. The ſame alſo is true of the centrifugal Forces of theſe Bodies.

Let the centripetal Force of one of the Bodies, moving in one of the Circles, be call'd *V*, in the other *v*; the Celerity in one Circle C, in the other *c*; the Arcs at the ſame time deſcribed in the firſt Circle *A*, in the ſecond *a*; the Radius, or

or Diſtance of the Body from the center, in the firſt Circle D, in the ſecond d; the Periodic Time in the firſt Circle T, in the ſecond t.

'Tis well known, that in comparing the motions of Bodies together, the Spaces run are in the compound ratio of their Times and Celerities directly. But the Radii of the Circles are proportional to the intire Circumferences, which are the Spaces run in the Periodic Times. And therefore $D : d :: C \times T : c \times t$, ſo that $C \times T \times d = c \times t \times D$: therefore $C^2 \times T^2 \times d^2 = c^2 \times t^2 \times D^2$. Therefore, dividing by $D^2 \times d^2$, 'tis $\frac{C^2 \times T^2}{D^2} = \frac{c^2 \times t^2}{d^2}$. Wherefore $\frac{t^2}{d} : \frac{T^2}{D} :: \frac{C^2}{D} : \frac{c^2}{d}$. But, (by Cor. 2. of the prec.) $V : v :: \frac{C^2}{D} : \frac{c^2}{d}$. And therefore $V : v :: \frac{t^2}{d} : \frac{T^2}{D}$. But the ratio of $\frac{t^2}{d}$ to $\frac{T^2}{D}$, is the ſame with that of $t^2 D$ to $T^2 d$; or of t^2 to T^2, and D to d. Therefore the ratio of V to v is equal to the ratio of t^2 to T^2, and of D to d.

What has been ſaid of the Centripetal Forces of two Bodies revolving about a Center, is true of their centrifugal Forces; ſince they differ only in the manner of conception; as was ſhown in the foregoing Propoſition. *Q. E. D.*

COROLLARY I.

From hence (and from Corol. 2. Prop. preced.) it follows, that if the Periodic Times are equal, then, as well the centripetal Forces, as the Velocities, will be as the Radii; and *vice verſa*.

For, by this Propoſition, $\frac{V}{v}=\frac{D\times t^2}{d\times T^2}$. Wherefore if $T=t$, or $T^2=t^2$, then will $V:v::D:d$. And *vice verſa*, if $V:v::D:d$, ſince $V:v::Dt^2:dT^2$, then $D:d::Dt^2:dT^2$; and conſequently $T^2=t^2$, or $T=t$. The ſecond part of the Corollary appears from hence. In the compariſon of Motions, the Velocities are as the Spaces run in the ſame Time. But, becauſe the Periodic Times are equal, the Spaces run in the ſame time are the Circumferences of the Circles; but they are as their Radii. Conſequently the Velocities are as the Radii. The converſe of this Propoſition is drawn after the ſame manner.

COROLLARY 2.

Hence alſo it follows, that if the Squares of the Periodic Times are as the Radii, the centripetal Forces are equal, and the Velocities are in the ſubduplicate ratio of the Radii; and *vice verſa.* For from this Prop. $\frac{V}{v}=\frac{D\times t^2}{d\times T^2}$: But by ſuppoſition, $T^2:t^2::D:d$, or $D\times t^2 = d\times T^2$. Therefore V and v, proportional to them, or the centripetal Forces themſelves, are equal. And *vice verſa*, if $V=v$, the quantities proportional to them will likewiſe be equal, namely, $D\times t^2=d\times T^2$: And therefore $D:d::T^2:t^2$. In like manner, becauſe by Cor. 2. of the preceding Prop. $\frac{V}{v}=\frac{C^2\times d}{c^2\times D}$, and in the preſent caſe, $V=v$, then $C^2\times D$ will be equal to $c^2\times D$, or $C^2:c^2::D:d$; that is, $C:c::\sqrt{D}:\sqrt{d}$. The converſe of this is drawn after the ſame manner as that of the former.

COROL-

COROLLARY 3.

If the Circles deſcribed are equal, the centripetal Forces are reciprocally as the Squares of the Periodic Times: For the ratio of the Radii, in this caſe, being that of equality, makes no alteration in the compoſition.

PROPOSITION XXVII.

THE ſame things being ſuppoſed, if the Bodies revolve in ſuch a manner, as to have the Squares of the Periodic Times as the Cubes of the Radii, then the Centripetal Forces are reciprocally as the Squares of the Radii: And the Celerities of the Bodies are reciprocally as the Square Roots of the Radii; and vice verſa.

Retaining the Characters uſed in the former Propoſition, 'tis evident from it, that $V : v :: \frac{t^2}{d} : \frac{T^2}{D}$. From whence it follows, that $\frac{VT^2}{D} = \frac{vt^2}{d}$, or $VT^2 d = vt^2 D$. And therefore $Dv : dV :: T^2 : t^2$. And by ſuppoſition, $D^3 : d^3 :: T^2 : t^2$; therefore $Dv : dV :: D^3 : d^3$. Conſequently $Dvd^3 = dVD^3$; or $vd^2 = VD^2$. And therefore $V : v :: d^2 : D^2$.

Again, in the demonſtration of the foregoing Propoſition, it was ſhewn that $C^2 \times T^2 \times d^2 = c^2 \times t^2 \times D^2$. Wherefore $c^2 \times D^2 : C^2 \times d^2 :: T^2 : t^2$. But by ſuppoſition, $D^3 : d^3 :: T^2 : t^2$. Therefore $c^2 \times D^2 : C^2 \times d^2 :: D^3 : d^3$. And ſo $c^2 : C^2 :: D : d$ (or by extracting the Square-root, and inverting the Proportion) $C : c :: d^{\frac{1}{2}} : D^{\frac{1}{2}}$; that is, $C : c :: \surd d : \surd D$. Which were to be demonſtrated.

The Converſes of theſe are evident. For as from the putting $T^2 : t^2 :: D^3 : d^3$, it follows that $V : v :: d^2 : D^2$, and that $C : c :: d^{\frac{1}{2}} : D^{\frac{1}{2}}$; ſo

converſely, from the ſuppoſition of any one of theſe, you may return to the former Proportion, *viz.* $T^2 : t^2 :: D^3 : d^3$; and from hence again to the other Concluſion. And conſequently any one of theſe three being ſuppoſed, the other two will follow.

SCHOLIUM.

The three præceding Propoſitions (and their Corollaries) are true, not only of Bodies revolving in concentric Circles, but of ſuch alſo as deſcribe Circles whoſe Centers are intirely different, (for the identity of Centers was not ſuppoſed in the Demonſtration;) and of thoſe that deſcribe ſimilar parts of any Figures, having their Centers ſimilarly poſited. For what was aſſumed in the Demonſtration of theſe Theorems in Circles, are equally true of all other Figures like thoſe we but juſt now deſcribed; namely, that the naſcent lineola GH (in *Fig.* 27.) is to the naſcent lineola EF, as MHq to MFq (for this is demonſtrated of any Figure that a Circle is equicurve to, in *Prop.* 24.) and that MH is to TD as SM to ST, is true of any ſimilar Figures; provided the Center S be ſimilarly poſited in both Figures, and MH, TD, ſimilar, and ſimilarly poſited parts of thoſe Curves in reſpect of the Center. And inſtead of an equable Motion in Circles, if the Figure be different from a Circle, the Motion of the Body in the Perimeter of it, muſt be ſuch, as that the Areas deſcribed, by rays drawn to their Centers ſimilarly poſited, be proportional to the Times; otherwiſe (by *Prop.* 12.) the Forces, whereby the Bodies are drawn aſide from their Rectilineal Motion, and kept in their Curvilineal Orbits, do not ſo much as tend to thoſe Centers.

PRO-

PROPOSITION XXVIII.

THE Forces, whereby the Primary Planets are perpetually retracted from their Rectilineal Motions, and retained in their Orbits, are reciprocally as the Squares of their Distances from the Center of the Sun, which they respect.

That these Forces respect the Center of the Sun, has been shewn in *Prop.* 13. But that the Law of these Forces is such, as that they are reciprocally as the Squares of their Distances from the Center, or that they increase as the Squares of the distance decreases, and decrease as the Squares of the distance increases: As for instance, that the Force whereby the Earth is made to tend towards the Sun, and is retained perpetually in its Orbit, so as not to fly off in a Tangent to it, is to the like Force in Mars, as 5 to 4, to that in Jupiter as 27 to 1; because the distance of the Earth from the Sun, is to the distance of Mars from the same, as 2 to 3, and to the distance of Jupiter, as 5 to 26; and that the Centripetal Force in the Earth is but half that in Venus, because the Earth's distance from the Sun is to the distance of Venus as 10 to 7; and so on in the others, is thus demonstrated. The Motion of the Planets revolving about the Sun, and tending towards it as a Center, is such, as that the Squares of the Periodic Times are as the Cubes of the distances from the Sun, by *Prop.* 22. Now by the preceding *Prop.* the Centripetal Forces of Bodies revolving according to this Law, are reciprocally as the Squares of their distances from the Center: Consequently 'tis evident that the Forces respecting the Sun, by which the Planets are acted upon, and retain'd in their Orbits, are re-

ciprocally as the Squares of their diſtances from the Center of the Sun.

PROPOSITION XXIX.

THE Forces whereby the Secondary Planets are retained in their Orbits, are reciprocally as the Squares of their Diſtances from the Center of their reſpective Primary, to which theſe Forces tend.

For ſince (by *Prop.* 21.) the Motion of the Circum-Jovials revolving about Jupiter, and Circum-Saturnials revolving about Saturn, is ſuch, as to have the Squares of the Periodic Times in the ſame ratio with the Cubes of their Diſtances from the Center of Jupiter and Saturn reſpectively; and ſince (by *Prop.* 23.) the Centripetal Forces of Bodies revolving by this Law are reciprocally as the Squares of their Diſtances from their Center: 'Tis evident that the Centripetal Forces, whereby the Secondary Planets are reſpectively urged towards Jupiter and Saturn, are reciprocally as the Squares of their Diſtances from the Center of their reſpective Primary Planet.

But this Force in any Secondary Planet is only one part of the whole Force, whereby the Satelles is urged; and the other Force is the whole accelerating Force whereby the Primary is urged; as has been ſhewn in *Prop.* 20.

As for the Moon, the Earth's attendant, becauſe it is ſolitary, it has no other Secondary to be compared withal, as to Diſtance and Periodic Times. So that the Law of its Centripetal Force, whereby it is urged towards the Earth, can't be deduced after this manner. Yet hereafter you will find it drawn from the Figure of the Orbit that the Moon deſcribes about the Earth; which hitherto we have conſidered as

not

not differing ſenſibly from a Circle. But if it really were a Circle concentric to the Earth, the Law of the Forces whereby it is retained in its Orbit, could not be from hence diſcovered: For ſuch an Orbit may be deſcribed by a Body, let the Law of the Centripetal Force be what it will.

COROLLARY.

It follows from theſe two Propoſitions, that among the Primary Planets, that neareſt the Sun, and among the Secondary, that next the reſpective Primary, moves faſteſt. For it has been demonſtrated, that in both of them, their Centripetal Forces are reciprocally as the Squares of their Diſtances from their Center: And in *Prop.* 27. it was demonſtrated that the Celerities in this caſe are reciprocally as the Square Roots of their Diſtances from the Center of Motion; or that a mean proportional between the Diſtances of that which was nearer, and that which was more remote, is to the diſtance of the nearer, as the Velocity of the nearer, to the Velocity of the more remote. And ſince the firſt of theſe Quantities is greater than the ſecond, the third will be greater than the fourth; that is, the Velocity of the nearer is greater than the Velocity of the more remote.

SECTI-

SECTION V.

Of the Motion of the Primary Planets about their proper Axes, and of the Phænomena thence ariſing.

PROPOSITION XXX.

TO *explain the Motion of the Celeſtial Bodies about their proper Axes, given in Poſition, and the Revolutions of them.*

When we ſay a Body is not moved, we mean by it, that every line, and every point in it, is at reſt. But if we ſay a Body is moved along a line, without any other Motion, we intend to ſignify, that the Center of Gravity of the Body deſcribes that line, while the parts of the Body, in reſpect of the whole, remain at reſt; that is, every line in that Body retains a ſituation parallel to that it had before it was moved; and conſequently every line in the Body thus moved, remains always parallel to it ſelf. For after this manner, and this manner only, will it happen, that the Sum of all the Motions will be the leaſt, (that is, the motion of the whole the leaſt, as is ſuppoſed) and that any point of the Body thus moved, is as much moved as any other, and deſcribes a line ſimilar and equal, and only differing in ſituation: For the line deſcrib'd by the Center of Gravity, is ſuppos'd to be deſcribed by the Body it ſelf, from the known nature of a Center of Gravity.

If a Body be ſaid to be moved about a given Axe, being in other reſpects not moved, that Axe is ſuppos'd to be unmov'd, and every point

out

out of it to deſcribe a Circle, to whoſe Plane the Axis is perpendicular. And for that reaſon, if a Body be carried along a line, and at the ſame time be revolved about a given Axe; the Axe, in all the time of the Body's motion, will continue parallel to it ſelf. Nor is any thing elſe required to preſerve this Parallelism, than that no other Motion beſides theſe two be impreſſed upon the Body; for if there be no other third Motion in it, its Axe will continue always parallel to the Right-line, to which it was once parallel.

Theſe things being premiſed, 'tis evident from Obſervation, that the greater Celeſtial Bodies, (if not all, yet at leaſt of every kind) beſides the Motions ſpoken of in general above, do revolve about their Axes. And, firſt, from obſervations, of Spots in the Sun, 'tis evident that the Sun, the only Body of its kind, in our Syſtem, revolves from Weſt to Eaſt about an Axe, inclin'd to the Plane of the Ecliptic, in an Angle of about 87$\frac{1}{2}$ Degrees, in the ſpace of about 25 Days. And ſince the Sun has no progreſſive Motion, its Axis continues unmoved. Among the Primary Planets, the Earth revolves with an equable Motion, from Weſt to Eaſt, about an Axe, inclin'd to the Plane of the Ecliptic, in an Angle of 66$\frac{1}{2}$ Degrees, in the ſpace of a natural Day. And ſince the Earth alſo is carried about the Sun in the ſpace of a Year, its Axe muſt however continue parallel to it ſelf, tho' it can't be intirely at reſt. And this will come to paſs by means of theſe two Motions alone, without the aſſiſtance of a third, which ſome have groundleſly invented and call'd by the name of a *Parallelism.* But the Axis of the Earth, tho' it continue parallel to it ſelf, as

to

to ſenſe, during one Revolution, yet after ſeveral, it changes this Site. Of the Cauſes and Phænomena whereof, we ſhall ſpeak in a proper place.

After the ſame manner Jupiter is revolved, from Weſt to Eaſt, about an Axe in the ſpace of about 10 Hours; Mars in $24\frac{2}{3}$ Hours, and Venus in 23 Hours, about their Axes; every one of which therefore continues always parallel to itſelf, if theſe Bodies are not moved by any other kind of Motion beſide their Revolution about the Sun, and Circumrotation about their own Axes.

Among the Secondary Planets, the Moon, the Earth's attendant, beſides its monthly Motion about the Earth, and annual about the Sun, (by which alone each line in it would be always parallel to its ſelf,) it likewiſe revolves about itſelf in the ſame ſpace of a Month, ſo as to turn always the ſame Face to the Earth.

We are not ſo certain yet of the other Planets, both Primary and Secondary, from Obſervations. Notwithſtanding, 'tis probable that they, like the former, revolve about a certain Axe; for this reaſon, that every part of them may be brought under, or removed from, the Rays of the Sun, more than once in one Revolution about the Sun, and may undergo Changes and Viciſſitudes ſuitable to their Nature.

PROPOSITION XXXI.

TO *deſcribe the Figure of the Sun and Planets, revolving about their own Axes.*

If the Sun and Planets were deprived of all Motion about their Axes, they would put on a Spherical Figure; ſuppoſing their parts like the parts of the Earth, to gravitate towards their Cen-

ter,

ter, (ſuch as really they are, as we ſhall afterwards demonſtrate,) and either are now or were formerly fluid. This is demonſtrated by *Archimedes* in Hydroſtatics. From a Motion about their Axes, it comes to paſs, that their Parts receding from the Axis as much as they can, endeavour to aſcend about the Circle that lies betwixt the two Poles, by a Centrifugal Force ariſing from their circular Motion. And therefore if the Matter of the Sun or Planet be fluid, it will increaſe the Diameters of that middle Circle between the two Poles, by its aſcent; and leſſen the Axe towards the Poles by its deſcent, and conſequently conſtitute an oblate Spheroid, generated by the rotation of an Ellipſe about its leſſer Axe; being depreſſed towards the Poles, and elevated towards the Circle that lies exactly between the Poles, more or leſs according to the quantity of the Centrifugal Force. This Spheroidical Figure is obſerved in Jupiter by the help of a Teleſcope; and is proved to be in the Earth by Experiments, (ſince we can take a view of her at a diſtance.)

SCHOLIUM.

As this Spheroidical Figure follows upon the rotation of the Body about its own Axis, the fluidity of its Parts; ſo, on the contrary, from this obſerved Figure, we may juſtly infer its Motion about its Axis, and the (former) fluidity of its Parts.

PROPOSITION XXXII.

TO explain the Phænomena ariſing from the Diurnal Motion of the Earth about its Axis, ſeen from the Earth.

In the foregoing Propoſitions I have ſhewn, what would be the Phænomena of the Motion of

of the Sun and Planets ſeen from the Earth, if the Earth were a Point; as that the Sun would ſeem to move among the Fixt Stars from Weſt to Eaſt; that the Planets would ſometimes ſeem to go forwards, at another time backward, according to the ſituation of the Sun and each Planet. But ſince the Earth is an opake Body, and large enough, in reſpect of the Obſerver, that ſmall part of its Surface, tho' Spherical, which comes at the ſame time under his confin'd view, will ſeem to be extended like a Plane. And the Eye taking a view of the Heavens all around itſelf, definesa concave ſpherical Superficies, concentric to the Earth, or rather to the Eye; which the abovemention'd Plane of the Earth's Superficies, becauſe drawn thro' the Center, will divide into two equal Parts, one of which will be viſible, but the other, by reaſon of the Earth's Opacity, will lie conceal'd. Becauſe the Earth moves upon its own Axis, the Spectator ſtanding upon it, together with the ſaid Plane he ſtands upon, call'd his *Horizon*, dividing the viſible from the inviſible Hemiſphere of the Heavens, is carried round the ſame way, *viz.* to the Eaſt. From whence it is, that Objects ſituated towards the Eaſt, and inviſible, become viſible, the Horizon ſinking as it were below them; and Objects ſituated towards the Weſt, are hid and become inviſible, the Horizon being elevated above them. And the former Objects therefore will appear to the Spectator, who uſually reckons the Place he ſtands in, to be immoveable, to aſcend above the Horizon or riſe, the latter to deſcend below the Horizon or ſet.

Since the Earth and the Horizon of the Spectator fix'd to it, continues to move always to-

wards

wards the ſame Quarter and about the ſame Axis equably; all Bodies, and all Appearances that don't partake of that Motion (that is, all ſuch things as are entirely ſeparate from the Earth) will ſeem to move in the like time uniformly, but towards the oppoſite parts, *viz.* from Eaſt to Weſt, excepting ſuch things as the Earth's Axis produced will meet withal, they will appear at reſt. And every one of theſe Objects according to Senſe, will deſcribe the Circumference of a Circle, to whoſe Plane the Axis of the Earth is perpendicular at the Center. And becauſe all theſe Circles, together with the viſible Objects deſcribing them, appear to be in the concave Spherical Superficies of the Heavens, concentric to the Earth; every viſible Object will ſeem to deſcribe a greater or leſſer Circle, according to its greater or leſſer diſtance from the Points above deſcribed, that ſeem at reſt. In ſhort, the Celeſtial Sphere will ſeem to be moved from Eaſt to Weſt, in the Space of a Day; about the ſame Axe in Poſition, about which the Earth really moves in that time from Weſt to Eaſt: And thoſe Points will be the Poles of this Motion, in which the Axis of the Earth produced meets with its Superficies; and that will be the middle Circle betwixt the Poles, and conſequently the greateſt, (call'd the *Æquinoctial* and *Equator*, for reaſons to be explain'd in the next Propoſition,) where the Plane of that Circle of the Earth, which lies exactly between the Poles, being produced, meets the concave Sphere. And univerſally, any Circle on the Surface of the Earth, mark'd by any Point of it, in the diurnal Motion of the Earth, has a correſpondent Circle in the Celeſtial Sphere, made by the interſection of the

Surface

Surface of the Celeſtial Sphere with the Superficies of a Cone, whoſe Baſe is a Circle marked on the Earth, and Vertex the Center of the Earth. The Axis of the World, and the above-mention'd Celeſtial Circles, have always the ſame ſituation among the Fix'd Stars; becauſe their primitive Terreſtrial ones, from whence they have their original, produced ſo far, fall upon the ſame Stars, as to Senſe, in what Point ſoever of its Orbit the Earth be placed.

PROPOSITION XXXIII.

TO explain the Phænomena of the Sun ſeen from the Earth, ariſing from the Diurnal Motion of the Earth about its Axis, in conjunction with its Annual Motion about the Sun; that is, to explain the changes of Day and Night, and their variety in different Places, and different Seaſons of the Year.

Tho' each place of the Earth be inlighten'd by every Star that is above its Horizon, yet the illumination of the Sun being ſo very great above all others, its being above the Horizon alone is called *Day*; and its abode below the Horizon is called *Night*: And becauſe the Sun is intirely ſeparate and remote from the Earth, and ſo does not partake of its Diurnal Motion, it will Riſe and cauſe Day, in reſpect of a given place; and being carried thro' the midſt of the Heavens, deſcribing a Circle to appearance, having the Axis of the World perpendicular to its Plane, will at laſt Set again; making the Night to continue, till by the apparent univerſal Revolution of all the Celeſtial Bodies, it come up again above the Horizon of the ſaid Place.

To explain the variety of the Days and Nights, and the Seaſons of the Year thence ariſing, imagine the Circle ♈ ♋ ♎ ♑ [*Fig.* 28.] to repreſent the

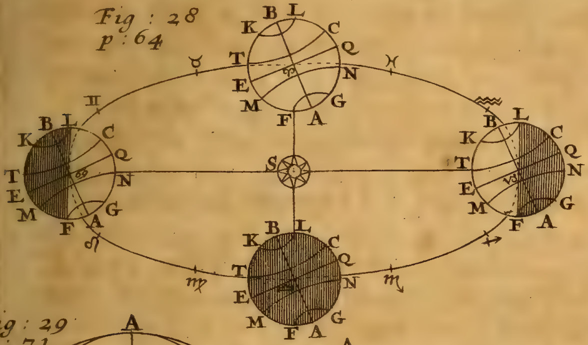
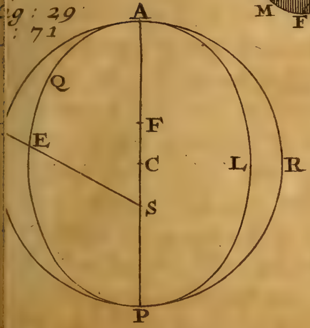


Fig : 28
p : 64



ig : 29
: 71

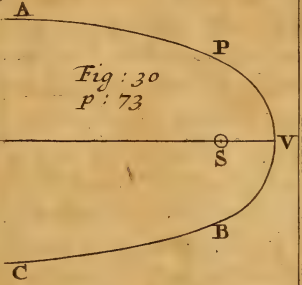


Fig : 30
p : 73

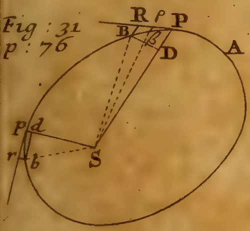


Fig : 31
p : 76

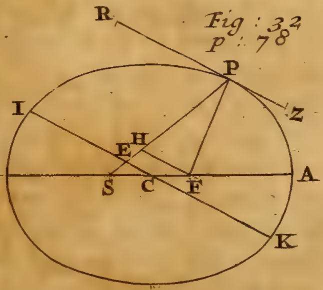


Fig : 32
p : 78

the Orbit deſcrib'd by the Earth, in a Year, about the Sun plac'd in the Center at *S*. The Circle in this Scheme is deſignedly reclined, and made to appear in the form of an Ellipſe, for the more commodious delineation of the various Poſitions of the Earth.

In this Orbit, let the Earth *A ♋ B E* be ſuppoſed to be carried after the manner deſcribed in *Prop.* 30. from ♈, ♉, to ♊, *&c.* revolving at the ſame time about its own Axis *B A*, towards the ſame quarter, or from ♋ upwards towards *E*. The extremities of the Axis of the Earth will be the Poles; *A* the Southern Pole, and *B* the Northern: *E ♋* the Equator, whoſe Plane is inclin'd to the Plane of the Ecliptic, by an Angle of 23½ Degrees; *viz.* the Complement of 66½ Degrees to a right Angle, by which the Axis of the Earth (perpendicular to the Plane of the Equator) is inclin'd to it.

As *A ♋ B E* repreſents the Earth, ſo it may be imagin'd to repreſent the Celeſtial Sphere, (deſcrib'd in the preced. Prop.) concentric to the Earth, and ſurrounding it as it were, and carried along with it; for this Sphere may be deſcrib'd at any diſtance. And the Place of the Sun or any Star or Celeſtial Point in this Sphere, will be there, where a Right line joining the Center of the Earth and the ſaid Point, meets its Surface, for there the Eye plac'd in the Center of the Earth refers it.

Let the Earth be ſuppos'd to be at ♎, where a Right line *S* ♎, joining the Sun and Earth's Centers, is perpendicular to the Axis of the Earth *A B*, that is, where the Sun appears in the Plane of the Earth's Equator produced, or where being ſeen from the Earth, it appears in the Æquinoctial Circle, marked in the Celeſtial Sphere;

and consequently declines neither to the North Pole, nor the South, but seems to describe, in its diurnal Motion, the Equinoctial it self.

The Sun always seems to be in the Plane of the Ecliptic, (as was shewn in *Prop.* 2.) it will therefore appear to be in the common intersection of the Circles of the Ecliptic and Æquator described in the Celestial Sphere; namely, in ♈. Now in this Situation, the illumination of the Earth, made by the Sun, reaches both the Poles *A* and *B*; (because the Boundary or Circle of illumination is a great Circle on the Earth, to whose Plane, the Right line joining the Centers of the Sun and Earth, is perpendicular) consequently half the Terrestrial Equator *E* ♋, or any other Circle parallel to it, is illuminated by the Sun, while the other half continues in darkness. And therefore every Plane of the Earth being carried round by the equable diurnal Motion, will be as long in darkness as it was in the light, that is, the Day, all over the Earth, will then be equal to the Night. From whence the Circle that the Sun then describes by its Diurnal Motion, lying between the two Poles in a Celestial Sphere, is called the *Æquinoctial Circle.*

The Earth moving a little forwarder towards ♏ and ♐ in its Annual Motion, has the Plane of its Equator *E* ♋ no longer directed towards the Sun; but subsiding below it towards the South: From whence the Sun seems by little and little to decline from the Celestial Equator, towards the North Pole. For the Earth being at rest to all appearance, its Equator must likewise be apparently at rest. Consequently the Celestial Equator corresponding to it, is moved only by the apparent Diurnal one: And there-

fore

fore the Sun, which changes its Situation to it, will ſeem to move. And the light of the Sun, which before reached to both Poles, *A* and *B*, will by degrees be ſpread beyond *B*, and fall ſhort of *A*. But when the Earth arrives at ♑ and the Sun hereupon appears in ♋, where it will ſeem to have its greateſt declination Northward, (equal to the inclination of the Ecliptic to the Equator,) and will afterwards return towards the Ecliptic. The Circle in the Celeſtial Sphere parallel to the Equator, and expreſs'd by *TC*, to which the Sun in its North declination ſeems to have arriv'd, and deſcribe by its Diurnal Motion at that time, is call'd the *Tropic of Cancer*; borrowing its name from the Sign of the Ecliptic the Sun is then in; with which name likewiſe the correſpondent Circle on the Earth is call'd. The Earth being in this Situation, 'tis evident that the rays of the Sun, inlightening half of it, will reach beyond the North Pole *B*, to *L*, and ſtop ſhort on this ſide the South Pole *A*, at *F*, ſo as that the Arc *B L* or *A F*, is equal to the Arc *ET*; the meaſure of the Sun's greateſt declination, or of the Inclination of the Ecliptic to the Equator. If Circles be imagined to be drawn thro' *L* and *F*, parallel to the Equator *E*♋, (namely *KL*, *FG*,) they will be the *Polar Circles*; the one the *Arctic*, and the other the *Antarctic*. Theſe things being ſuppos'd, 'tis plain that all that Tract of the Earth, which is included in the Arctic Polar Circle *VL*, notwithſtanding the Diurnal revolution, is all the while inlightened, and injoys therefore a continual Day; and on the contrary, that all within the compaſs of the Antarctic Circle *FG*, have continual Night or Darkneſs covering it. 'Tis further evident (from *Prop.* 19. *Book* 2. of the *Sphærics*

of Theodosius,) that the greater part of any Circle, parallel to the Equator *E ♎*, lying between it and the Arctic Circle *K L*, is inlightened; the greater part of any one lying between the Equator and the Antarctic Circle *FG*, is in darkneſs; and that ſo much the more (by *Prop.* 20. of that Book) as the Circle is farther diſtant from the Equator; and (by *Prop.* 11. *Book* 1.) one half of the Equator is always in darkneſs, and the other half always inlightened. And therefore in that Situation of the Earth, wherein the Sun appears in ♋, to the Inhabitants of the Northern parts of the Earth, the Days are longeſt, and Nights ſhorteſt; conſequently it is Summer: But to the Inhabitants of the Southern part, the Days are ſhorteſt, and Nights longeſt, conſequently 'tis Winter. And theſe longeſt Days are ſo much the longer, and ſhorteſt Nights ſo much the ſhorter, according as the Place is more Diſtance from the Equator: But to ſuch as inhabit the Equator, the Days and Nights are ſtill equal; and conſequently are ſo all the Year long.

The Earth going forward thro' ♒, ♓, to ♈; during which time, the Sun ſeems to move thro' the Signs ♋, ♌, and ♍, the Sun returns towards the Equator; till at laſt at ♎, the Sun appears in the Equator of the Heavens, (becauſe at that time, the Plane of the Earth's Equator produced does again paſs thro' the Sun,) making the Days equal to the Nights all over the Earth. And now again the extremities of illumination reach to both Poles; ſo that the Day under the Pole *B*, which has been all the while inlightened, is equal to the ſpace of half a Year, (the time the Earth has ſpent ſince its departure from ♎;) and the Night under the Southern Pole *A*, as long.

The

The Earth moving forwards thro' the Signs ♈, ♉, ♊, the Sun will ſeem in the mean while, to go thro' ♎, ♏ and ♐, and to decline gradually from the Equator towards the South; till at laſt, the Earth being really in ♋, and the Sun appearing to be at ♑, all the ſame Appearances will happen to the Inhabitants of the Southern Hemiſphere, as have happen'd to the Inhabitants of the Northern Hemiſphere, when the Earth was at ♑; and what happen'd to the Inhabitants of the Southern Hemiſphere before, will now happen to them of the Northern. And while the Earth is carried thro' ♋; ♌, and ♍, the Sun will ſeem to move thro' ♑, ♒, and ♓ to the Equator, and ♈, compleating the Year: And the Sun will cauſe the ſame Phænomena of Day and Night, the former decreaſing, the latter increaſing, to the Inhabitants of the Southern Hemiſphere, as it did to the Inhabitants of the Northern Hemiſphere, while it went thro' the three oppoſite Signs.

SECTION VI.

Of the Orbits of the Planets and Comets, and their Figure; together with the Law of Centripetal Forces, requiſite to make them move in ſuch Orbits.

PROPOSITION XXXIV.

Each Primary Planet deſcribes the Perimeter of an Ellipſe, having the Sun in one of its Focus's.

Having hitherto been treating of what was general, 'twas ſufficient to conſider the way of each

each Planet, as if it were the Circumference of a Circle, having the Sun for a Center, ſince it does not differ very much from a Circle. He that conſiders the Phænomena of the Planets mov'd about the Sun more cloſely, will find, that they are not every where equally diſtant from the Sun: For inſtance, the Earth is more diſtant from the Sun in Summer than in Winter; as is evident from the apparent Diameter of the Sun, which is greateſt about the middle of *June*, and leaſt about the middle of *December*. 'Tis further evident from the ſwifter Motion of the Earth about the Sun, in the midſt of Winter, than in the midſt of Summer, near a fifteenth Part. And for this reaſon it is, that there are about eight Days more from the Vernal Equinox to the Autumnal, the Summer Seaſon, than from the Autumnal Equinox to the Vernal, the Winter Seaſon; tho' in both caſes the Earth or Sun ſeems to have moved but a Semicircle. Since therefore the Earth moves ſwifter about the Sun in Winter than in Summer, it muſt then be nearer to the Sun, by *Prop.* 12. to make the Area, deſcribed by a Right-line drawn from the Sun, augment equably as the Time, the greateſt length of the Baſe compenſating for the leſſer Altitude of the Trilineal Figure. The ſame holds likewiſe in the Motion of the other Primary Planets about the Sun: Nay the Orbit of any of them (except Venus) is more different from a Circle, having the Sun for its Center, than the Orbit of the Earth, as ſhall be ſhewn hereafter.

This deviation of the Planets in their Motions about the Sun, from the Circumference of a Circle concentric with the Sun, is ſo great and ſenſible, that all Aſtronomers have been oblig'd to

ſuppoſe

ſuppoſe *Excentric Orbs:* And becauſe they thought it was paſt diſpute, that all Celeſtial Motion muſt be perfectly Circular, they imagined each Planet to be moved in a Circle, as *AIPR* [*Fig.* 29.] having its Center C, at ſuch a diſtance from the Sun *S*, as might agree with its Motions. The remoteſt Point *A*, of this Excentric Orb from the Sun, they call'd the *Aphelium*; *P*, the *Perihelium*; both the *Apſides*; the Right-line *A P*, connecting the Apſides, and in which they placed the Sun, the *Line of the Apſides*; the part of it *C S*, the *Excentricity*. And by the help of ſuch a *Theory* of the Planets as this ſuppoſe, the Places of the Planets found by Calculators, agreed exactly enough with their true ones in the Heavens; eſpecially of them whoſe Orbits were very little different from Circles; for inſtance, the Earth. But when they came to Mars, its Place, by computation from this Theory, was ſo different from its Place by obſervation, that they concluded its Orbit was not Circular, but ſomething depreſs'd in the Points *I* and *R*, which are remoteſt from the Apſides, and therefore, as it were, an Oval. This Oval the ſagacious *Kepler*, (in his *Commentaries upon Mars*,) after indefatigable pains in Calculations, found to be a perfect Ellipſe, having the Points *A* and *P* for its Vertices, which were the Apſides in the Circular Orbit, and ſtill retain the ſame Names, and the Center of the Sun for one of its Foci: Becauſe this Ellipſe alone exhibits, the Diſtances from the Sun, (or Right-lines *S E*) agreeable to obſervation, as it does the true Equation for determining the Angle *A S E*, from the mean Motion of the Sun given: That is, only the before mentioned Ellipſe is of that nature, as to have the Periodic Time, or intire Revolu-

 tion,

tion, in the ſame ratio to the Time ſince its departure from the Aphelium in *A*, as the intire Area *A E P L A*, is to the Area *A S E Q A*, intercepted between the line of the Apſides *S A*, and the line *S E* (known by obſervation) drawn from the Sun to the Planet: But that this is a circumſtance in the Motion of every Planet about the Sun, is evident from *Prop.* 13. Beſides, the Place of the Earth in an Elliptic Orbit is ſuch, as that the Places of the other Planets about the Earth, found in their reſpective Orbits by Calculation, and ſeen from the Earth, agree very exactly with their Places found by Obſervation in the Heavens; which happens ſo exactly in no other Orbit whatever. Since therefore the Places of the Planets, found by a Calculus accomodated to Elliptic Orbs, agree ſo exactly with their Places by Obſervation; but do not at all agree with a Calculus founded upon Orbs of any other Figure, but upon the account of their being ſomething near, or approaching to an Elliptic form; 'tis evident that the Orbits deſcribed by the Planets are Elliptic.

The Elliptic Orbits of the Primary Planets deſcribed about the Sun in the common Focus, are almoſt immovable; that is, when the Planets are carried to the Apſides of their Orbits, or any other given Points, they return to the ſame Points of the Mundane Space nearly: For ſuch kind of Orbits only, ſolve the obſerved Phænomena of the Places of the Planets among the Fixt Stars, ſeen from the Earth.

PROPOSITION XXXV.

A Comet moves in a Conic Section, that has the Sun in one of its Foci.

For ſuch Orbits, and ſuch only, agree with the

the Places of the Comets obſerved in the Heavens. 'Tis true that *Kepler*, and ſeveral Philoſophers after him, ſuppoſed the Trajectories of Comets to be Right lines; and have Calculated their Places upon that Hypotheſis, agreeing pretty exactly with their Places by Obſervation. But this might have been done, tho' the Comet were moved in a Conic Section, if the Obſervation were made only while it is in that part of its Orbit which is not much different from a Right line. Let *A P V B C* [*Fig.* 30.] be a very excentric Section of a Cone; that is, the Diſtance of whoſe Foci is almoſt equal to its Tranſverſe Axe: Let *S*, the Center of the Sun, be one of the Foci; it may be the Comet is obſerv'd while it is deſcribing the part *A P* of its Orbit, but during the reſt of the time, while it paſs'd from *P*, thro' *V*, *B*, and *C*, (where it diſappears, and goes off into very diſtant Regions,) it lies hid by the Rays of the Sun, in reſpect of the Obſerver upon the Earth, moved about the Sun *S*: Or it may be the Comet was hid during its Motion along *A P V B*, under the Beams of the Sun, (the Situation and Motion of the Earth allowing of it, and concurring;) and then at laſt is obſerved, when it is come to *B*, and is about to deſcribe the line *B C*; and in both caſes, the Path of a Comet differs not ſenſibly from a Right-line. In the former caſe, the Comets will ſeem to be ſwallow'd by the Sun; becauſe they were ſeen before their acceſs to the Sun, and afterwards were ſeen no more; and conſequently are look'd upon as deſtroy'd: But in the latter caſe, they will ſeem to ariſe out of the Sun, becauſe they were firſt ſeen in their aſcent from the Sun, towards the remote Regions. Beſides, when a Comet in its

deſcent

descent towards the Sun (as in *AP*, comes under our Observation, approaching afterwards to the Sun, and lying hid for some time in his Rays (namely, while it describes *PVB*,) and unexpectedly emerges on the other side of the Sun, with a Tail considerably increased, (for reasons afterwards to be assigned in their proper place;) it is commonly taken for a new Comet, and different from the former; the Trajectory of the one being the Right line *AP*, but of the other *BC*. Now the difference and seeming contrariety of their Trajectories, have deceived some Astronomers, who have asserted that it was not the same Comet that described them both. Nor is this Mistake easily corrected in a Comet that never returns to the same place, as we know of, by Observation. From all which, the reason is plain, why Rectilineal Trajectories, for the most part, agree with the Motion of Comets; namely, because only one portion of it, is taken for the whole Trajectory: But if the whole be considered together, as well in its descent towards the Sun, as in its ascent from it, no other line but a Conic Section would be found to agree with it.

Of the three Sections of a Cone, an Ellipse, having the Center of the Sun in one of its Foci, is most agreeable to the Motion of a Comet, as shall be demonstrated in its proper place: And if the Comets be durable Bodies, and return, none but an Ellipse will suit their Motion. But instead of a portion near the Vertex of a very excentric Ellipse, which the Motion of a Comet requires, a portion of a Parabola, having the same Vertex and Focus, may be substituted; because not sensibly differing from it. As the Parabolic Curve is its true Trajectory, if it never

ver return; or else perhaps an Hyperbolic one, if there be any such Comet, whose Motion an Hyperbolic Trajectory will better agree to, than a Parabolic.

PROPOSITION XXXVI.

Each Secondary Planet revolves about its Primary, in the Perimeter of an Ellipse, having the Center of the Primary in one of its Foci.

As for the Moon, the Earth's Secondary, it appears once, in its Revolution about the Earth, lesser, and at the same time slower; and in the opposite point greater, and swifter: 'Tis plain therefore, that its Orbit is excentric to the Earth. That it is an Ellipse having a Focus in the Earth's Center, may be proved by reasons like those by which (in the præced. *Prop.*) we demonstrated the Orbits of the Primary Planets about the Sun, to be Elliptic: namely, because assuming an Elliptic Orbit for the Moon, its distances from the Earth, come out agreeable to Observation, and the Æquations of its Motion, such as are not to be found from the Supposition of any other kind of Orbit. All this agrees likewise with the Orbits of the Satellites of Jupiter and Saturn; tho' they are nearly Circular and concentric with Jupiter and Saturn.

Tho' the Elliptic Orbits of the Primary Planets are at rest, as was said in the præceding Proposition, yet 'tis quite otherwise in the Secondary: For their Motions are so disturbed by causes to be explain'd hereafter, that the same Ellipses are not described again, abstracting from the Motion of the primary Planet. But to reduce their Orbits to Ellipses, we must imagine the Ellipse of each Secondary Planet, to be moved so, as that the Plane of the Ellipse, and

and Focus fixed in the Center of the Primary, continuing the ſame, the Axis of the Ellipſe is carried, *in conſequentia*, by an angular Motion, round the immoveable Focus. But this Motion is ſo ſlow, and inſenſible, (being in the Moon an hundred times ſlower than the Moon it ſelf,) that it may very well be neglected in this place; and the Ellipſes deſcribed by the Secondary about their Primary, may be look'd upon as unmoved, abſtracting from their Motion round the Sun in common with their Primary.

Proposition XXXVII.

If a Body be projected according to the direction of any Right line, and at the ſame time urged by a Centripetal Force tending to the Center S, [*Fig.* 31.] *ſo as by a Motion compounded of both, it deſcribes the Curve* APp; *and the Right line* PR *be a Tangent to it, in a point* P; *and from the point* B, *neareſt to* P, *a line* BD *be drawn perpendicular to the Right line* SP, *and the Right line* BR *parallel to the ſame line* SP; *and if the like conſtruction be made at any other Point* p, *of the Curve: I ſay the Centripetal Force in* P, *is to the Centripetal Force in* p, *as* $\frac{\mathrm{Sp}^q \times \mathrm{bd}^q}{br}$ *is to* $\frac{\mathrm{SP}^q \times \mathrm{BD}^q}{\mathrm{BR}}$: *Or that the Centripetal Force in* P, *is reciprocally as the Solid* $\frac{\mathrm{SP}^q \times \mathrm{BD}^q}{\mathrm{BR}}$, *when the Figure* PRBD *is indefinitely ſmall.*

Let the Force in *P*, that tends to the Center *S*, be called *V*. Let the Time wherein the indefinitely ſmall Arc *PB*, is run, or in which a Body, by its *Vis inſita* alone, would run the indefinitely ſmall Tangent *PR*, be called *T*. Let the Centripetal Force acting in *p* be called *v*: and

and the Time that $p\,b$, or the Tangent $p\,r$, is run in call t. Take an Arc $P\beta$, run in the same time with the Arc pb; and draw $\beta\rho$ parallel to PS. These things being supposed, 'tis evident that the ratio of the lineola BR to the lineola $b\,r$, is compounded of the ratio of BR to $\beta\rho$, and of the ratio of $\beta\rho$ to $b\,r$: But (by *Prop.* 24.) BR is to $\beta\rho$ in the duplicate ratio of the Arc PB to the Arc $P\beta$; and these Arcs, since they are indefinitely small, are as the Triangles BSP, βSP, of the same Altitude; that is, (by *Prop.* 12.) as the Times in which they are described by a radius drawn to S; or, by Construction, as the Times wherein the Arcs PB, $p\,b$, are described. Again, the lineola $\beta\rho$, is to the lineola $b\,r$, as the Causes producing them; that is, as the Centripetal Force in P, to the Centripetal Force in p. And therefore the ratio of the lineola BR to the lineola $b\,r$, is compounded of the duplicate ratio of the Times wherein PB and $p\,b$ are described, and of the ratio of the Centripetal Force in P to the Centripetal Force in p: That is, in Symbols, $\frac{BR}{br} = \frac{T^2}{t^2} + \frac{V}{v}$; or $\frac{BR}{br} = \frac{T^2 \times V}{t^2 \times v}$. And consequently $\frac{V}{v} = \frac{BR \times t^2}{br \times T^2}$ But (by *Prop.* 11.) T is to t, as the Area SBP to the Area $S\,b\,p$; or as its double $SP \times BD$ to $S\,p \times b\,d$ the double of the latter: Substituting therefore instead of the ratio $\frac{t^2}{T^2}$, its equal $\frac{Sp^q \times bd^q}{SP^q \times BD^q}$, and then $\frac{V}{v} = \frac{BR \times Sp^q \times bd^q}{br \times SP^q \times BD^q}$, or V to v, as $BR \times Sp^q \times bd^q$ to $br \times SP^q \times BD^q$; that is, (dividing them by $BR \times br$) as

as $\frac{Sp^q \times bd^q}{br}$ to $\frac{SP^q \times BD^q}{BR}$. The Force therefore in P tending towards S, is reciprocally as the nafcent or evanafcent Solid $\frac{SP^q \times BD^q}{BR}$. Which was to be demonftrated.

COROLLARY.

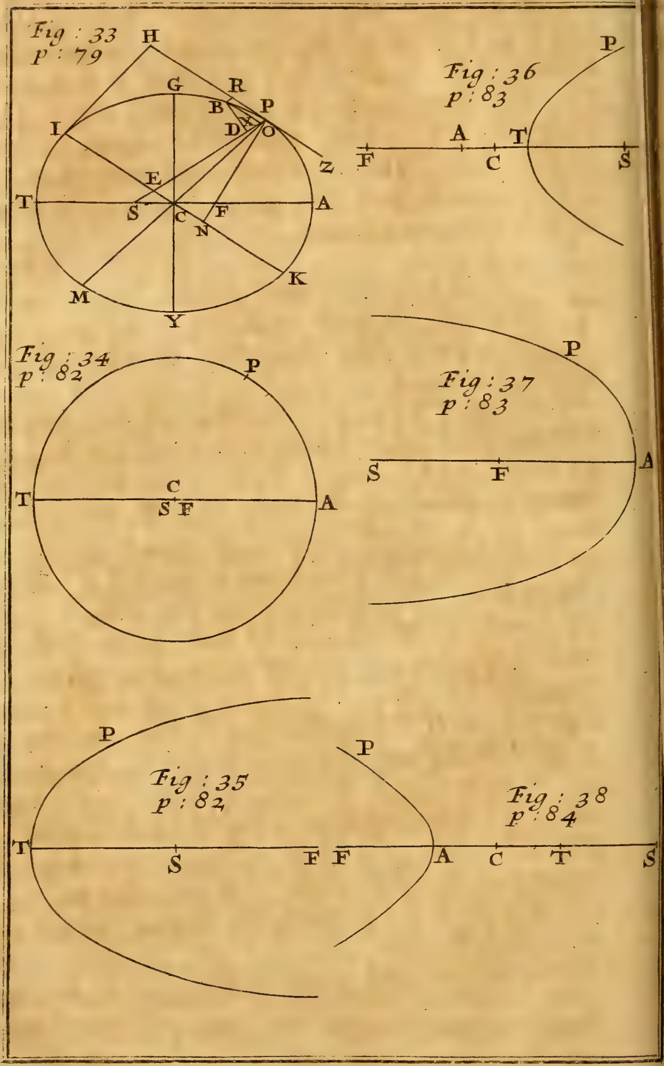
From whence, if any Figure (APp) be given, and in it a Point S, to which the Centripetal Force is directed; the Law of the Centripetal Force, whereby a Body drawn from its rectilineal courfe, fhall be made to keep in the Perimeter of that Figure, and defcribe it in its revolution, may be found; provided this Force be compounded of a proper one impreſs'd along the Tangent of the Figure. For the Law fought (by this *Prop.* 37.) is, that the Force be reciprocally as the nafcent Solid $\frac{SP^q \times BD^q}{BR}$, which is to be computed from the property of the Figure being given. A remarkable inftance of which, we fhall give, taken from nature.

PROPOSITION XXXVIII. LEMMA.

If a Right line ZPR [*Fig.* 32.] *touch the Ellipfe* API, *whofe Foci are* S *and* F, *in any Point* P, *and a Diameter as* IK *be drawn parallel to it, thro' the Center* C; *the portion* EP, *of the Right line* SP, *joining either of the Foci (as* S) *and the point of Contact, is equal to half the greater Axe* CA.

Thro' the other Focus F, draw FH parallel to PR, meeting SP in H. Becaufe (by *Prop.* 48. *lib.* 3. *El. Conic. Apollonius*) the Angles FPZ, HPR, are equal, and confequently their alternate ones PFH, PHF are equal; therefore PH is equal to PF. Again, (by 2. *Elem.* 6.) SE is equal to EH, becaufe

Plate 7 Book 1.



cause SC is equal to CF; and since SH is the difference of the Right lines PS, PH, it will likewise be the difference of the lines PS, PF; and EH, half this difference. And therefore PE, (made up of the lesser and half the difference) is equal to half the Sum of PS, and PF: But (by *Prop.* 52. *lib.* 3. *El. Conic.*) the Right lines SP, PF, taken together are equal to the greater Axe: Wherefore half their Sum (that is, EP) is equal to half the greater Axe CA. Which was to be demonstrated.

PROPOSITION XXXIX.

Let a Body revolve in an Ellipse; the Law of the Centripetal Force tending to the Focus of the Ellipse is required.

Let APM [*Fig.* 33.] be the Ellipse, whose Perimeter is described by the revolving Body; its Focus or Umbilical Point S, to which the Force is directed, whereby the Body drawn from its rectilineal Motion is retained in the Elliptic Orbit: required the Law of this Force.

Draw the conjugate Axes of the Ellipses TA, GY, crossing one another in the Center C. And thro' any Point P, in its Perimeter, draw a Tangent PZ, and a Diameter PM, and a conjugate to it ICK parallel to PZ, to which, from P, let fall the perpendicular PN. Compleat the Parallelogram $PCIH$, and the Right line IH will touch the Ellipse: (by 17. *Book* 1. *Conic.*) Join SP cutting IK in E. Thro' B, the nearest Point possible to P, draw BR parallel to PS, and BX to PZ, meeting the Right line SP in X, and MP in O.

The centripetal Force tending to S, is (by *Prop.* 37.) reciprocally as the solid $\frac{SPq \times BDq}{BR}$.

Which

Which therefore must be computed from the nature of the Ellipse. In order to which, let the *Principal Parameter* of the Ellipse, or that which belongs to the greater Axe TA, be call'd L. The Ratio of $L \times BR$ to BDq is compounded of the Ratios of $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BOq, BOq to BXq, and BXq to BDq: But the Ratio of $L \times BR$ to $L \times PO$, or (by *Prop.* 1. *El.* 6.) of BR or PX to PO, is equal to the Ratio of PE to PC, (by *Prop.* 2. *El.* 6.) because XO is parallel to EC; and (by præced.) PE is equal to AC: Therefore the Ratio of $L \times BR$ to $L \times PO$ is equal to the Ratio of AC to PC. The Ratio likewise of $L \times PO$ to $MO \times PO$ is the same with the Ratio of L to MO. And the Ratio of $MO \times PO$, or the rectangle MOP to BOq, is the same (by *Prop.* 21. *B.* 1. *Conic.*) with the Ratio of the rectangle MCP (that is, CPq) to ICq. And in the present case, the Ratio of BOq to BXq is the Ratio of equality; for the Point B coinciding with the Point P, BO becomes equal to BX. Lastly, the Ratio of BXq to BDq is equal to the Ratio of PEq to PNq: (For the Triangles BDX, PNE are equiangular, the Angles at D and N being right; and BXD, PEN, being alternate, and therefore equal, because of the two Parallels BX, EN.) And PEq is equal to CAq, because the lines themselves are equal (by the præceding Proposition.) Therefore the Ratio of BXq to BDq is the same with the Ratio of CAq to PNq. Besides, (by *Prop.* 72. *Book* 4. of *Gregory St. Vincent*,) the rectangle under GC and CA is equal to the Parallelogram $PCIH$, that is, to the rectangle under IC, PN: Wherefore (by 16. *El.* 6.) CA is to PN as IC to GC; therefore CAq

is

is to PN^q as IC^q to GC^q: The ratio therefore of BX^q to BD^q, is the ſame with the ratio of IC^q to GC^q. Since therefore the ratio of $L \times BR$ to BD^q is compounded of the ratios of $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BO^q, BO^q to BX^q, and BX^q to BD^q; 'tis compounded alſo of ratios reſpectively equal to theſe; namely, of the ratios of AC to PC, L to MO, CP^q to IC^q, and IC^q to GC^q; and therefote will be equal to the ratio of $AC \times L \times PC^q$ to $PC \times MO \times GC^q$: But (by *Prop.* 13. *Book* 1. *Conic.*) $AC \times L = 2GC^q$. Subſtituting therefore the one in the room of the other, the ratio of $L \times BR$ to BD^q, is equal to the ratio of $2GC^q \times PC^q$ to $PC \times MO \times GC^q$; that is, to the ratio $2PC$ to MO: But (in the preſent caſe, ſince the point B is the neareſt poſſible to P,) the point O muſt be the leaſt diſtance poſſible from the point P; that is, MO is equal to $2PC$; and therefore, in this caſe, $L \times BR$ is equal to BD^q: Becauſe therefore, in an Ellipſe, $BD^q = L \times BR$, 'tis $\frac{SP^q \times BD^q}{BR} = \left(\frac{SP^q \times L \times BR}{BR}\right) = SP^q \times L$: The Centripetal Force therefore is reciprocally proportional to this Solid; and L in it is a conſtant invariable Right line.

The Law therefore of the Centripetal Force tending to the Focus of an Ellipſe, (whereby a Body, turn'd off perpetually from its Rectilineal courſe, is kept in the perimeter of that Figure, and deſcribes it in its Motion,) is, that it is reciprocally proportional to the Square of the diſtance of the Place P from the Center S. Which was to be found.

COROLLARY 1.

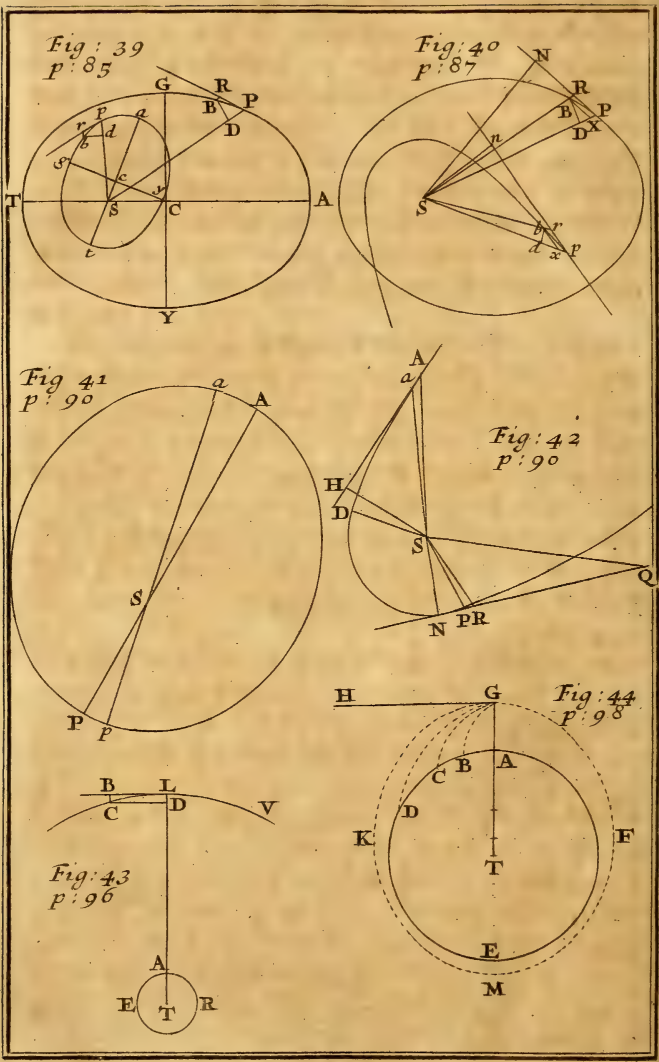
If the Focus *S*, of the Ellipse, to which the Force tends, together with the next Vertex *T*, continue as before; [*Fig.*34.] but the Focus *F*, approach it, and at last coincide with it; the Body *P*, will now be kept revolving in the Perimeter of a Circle, by the Centripetal Force tending to *S*, after the same manner, and by the same Law, as it was in the Perimeter of the Ellipse. And as a Circle may be described by a Body, acted upon by any Centripetal Force tending to a point situated without the Right line, according to the direction of which it would have moved by its *Vis insita* alone, (as was said in *Prop.* 29.) so a Right line will be described by a Body that is urged by no Force tending to a point situated without the said Right line: For if the two Vertices continue the same, and the Foci approach to them, the Ellipse will be turned into a Right line. But to describe any other Lines, besides a Right one and a Circular, a particular Law of the Centripetal Force is necessary.

COROLLARY 2.

If the Focus *S*, [*Fig.* 35.] of the Ellipse, to which, as a Center, the Force be directed, together with the next Vertex *T*, remain as before, and the other Focus *F*, go farther off *in infinitum*, the Body *P* will be kept in the Perimeter of the Figure *T P*, by the Centripeaal Force tending to *S*, after the same manner as it was before, and by the same Law. But in this case, the Ellipse will be turn'd into a Parabola: The Law therefore of a Centripetal Force tending to the Focus of a Parabola, retaining a Body in the Perimeter of such a Figure, is this, that it is reciprocally proportional to the Square of the Distance from the Focus.

CO-

Plate 8 Book 1.



COROLLARY 3.

If, all the reſt continuing as before, the Focus *F*, [*Fig.* 36.] go off to a more than infinite Diſtance; (that is, if it appear on the other ſide of *T*, and make the Vertices *A* and *T* lie between the Foci *S* and *F*; whereas before, the Foci *S* and *F* were between the Vertices *A* and *T*,) in which caſe, the Ellipſe is turn'd into an Hyperbola; the Body acted upon by the ſame Law of the Centripetal Force as before, (that is, ſo as to increaſe, as the Square of the Diſtance from the Center decreaſes,) will then move in the Perimeter of this Hyperbola.

COROLLARY 4.

But if the Focus *F*, [*Fig.* 37.] of the former Ellipſe, and the Vertex *A* continue fixt, and in the mean while the other Focus *S*, to which the Force is directed, goes off further and further, to an infinite Diſtance, (that is, if the Ellipſe be turned into a Parabola;) the Centripetal Force directed to the Point *S* at an infinite Diſtance, or according to Right lines parallel to the Axe *AF*, by which the Body moves in its Perimeter, will become equable: For Diſtances from an infinitely diſtant Center, whoſe intervals are finite, are equal. And therefore the Force, which is reciprocally as the Square of that Diſtance, is equable. And converſely, a Body acted upon by an equable Force, tending to a Center infinitely diſtant, and which conſequently is directed according to Right lines parallel to a given one, will deſcribe in its Motion a Parabola, whoſe Axis is parallel to the given Right line. And this is the famous Theorem of *Galileo*, concerning the path or tract of a Projectile.

COROLLARY 5.

All things continuing as before, if the Focus *S*, (or Center to which the Force is directed) go off to more than an infinite Diſtance, as in *Corol.* 3. it happen'd to the Focus *F*, then the Ellipſe will be turned into an Hyperbola, [*Fig.*38.] and the Body made to move in the Perimeter of the Figure thus changed, by a Force acting according to the direction of the Right line *S P*, that is, reciprocally proportional to the Square of the Diſtance of the Place from the Center *S*: and that conſequently is a Centrifugal Force, in regard of the Point *S*, becauſe the Curve *A P*, is convex towards *A*.

As an Ellipſe is deſcribed by a Body urged by a Force tending to either of the Foci (becauſe within it,) and acting according to the Law of the preceding Propoſition; and an Hyperbola, while the Centripetal Force of the ſame Law tends to a Focus within it, or while the ſaid Centripetal Force is changed into a Centrifugal one, tending to a Focus, without the Curve: So a Parabola, which, as it were a mean betwixt theſe two Sections, partakes of the nature of both; and is deſcribed by the action of a Centripetal Force of the ſame Law, tending to a Focus within the Curve, (as in both the preceding,) and by the action of a Centripetal Force tending to one Focus, or a Centrifugal Force tending from the other, (after the manner of an Ellipſe and Hyperbola reſpectively,) according as that other Focus is ſuppos'd to be placed towards the convex or concave parts of the Curve: For both ways it may be conſidered as infinitely diſtant.

COROLLARY 6.

From this Propoſition and its Corollaries, it follows, that if any Body be projected from any Point

Point P, according to the direction of any Right line PR, with any Velocity, and be acted upon at the same time by a Centripetal Force, which is reciprocally proportional to the Square of the distance of the Place from the Center S, situated without the abovemention'd Right line; it will be moved in one of the Conic Sections, having its Focus, as the Center to which the Force is directed. For the Ellipse of the former Proposition can be changed into no other Line but a Right line and a Conic Section; (by the motion of either Focus in the Axe produced if need be.) And the principal Parameter of this Section or L, by the demonstration of the Proposition, (where it was shewn that $L \times BR = BDq$,) is a third proportional to BR and BD considered as nascent.

PROPOSITION XL.

IF several Bodies revolve about a common Center, and the Centripetal Force be reciprocally as the Square of the Distance from the Center; I say the Squares of the Periodic Times in Ellipses are as the Cubes of the Transverse Axes.

Let there be any two Orbits APT, apt, about a common Focus S, (*fig.* 39.) in which the former Construction is suppos'd to continue. Their Parameters L and l, (by Corol. 6. of the preceding Proposition) are respectively equal to $\frac{BDq}{RB}$, $\frac{bdq}{rb}$; the Points B and P, likewise b and p, coinciding. But RB is to rb (being generated in the same time) as the Centripetal Force in P, to the Centripetal Force in p; that is, by supposition, as Spq to SPq: And therefore $L : l :: \left(\frac{BDq}{Spq} : \frac{bdq}{SPq} ::\right) BDq \times SPq :$

 bdq

$bd^q \times Sp^q$. And $BD \times SP$ is to $bd \times Sp$, as their halves; that is, as the Areas described at the same time by the Bodies P and p: And therefore L and l are as the Squares of the Areas, that are described in the same time, by Radii drawn to the common Center S; that is, these Areas, described at the same time, are in the subduplicate ratio of the Parameters. And since the intire Areas of the Ellipses are as any lesser parts of them, described at the same time, drawn into the respective Periodic Times; the intire Areas will be to one another, in a ratio compounded of the subduplicate of the Parameters, and simple ratio of the Periodic Times.

Since (by *Prop.* 191. *Book* 4. of *Gregory St. Vincent*) the Ellipse $AGTY$ is to $agty$, as $AT \times GY$ is to $at \times gy$: And therefore the rectangles under the Axes of the Ellipses, are in a ratio compounded of the subduplicate ratio of the Parameters, and of the simple ratio of the Periodic Times; that is, as $L^{\frac{1}{2}} \times$ Periodic Time in APT to $l^{\frac{1}{2}} \times$ Periodic Time in apt. And since from the nature of an Ellipse, GY is a Geometric mean between L and AT, and gy betwixt l and at; or $GY = L^{\frac{1}{2}} \times AT^{\frac{1}{2}}$, and $gy = l^{\frac{1}{2}} \times at^{\frac{1}{2}}$; Therefore $L^{\frac{1}{2}} \times AT^{\frac{3}{2}} : l^{\frac{1}{2}} \times at^{\frac{3}{2}} :: (GY \times AT : gy \times at ::) L^{\frac{1}{2}} \times$ Periodic Time in $APT : l^{\frac{1}{2}} \times$ Periodic Time in apt. Dividing therefore the Antecedents by $L^{\frac{1}{2}}$ and the Consequents by $l^{\frac{1}{2}}$, and squaring the Quotients you will have the Cube of AT, to the Cube of at, as the Square of the Periodic Time in the Ellipse APT, to the Square of the Periodic Time in the Ellipse apt. Which was to be demonstrated.

COROLLARY I.

Because a Circle is a Species of an Ellipse; where both the Foci coincide with the Center, and the greater Axe is equal to any Diameter; 'tis

'tis manifeſt that the Squares of the Periodic Times of Bodies revolving in an Ellipſe and Circle about the ſame Center, are as the Cubes of the Tranſverſe Axes: And therefore, if the Axes and their Cubes are equal, the Squares of the Periodic Times, and conſequently the Periodic Times themſelves, will be equal: that is, when the Center of the Forces is the ſame, the Periodic Times in Ellipſes, will be the ſame with thoſe in Circles, whoſe Diameters are equal to the greater Axes of the Ellipſes.

COROLLARY. 2.

'Tis evident from the Demonſtration of the Propoſition, that the Parameters of any Orbits, deſcribed by Bodies revolving about a common Center, and acted upon by a Centripetal Force, which is reciprocally as the Squares of the Diſtances from the Center, are as the Squares of the Areas deſcribed in the ſame time, by Radii drawn to the Center: For this was demonſtrated univerſally of any Orbits, whether Elliptic, Parabolic, or Hyperbolic, before the reaſoning was confin'd to Ellipſes, in which only, the Periodic Time has any place.

PROPOSITION XLI.

THE *Law of the Centripetal Force being as before; I ſay the Velocities of any Bodies revolving about the ſame Center, in any points whatever, are in a ratio compounded of the ſubduplicate ratio of the principal Parameters belonging to the Lines they deſcribe, and of the reciprocal ratio of the perpendiculars let fall from the common Center upon Right lines that are Tangents at the points where the Bodies are.*

Let the Bodies *P* and *p* [*Fig.* 40.] revolve about the common Center *S*, in any kind of Lines *PB*, *pb*. Becauſe the Bodies, by ſuppoſi-

 tion,

tion, are acted upon by a Centripetal Force, reciprocally proportional to the Square of the Distance of the Place from the Center; these Lines will (by *Corol.* 6. *Prop.* 39.) be Conic Sections, having S for a Focus. Let their principal Parameters be call'd L and l. From S, let fall SN, sn, perpendicular upon the Right lines PR, pr, Tangents in P and p. I say the Velocity of the Body P, is to the Velocity of the Body p, in a ratio compounded of the ratio of $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$, and of the ratio of Sn to SN; or as $L^{\frac{1}{2}} \times Sn$ to $l^{\frac{1}{2}} \times SN$.

Take the Arcs PB, pb, described in the same infinitely small time; and thro' B draw BX parallel to RP, and BD perpendicular to SP; let the same be done at b. The Velocities in P and p are as the nascent Arcs PB, pb; because they are the Spaces run in the same time; that is, as the Lines PR and pr equal to them. The right angled Triangles BDX, SNP, having the Angles BXD, SPN equal, because BX, NP are parallel; so that $SN : SP :: BD : BX$; and therefore BX, or what is equal to it $PR = \frac{SP \times BD}{SN}$. And $pr = \frac{Sp \times bd}{Sn}$, because of similar ratios. But $SP \times BD$, $Sp \times bd$ are as their halves, that is, SPB, Spb; or the Areas described at the same time by the Bodies P and p, which again (by *Cor.* 2. *Prop.* preced.) are as the Square Roots of the Parameters L and l; or as $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$. And therefore PR is to pr, or the Velocity of the Body in P, is to the Velocity of the Body in p, as $\frac{L^{\frac{1}{2}}}{SN}$ to $\frac{l^{\frac{1}{2}}}{Sn}$, or as $L^{\frac{1}{2}} \times Sn$, to $l^{\frac{1}{2}} \times SN$: That is, in a ratio compounded of the subduplicate ratio of the Parameters to the Figures PB, pb, and of the reciprocal ratio of the

the Perpendiculars, let fall from the Center of the Forces *S*, upon the Tangents. Which was to be demonſtrated.

COROLLARY 1.

Hence it follows, that the Velocities of the Bodies, in their greateſt and leaſt Diſtances from their common Focus, about which they revolve, are in a ratio compounded of the ſubduplicate ratio of the Parameters, and of the reciprocal ratio of the Diſtances from the common Focus. For, in their greateſt and leaſt Diſtances from the Focus, the Perpendicular to the Tangent is the Diſtance it ſelf. If theſe Diſtances from the Center are equal, the Velocities, in that moment of time, are in the ſubduplicate ratio of the Parameters; that other ratio of the diſtances becoming a ratio of equality. And if one of the Figures be a Circle, the Velocity in the Conic Section, in the greateſt or leaſt diſtance from the Focus, will be to the Velocity in the Circle, as the Square Root of the Parameter of the Conic Section is to the Square of the Diſtance.

COROLLARY 2.

In the ſame or different Figures, having equal Parameters, the Velocity of the Body is reciprocally as the Perpendicular let fall from Focus, or Center of the Forces, on the Tangent. For the ſubduplicate ratio of the Parameters, in this caſe, is a ratio of equality. And therefore the Velocities of a Body revolving in an Ellipſe, in the greateſt and leaſt diſtances from the Focus, are reciprocally as the diſtances. For, in this caſe, the diſtances are the perpendiculars to the Tangents of the Orbits.

CO-

COROLLARY 3.

The apparent Motions of a Body in the principal Vertices A and P, [*Fig.* 41.] viewed from the Center of the Forces S, are reciprocally as the Squares of the Diſtances $S A$, $S P$. Take at the Vertices A and P, the indefinitely ſmall Arcs $A a$, $P p$, deſcribed in equal indefinitely ſmall Times, by a Body revolving about S; and join $S a$, $S p$. The apparent Motions of the Body from S, are the Angles $A S a$, $P S p$. The ratio of theſe Angles $A S a$, $P S p$, is compounded of the Ratio of $A a$ to $P p$ (ſince theſe Arcs do not differ from Circular Arcs deſcribed upon the Center S,) and of the ratio of $S P$ to $S A$; or the reciprocal of the Radii. But becauſe $A a$, $P p$ are deſcribed in equal Times, they will be as the Velocities in the points A and P, which (by *Corol.* preced.) are as $S P$ to $S A$. And therefore $A S a$ is to $P S p$, as $S P^q$ to $S A^q$.

COROLLARY 4.

Hence it follows, that, if the Orbit be very nearly a Circle, the apparent Motions of the Body viewed from the Center S, are very nearly reciprocally as the Squares of the Diſtances from it. For, in this caſe, all the Right lines that fall upon the Orbit from S, are nearly perpendicular to it.

COROLLARY 5.

From *Corol.* 2. it follows, that if a Body, moved in any Conic Section $A D S$ [*Fig.* 42.] as before, ſhould leave the Curve, and go on uniformly in a Right line $P Q$, touching the Curve in P, with the Velocity it had in the point P; the Area $S P Q$, which it would deſcribe by a radius drawn to S, would be equal to the Area $S A D$, which (in the ſame or equal time,) it would deſcribe, being kept ſtill moving

in

in the Conic Section. Take the lineola *A a*, *P R*, described in the same indefinitely small time, by two Bodies; and from the Focus *S* of the Section, to the Tangents *A H* and *P Q*, let fall the Perpendiculars *S H*, *S N*. By *Corol.* 2. the Velocity in *A* is to the Velocity in *P*, as *S N* to *S H*. But as the Velocities in *A* and *P* are, so are the Spaces run in the same time, by the Bodies; namely, *A a* and *P R*. And therefore *A a* is to *P R*, as *S N* to *S H*. Therefore the Triangle *S A a* is equal to the Triangle *S P R*. And since this holds in the indefinitely small Triangles that constitute the Trilineal Figures *S A D*, and *S P Q*, and the Spaces *S A D*, *S P Q* are made up of an equal number of Triangles, the Times of running *A D* and *P Q* being taken equal; the Triangle *S P Q* is equal to the Area *S A D*.

PROPOSITION XLII.

THE Primary Planets and Comets moves about the Sun according to this Law, that the Motion of each is compounded of an equable one along the Tangent to the Orbit, and another tending towards the Center of the Sun, in which the accelerating Force is reciprocally proportional to the Square of the distance from that Center.

Each of the Primary Planets (by *Prop.* 13.) is acted upon by a Force tending toward the Center of the Sun; and (by *Prop.* 34.) describes the Perimeter of an Ellipse, having the Sun in its Focus; and all this is true also of any Comet, (by Propositions 14 and 35;) or at least it describes some other Conic Section having the same Focus, seeing it is acted upon by a Force tending towards the Sun. And therefore (by *Prop.* 39. and its *Corollaries*) every Comet is acted upon by an equable Force impress'd along a

Tangent

Tangent to the Orbit, and another whereby it is drawn off from the Tangent and kept in a Curvilineal Orbit, which tends towards the Sun's Center, placed in the Focus of the Section, and is reciprocally proportional to the Square of the distance from it. Which was to be demonstrated.

The Planets and Comets therefore are to be conceiv'd as so many Projectiles, which are acted upon by two Forces. For in this manner, and this only, do they describe Orbits, such as Observation finds them to be. And the Law of the Force that draws off these Projectiles from their Rectilineal Motion, is, that it increases as the Square of the distance of the Comet or Planet projected, from that Center of the Sun, increases: Which is the Law whereby the Planets tend to the Sun, as we have demonstrated, otherwise than from the Figure of its Path, in *Prop.* 28.

PROPOSITION XLIII.

THE Nodes and Apsides of the Orbits and Planets are at rest.

For, by *Prop.* 40. every Planet revolves in an immoveable Plane, and the common Intersection of any two Planes continues immoveable: But the Intersection of the Plane of any Planet, with the Plane of the Earth's Orbit, is the Line of the Nodes of that Planet. The Nodes therefore of that, and in like manner of all the Planets, are at rest. Besides, every Planet describes, in a Plane that is at rest, always the same Ellipse, which therefore is at rest, (by *Prop.* 34.) and consequently its Apsides are at rest. Which was to be demonstrated.

COROL-

COROLLARY.

Since the office of every Fixt Star is like that of the Sun, (as far as Men are capable of judging;) namely, to have ſeveral leſſer Bodies like ſo many Planets revolving round its vaſt Body; its other affections likewiſe will be ſimilar; as, that its Motion, like the Sun's, will be none or inſenſible; which is farther evident from the conſtant and always the ſame mutual Diſtance of the Fixt Stars. Conſequently the Situation of the Nodes and of the Apſides of the Planerary Orbits, in reſpect to the Fixed Stars, continues the ſame.

SCHOLIUM.

This Propoſition is true, if the Centripetal Force of the Planets towards the Sun be only conſidered, as has hitherto been done: But if the mutual Actions of the Planets and Comets upon one another, be taken into the conſideration, things will be a little otherwiſe; as ſhall be ſhewn in its proper place. But the Effects of theſe Actions, by reaſon of the ſmallneſs of the Forces that produce them, are very ſmall, and as it were none at all, and therefore to be neglected in this place. Nay, the Motions of the Nodes and Apſides of the Primary Planets are ſo ſmall, that no notice at all have been taken of them by very conſiderable Aſtronomers, in their Calculations, and the Nodes and Apſides look'd upon to be at reſt.

PROPOSITION XLIV.

THE *Planets and Comets move in the Heavens with the greateſt Freedom; and conſequently their Motion may be preſerved for a very long ſpace of time.*

For, in *Prop.* 39. (where we determined the Law of the Centripetal Force, whereby a Body is

is urged, that revolves in the Perimeter of an Ellipſe, or any other Conic Section, having the Center to which the Force tends for a Focus,) we ſuppoſed both the Motions, of which the Motion in an Elliptic, Parabolic, or Hyperbolic Curve is compounded, to be perfectly free, and having nothing to hinder or reſiſt it. Since therefore the Planets deſcribe in their Motion, perfect Ellipſes, whoſe common Focus is the Sun, and continue their revolutions in the ſame Ellipſes; 'tis manifeſt from the converſe, that the Motions of the Planets are intirely free from all reſiſtance; that is, that the Planets move in the Heavens very freely. And if the Planets move freely in the Heavens, without any reſiſtance, the Comets will do ſo too; becauſe they are moved along the regions of the Planets, during their being viſible to the Inhabitants of the Earth, in Orbits that are either true Ellipſes or ſpecies of Ellipſes; namely, Parabola's or Hyperbola's, having the Sun in the Focus; to the deſcription of which the ſame freedom from all reſiſtance is neceſſary. And that the reſiſtance of the medium in the regions of the Solar Syſteme, beyond the Planets, is not greater than in the Planetary regions, is very probable. For if there were the leaſt ſenſible reſiſtance in the Heavens, the ſame Orbits could not be deſcribed by the Planets carried about the Sun, like ſo many Projectiles, as have been demonſtrated to agree to them, moving freely: But, juſt as in our Air, the way of a Projectile is found to be very different from a Parabola, which it would deſcribe, if there were no reſiſtance; ſo the ways of the Planets at leaſt after ſeveral revolutions, would be found to be very different from Ellipſes; and the Ellipſes the ſame, for Magnitude and Poſition, with ſuch as are to be

de-

described by Planets, moved in free Spaces, would not so exactly agree with the Places observed in former times, as well as in the present Age, as we find by a due Calculation they do. Consequently there is no resistance at all: But the Planets and Comets move in the Heavens very freely, and therefore their Motion may be continued, so as to endure for a very long time. Which was to be demonstrated.

PROPOSITION XLV.

THE Secondary Planets move about the Primary according to this Law, that besides the accelerating Force of the Primary, the Motion of each is compounded of an equable one, along the Tangent to the Orbit, and of another tending to the Center of the Primary, in which the accelerative Force is reciprocally proportional to the Square of the distance from the Center.

Laying aside all the accelerating Force whereby the Primary is urged, (as was done in *Prop.* 20.) each Secondary Planet (by *Prop.* 37.) revolves about its Primary in the Perimeter of an Ellipse, having a Focus in the Center of the Primary, to which (by *Prop.* 20.) the Centripetal Force tends, by which it is drawn off from its rectilineal Motion: (For the deformity of the Ellipse arising from the Motion of the Apsides, or from any thing else, is so small that it may well be neglected in this place:) And therefore (by *Prop.* 32.) the Law of the Centripetal Force tending to the Center of the Primary, is, that it is reciprocally proportional to the Square of the distance from the Center: For by this Force only is a Body drawn off from a Rectilineal Motion in a Tangent, arising from its *Vis insita*, and kept in an Elliptic Orbit. But these two Motions (the one equable in the Tangent, the other

other tending towards the Center of the Primary, and increasing as the Square of the distance of the Satellite from it lessens,) being compounded, make the Secondary Planet to describe an Elliptic Orbit like a Projectile.

SECTION VII.

The Planets and Comets are kept in their Orbits by Gravity, which is propagated thro' the whole Solar Systeme according to the same Law.

PROPOSITION XLVI.

THE *Force whereby the Moon tends to the Center of the Earth, is the same with the Force of Gravity, whereby all Bodies about the Earth tend to the same Center.*

Let *R A E* [*Fig.* 43.] represent the Earth, *T* its Center, *V L* the Orbit of the Moon, *L C*, a part of it run by the Moon in the space of a Minute. Because (by *Prop.* 15.) the Moon finishes its Course or whole Circle in 27 days, 7 hours, 43 minutes; that is, 39343 minutes of Time; therefore *L C* is $\frac{1}{39343}$ of the whole periphery, or 33 Seconds of a degree. Now the Circumference of the Earth (according to the late mensuration of Mr. *Picart*) is 123249600 *Paris* Feet; consequently its Semidiameter *TA* is 19615800 Feet: wherefore *TL* the Semidiameter of the Moon's Orbit is 1176948000 Feet; or sixty times *TA*, (by *Prop.* 15.) And the versed Sine *L D* of the Arc *L C*, of 33″, and *B C*, which is equal to it, is 15 $\frac{1}{12}$ Feet, nearly. The Force therefore, whereby the Moon tends toward the Center of the Earth (as by which it is drawn

drawn off from its Rectilineal Motion along the Tangent *LB*, and retained in its Orbit,) is ſuch, that it impels the Moon towards the Center of the Earth, about $15\frac{1}{12}$ *Paris* Feet in a Minute. By the preceding Prop. this Force in the Moon is increas'd, as the Square of the diſtance from the Center of the Earth is leſſen'd; if therefore it deſcend to the Surface of the Earth, where the Square of the diſtance from the Center is diminiſhed 60×60 times, it will be increaſed 60×60 times, and therefore 'twould impel the Moon placed on the Surface of the Earth, $60 \times 60 \times 15\frac{1}{2}$ Feet, towards the Center in a minutes time. But Gravity is that Force, which impels any Body placed on the Surface of the Earth, the length of $15\frac{1}{2}$ *Paris* Feet towards the Center, in the ſpace of a ſecond of time, as Mr. *Hugens* has determined by very accurate Experiments, in *Prop.* 25. *Part* 4. of his *Horolog. Oſcil.* Therefore the ſame Force of Gravity, in the ſpace of a minute, impells a Body $60 \times 60 \times 15\frac{1}{12}$ Feet towards the Center; becauſe the Spaces run by a heavy Body, in its fall, are as the Squares of the Times. Since therefore the Force, whereby the Moon is kept in its own Orbit, and the Force of Gravity, perform the like and equal things, and would produce the very ſame Effects, in the ſame Circumſtances, and tend toward the ſame point, (namely, the Center of the Earth,) they are the ſame Forces; that is, the Force whereby the Moon is drawn off from its Rectilineal Motion, and kept in its Orbit, is the ſame Force with that we call Gravity. Which was to be demonſtrated.

A diſtance of the Moon, a little greater than ſixty times the Semidiameter of the Earth, would fit the preceding Calculation better, as $60\frac{1}{3}$. Which number likewiſe is more agreeable

to Aſtronomical accounts: Though we have thought it ſufficient to give the meaſure in round Numbers.

SCHOLIUM.

After the ſame manner as the Moon revolves about the Earth, any other heavy Body projected with a Force ſtrong enough, from a given Point without the Surface of the Earth, according to the direction of an Horizontal Right line, would deſcribe an Orbit, and compleat a revolution, like a Planet, without touching the Earth; and that continually. Let the Circle *ABCD* [*Fig.* 44.] repreſent the Earth, *T* its Center. Take any Point without the Surface of the Earth, as *G*, from which, let any heavy Body be projected according to the direction of the Line *GH*, perpendicular to the Line *TG*. 'Tis evident, that if the Body be projected with little or no Force, (that is, if it be let fall freely) it will fall down to the Point *A*, directly under *G*; but if with ſome Force, it will reach the Earth at the Point *B*, diſtant from *A*, towards *H*; if with a greater, the Body projected will meet the Earth at *C*, beyond *B*. If the projectile Force be increaſed, ſo as that the Body does not reach the Earth, till it has got beyond the Point *E*, oppoſite to the Point *A*, then aſcending again towards *G*, it will compleat the Ellipſe *GKMF*; and, if not hindered, will perpetually deſcribe it, and ſo become a Planet. If the Projectile in the Point *M* of its Way, oppoſite to *G*, be leſs diſtant from the Earth, than at *G*; then the Center of the Earth, *T*, is the Focus of the Ellipſe, more remote from *G*. If the projectile Force increaſed ſtill, till the Points *M* and *G* are equally diſtant from *T*, the Path or Orbit will be a Circle; and, ſtill augmenting the projectile Force, (all other things continu-

ing

ing the same,) it will become an Ellipse, in which *T* is the Focus that is nearest *G*, and the other Focus will be removed from *T*, proportional to the augment of the projectile Force, till at length, it turns into a Parabola, and then into an Hyperbola. The same things will happen tho' *H G* be not perpendicular to *T G*, excepting that in this case, *G* will not be the principal Vertex of the Orbit. The higher the Point *G* is above the Earth, the less Force is needful to turn the Projectile into a Planet; and the lower it is, the greater: So that if the Moon should be projected with the same Celerity as it is carried now in its Orbit, from a height above the Earth of but a mile, like a common Projectile, it would meet and strike the Earth before it had run the distance of 15 miles. For the Arc which the Moon runs in 20″, is less than 15 miles, and a heavy Body in 20″ near the Earth in falling, runs $20 \times 20 \times 15\frac{1}{12}$ or 6033 Feet; that is, more than a mile; therefore the Moon would strike the Earth, before it could be projected to the distance of 15 miles. But if, the action of Gravity continuing the same, and the Earth should be annihilated, all Projectiles (like the Planets) would describe Ellipses, or some other Conic Sections, whose common Focus would be that point which now is the Center of the Earth.

PROPOSITION XLVII.

THE Secondary Planets of Jupiter gravitate towards Jupiter, the Satellites of Saturn towards Saturn, and both the Primary and Secondary Planets gravitate towards the Sun; and all of them are drawn off from their Rectilineal Motions, and kept in their Orbits by the Force of their Gravity.

By reason of that Simplicity of Nature which

is obſervable in all her Works, more Cauſes ought not to be admitted, than are ſufficient to explain the Phænomena of Natural things; and the natural Effects of the ſame kind are to be looked upon as having the ſame Cauſes: Since therefore the revolution of the Medicean Stars about Jupiter, and Saturn's Attendants about Saturn, and of the Primary Planets about the Sun, are Phænomena of the ſame kind with the revolution of the Moon about the Earth; their Cauſes will be the ſame. For the Forces whereby the Secondaries of Jupiter and Saturn, and the Primary Planets, are drawn off from their Rectilineal Motions, and kept in their Orbits, reſpect the Centers of Jupiter, Saturn, and the Sun, reſpectively, after the ſame manner, as the Force, whereby the Moon is kept in its Orbit, reſpects the Center of the Earth; and the Forces whereby they are urged, increaſe and decreaſe by the ſame Law, according to their different Diſtances from the Centers of Jupiter, Saturn, and the Sun, reſpectively, as the Force acting upon the Moon, placed in various Diſtances from the Center of the Earth; as has been demonſtrated before at large. Conſequently, ſince this Force in the Moon has been ſhewn (in *Prop.* preced.) to be the ſame with the Force, which we call Gravity; that Force, whereby the abovemention'd Planets, both Primary and Secondary, tend reſpectively towards the Sun, and their Primary Planets, muſt be the ſame with the Force of Gravity. Beſides, all the Secondaries gravitate alſo towards the Sun, and their Gravity towards the Sun, is the accelerative Force that is common to each Secondary and its Primary; concerning which, ſee *Prop.* 20. and 45. Which was to be demonſtrated.

CO-

COROLLARY.

Hence it is evident, that there is ſuch a thing as Gravity towards the Sun and all the Primary Planets. For ſince the Moon and all Terreſtrial Bodies gravitate towards the Earth, and the Satellites of Jupiter and Saturn, towards Jupiter and Saturn, reſpectively; and ſince the three other Primary Planets, Mercury, Venus and Mars, are Bodies of the ſame kind with Saturn, Jupiter and the Earth, (which alſo is evident from their Gravity towards the Sun, before demonſtrated,) 'tis certain, from the ſimilitude of Cauſes of the like natural Effects before laid down and eſtabliſhed, that there is a Gravity alſo towards them, whoſe accelerative Force is reciprocally proportional to the Square of the diſtance from the Center. And becauſe to any kind of Action, there is a Re-action, equal and contrary, the Sun likewiſe muſt gravitate towards all the Planets, both Primary and Secondary. Beſides, becauſe it is certain, from Experiments accurately made, that all kinds of Bodies, at the ſame Diſtance, and in equal Times, run equal Spaces in their Falls towards the Earth, and that the quantity of Motion of unequal Bodies, equally accelerated, is as the quantity of Matter. All Bodies, of what Nature ſoever, in equal Diſtances, gravitate towards the Earth proportionally to their quantities of Matter. And ſince no doubt but the nature of Gravity towards the Sun and Planets, is the ſame as towards the Earth, (which is ſufficiently manifeſt from what has been already ſaid, as alſo from hence, that the Primary Planets and their Satellites, that is, Bodies very unequal, deſcend toward the Sun with an equal accelerative Force, juſt as Bodies unequal in Magnitude and Denſity fall with equal Celeri-

ties towards the Earth ;) 'tis evident that all Bodies gravitate towards the Sun, and each of the Planets; and their Gravities towards any one, at equal Diſtances from its Center, are proportional to the quantities of Matter in each. And becauſe all Action is mutual, the Sun and Planets gravitate according to the ſame Law towards all Bodies, and conſequently all Bodies towards one another.

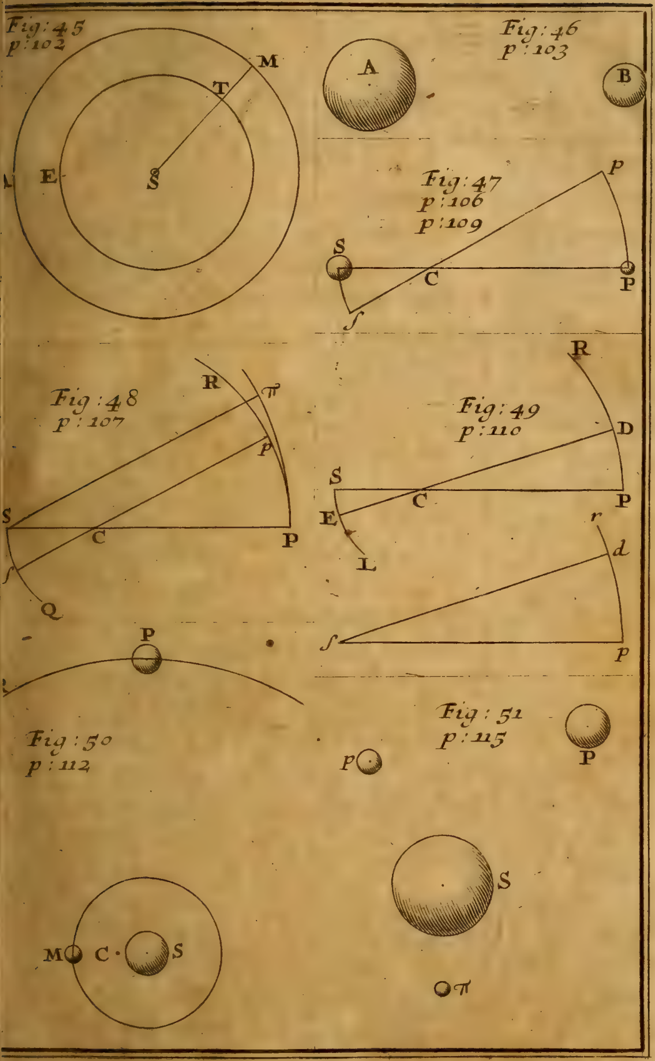
PROPOSITION XLVIII.

THE Force or Efficacy of any Virtue propagated from a Center, or to a Center, in Right lines every way round about, in different places, is reciprocally proportional to the Square of the Diſtance of the place from the Center.

Let *S*, [*Fig.* 45.] be the Center, from which, or to which, the Virtue is propagated, round it deſcribe two Spherical Surfaces *TE*, *MA*, at any diſtances, *ST*, *SM*. I ſay the Force or Efficacy of the Virtue in *T*, is to the Force of the ſame in *M*, as *SM*q, to *ST*q.

The ſame Virtue equally diffuſed and ſpread thro' a double Space, will be twice as little in any given part: And if it be diffuſed thro' a triple Space, it will be three times as little: And, univerſally, the Efficacy of the Virtue is reciprocally as the Space into which it is diffuſed ; becauſe 'tis directly as the conſtipation of the Virtue. But any Virtue, which in the diſtance *ST* from the Center, is equally diffuſed thro' the Spherical Surface *TE*, in the diſtance *SM*, is diffuſed thro' the Spherical Surface *MA*, after the like manner. The Efficacy therefore of that Virtue, in the diſtance *ST*, is to the Efficacy of the ſame in the diſtance *SM*, as the Spherical Superficies *MA*, is to the Sphe-

Plate 9 Book 1.



Spherical Superficies TE; that is, as SM^q to ST^q. Which was to be demonſtrated.

Of this kind are the Virtues or Effects of Light, Heat, &c. and alſo of Gravity, as is evident from what has been already demonſtrated. But ſome other Virtues, as thoſe of the Loadſtone and Amber, are of another kind.

PROPOSITION XLIX.

THE accelerative Gravities towards different Bodies, in equal diſtances, are as the Bodies themſelves toward which they tend.

Let there be any two Bodies A and B [*Fig.* 46.] in any places A and B; they (by *Corol. Prop.* 47.) will gravitate towards one another: And ſince the Reaction is equal and contrary to the Action, the Gravity or Motive Force of the Body A towards B, is equal to the Gravity or Motive Force of B towards A. But the Motive Force reſults from the Quantity of Matter or Maſs drawn into the Accelerative Force, juſt as the Quantity of Motion from the Quantity of Matter drawn into the Celerity: The Quantity of Matter therefore, or Maſs of the Body A, drawn into its Accelerative Force towards B, is equal to the Maſs of the Body B drawn into its Accelerative Force towards A. Conſequently the Accelerative Force of the Body A towards B, is to the Accelerative Force of the Body B towards A, as the Maſs of the Body B, to the Maſs of the Body A: But the Forces (treated of here,) are the Gravities, and the Maſſes of the Bodies are the Quantities of Matter or the Bodies themſelves: And therefore the Accelerative Gravity of the Body A towards B, is to the Accelerative Gravity of the Body B towards A, as the Body B, to the Body A. But (by what has been demonſtrated above,) all

Bodies placed in an equal diſtance from the Body *B*, with that of the Body *A*, have an accelerative Gravity towards *B*, equal to that of *A*; and all Bodies placed in a diſtance from the Body *A*, equal to that of *B*, have an Accelerative Gravity towards *A*, equal to that of *B*; and the diſtance of the Body *A* from the Body *B*, is equal to the diſtance of the Body *B* from the Body *A*, and the Diſtances and Bodies are taken at pleaſure. Therefore the Accelerative Gravities towards different Bodies, in equal diſtances, are as the Bodies themſelves. Which was to be demonſtrated.

COROLLARY.

From hence it follows, that if Gravity be conſider'd as the Attractive Force of the Body towards which it is directed, propagated as in the preceding Propoſition; the Abſolute Forces of the attracting Bodies are as the Bodies to which they belong. For the Abſolute attractive Force of the Body *A*, is to the Abſolute attractive Force of the Body *B*, as the Accelerative attraction of all Bodies towards *A*, is to the Accelerative attraction of all Bodies towards *B*, in equal diſtances. But, in equal diſtances, the Accelerative attraction of all Bodies is the ſame with the Accelerative attraction of any Body, and theſe Accelerative attractions, by this Propoſition, are as the Bodies toward which they tend; that is, as the Attracting Bodies themſelves: And therefore, *ex æquo*, the Abſolute Forces of Attracting Bodies are as the Bodies themſelves, to which they belong. And hence again, and from what goes before, (by *Prop.* 23. *El.* 6.) we may conclude that the Accelerative Force of the Body *A*, gravitating towards *B*, is to the Accelerative Force of the Body *C*, gravitating towards *D*; in

in a ratio compounded of the ratio of the Body *B* to the Body *D*, and of the duplicate ratio of the diſtance between *C* and *D* to the diſtance between *A* and *B*: And that the ratio of the weight of the Body *A* to the weight of the Body *C*, is compounded of the abovemention'd compound ratio, together with the ratio of the Body *A* to the Body *C*.

SECTION VIII.

Of the Motion of Bodies attracting one another, and the Affections; which are all applied to the Syſtem of the Sun and Primary Planets.

PROPOSITION L.

THE Bodies attracting one the other, and revolving about one another, deſcribe ſimilar Figures, both about themſelves, and about the common Center of Gravity.

In *Prop.* 29. and the following ones, we have explain'd the Motions of Bodies attracted to an immovable Center; for it is moſt proper to begin with the moſt ſimple Caſes. But becauſe the Attractions are made to Bodies, and Bodies attract one another; as has been before demonſtrated: When a Body revolves about another, the attracting Body it ſelf can't be at reſt; but both being acted upon by a mutual Attraction, do revolve about the common Center of Gravity. And if there were more Bodies that mutually attracted one another, they would ſo move among themſelves, that the common Center of Gravity might be either at reſt, or in an uniform direct Motion. For it is well known to Geometricians, that the common Center of Gravity does not change its ſtate of

Motion

Motion or Reſt, by reaſon the action of Bodies upon one another; as has been demonſtrated by Mr. *Wren* and Mr. *Hugens*.

Method therefore requires us to conſider the Motions of Bodies, as was ſaid, attracting one another; namely, the Orbits they deſcribe, and the other generals, wherein a Syſtem of Bodies acting upon one another mutually differs from the Syſtem ſuppoſed above; and that in the firſt place univerſally, whatever the Law of mutual Attraction be.

Let *S* and *P* [*Fig.* 47.] be the two attracting Bodies, which, revolving mutually about one another, deſcribe the Curves *S ſ*, *P p*, while their Center of Gravity *C*, is either at reſt, or (which does not diſturb the Demonſtration,) moves uniformly in a Right line: I ſay that the four Figures, namely, *S ſ C*, *P p C*, that which *S* deſcribes about *P*, conſider'd as immovable, and that which *P* deſcribes about *S*, conſider'd after the ſame manner, are ſimilar.

Becauſe *S* and *P* continuing the ſame, the ratio of *CP* to *C S*, during the Motion, will likewiſe continue the ſame; becauſe, from the nature of the Center of Gravity, it is the ſame with the reſpect to the Bodies *S* and *P*: And ſince the Angles *P C p*, *S C ſ*, are every where equal, (becauſe *C* is in the Right line connecting the Bodies,) the Figures *S ſ C*, *P p C* will be ſimilar. For the Figures *P p C*, *S ſ C*, are ſimilar, which the Right line *p C ſ*, deſcribes with an Angular Motion about the Point *C*, conſider'd as at reſt, provided every moment, the ratio of *C p* to *C ſ*, be the ſame with that of *C P* to *C S*.

Again, becauſe the ratio between *S C* and *C p* is always given, by compounding, the ratio of *ſ p* to *C p* is likewiſe given, and the Line *p C ſ* continues a Right one: And therefore (by what

has

has been said) the Right line Cp about the Point C, and the Right line sp about the Point s, considered as at rest, describe similar Figures. Besides, the Figures whereof one is described by p about s, considered as at rest, the other described by s about p, considered likewise as at rest, are similar and equal, (as is demonstrated in Optics,) because sp is the same in Magnitude and Position with ps. Wherefore the four Figures proposed are similar. *Q. E. D.*

PROPOSITION LI.

If two Bodies S *and* P *attract one another by any Forces, and in the mean while revolve about the common Center of Gravity* C; *I say, that a Figure may be described, by the same Forces about either of the Bodies consider'd as at rest, similar and equal to the Figures, which the Bodies so moved describe about one another.*

Suppose the Orbits PR, SQ [*Fig.* 48.] to be described first about the Point C at rest, and the Points s and p any places at pleasure, to which the Bodies S and P arrive at the same time. Let the Right line sCp be suppos'd to move parallel to its self, till the point s coincide with S, and let this be done in all the corresponding points of the Curves SQ, PR: 'Tis evident, that when s comes to S, the other extreme of the Right line sp, thus moved parallel to it self, will arrive at the point ϖ, in the Curve $P\varpi$, which the Body P would describe about the point S, if (S being at rest) P should revolve about it, being attracted by the same Force as the Bodies did before mutually attract each other: For the parallel Motion of the Right line sp, makes no alteration in the Forces, with which the Bodies attract each other. But the same $P\varpi$ is the Line which P describes about S, considered as at rest:

For

For thus it really continues at reſt; and the length and inclination of the Right line $S\pi$ to SP, is the ſame of that with $ſp$. Since therefore theſe Figures are congruous, they are equal and ſimilar. And what has been demonſtrated concerning the Figure, which P deſcribes about S, is true of the Figure, that S deſcribes about P, looked upon as at reſt, namely, that it is equal and ſimilar. But if the Center of Gravity C go forward uniformly, by ſuperinducing upon the Syſteme of the Bodies S and P, an equal and contrary Motion, all things will be reduced to the former Caſe: But there is no change made by this uniform Motion along parallel Right lines, in the Forces of the Bodies S and P. Therefore the Propoſition is certain in both Caſes. Which was to be demonſtrated.

COROLLARY 1.

Two Bodies mutually attracting one another, by any Forces, and revolving about the Center of Gravity, deſcribe, by Radii drawn to that Center and to one another, Areas that are proportional to the Times.

COROLLARY 2.

Becauſe a Body revolving about another immovable one, and attracted to it by a Force reciprocally proportional to the Square of the Diſtance, would deſcribe (by *Prop.* 39.) an Ellipſe, or ſome other Conic Section, one of whoſe Foci coincides with the Center of the Body that is at reſt; it follows, from this Propoſition, that two Bodies attracting one another, by Forces reciprocally proportional to the Square of the Diſtance, deſcribe, both about the common Center of Gravity, and about one another, Conic Sections, having their Foci in the Center, about which they are deſcribed. And conſequently, if ſuch Figures are deſcribed, the Cen-

tripetal

tripetal Forces are reciprocally proportional to the Squares of the Diſtance.

PROPOSITION LII.

IF two Bodies attracting one another by any Forces whatever, mutually revolve about one another, their Motion will be the ſame, as if they did not attract one another, but both were attracted with the ſame Forces by a third Body placed in the common Center of Gravity. And the Law of the attractive Forces will be the ſame, in reſpect to the diſtance of the Bodies from that common Center, and in reſpect to the whole diſtance between the Bodies.

For the Forces, whereby the Body *P* [*Fig.*47.] tends to *S*, are directed to any point whatever, in the Right line *P S* produced towards *S*. And in like manner, the Forces of the Body *S*, are directed to any point whatever, of the ſame Line produc'd towards *P*. But *C* is the only point of the Right line *SP*, produced both ways, that continues at reſt, in every ſituation of the Bodies; and therefore *C* is that point, to which the Bodies *P* and *S* tend, with the ſame Forces, as they attract one another.

Again, becauſe the Bodies that revolve are given, and the ratio of the diſtance of either of the Bodies from the common Center, to the diſtance of the ſame Body from the other. And conſequently the ratio of any Power of the diſtance of the one, to the like Power of the diſtance of the other, is given; as alſo the ratio of any Quantity, which is any way compounded of one diſtance and any given Quantities, to another Quantity, which is in the like manner compounded of the other diſtance and as many given Quantities, having that given ratio of the Diſtances to the former. Wherefore, if the Force, whereby one Body is attracted by the

the other be directly or inversly as the distance of the Bodies from one another, or as any Power of this distance; or lastly, as any Quantity compounded any how of this distance and the Quantities given; this same Force, whereby the same Body is attracted to the common Center of Gravity, will be in like manner directly or reciprocally as the distance of the attracted Body from that common Center, or as the same Power of that distance, or lastly, as a Quantity compounded after the same manner of this distance and of the analogous given Quantities: And therefore there will be the same Law of the attractive Force in regard of the distance from the Center of Gravity, as there is in regard of the distances of the Bodies. Which was to be demonstrated.

Proposition LIII.

The Periodic Time of two Bodies, S *and* P, [Fig. 49.] *revolving about the common Center of Gravity* C, *is to the Periodic Time of any other Body* P, *revolving about the other immovable point* S, *(attracting with the same Forces,) in the subduplicate ratio of the Body* S, *to the aggregate of the Bodies* S *and* P.

Let the Body *p*, equal and similar to *P*, be imagined to describe the Orbit *p d r*, about *ſ*, at rest, and similar and equal to *S*. The Figures *C P D R*, *ſ p d r*, (by Prop. 50.) are similar; and the Forces whereby the Bodies being in any Points (*D* and *d*) similarly posited, tend toward the Centers *C* and *ſ*, are equal; because, (by *Prop.* 51.) the Force whereby the Body *D* tends to *E*, is equal to the Force whereby the Body *d* tends to *ſ*. And, by the preceding Proposition, the Body *D* tends to the *C*, with the same Force as to *E*. And therefore, (by

Corol.

Corol. 2. *Prop.* 26, and *Schol. Prop.* 27.) the Square of the Periodic Time of the Body *P*, moving in the Orbit *P D R*, is to the Square of the Periodic Time of the Body *p*, in the Orbit *p d r*, as the Right line *C P* to the homologous Right line *ſ p*; that is, as *C P* to *S P*: that is, (from the nature of a Center of Gravity) as the Body *S* to *S* + *P*. Wherefore the Periodic Time of the Body *P* in the Orbit *P D R*, (or of the Body *S* in the Orbit *S E L*) is to the Periodic Time of the Body *p* in the Orbit *p d r*, in the ſubduplicate ratio of the Body *S*, to the aggregate of the Bodies *S* and *P*. Which was to be demonſtrated.

PROPOSITION LIV.

If two Bodies S *and* P, *attracting one another by Forces reciprocally proportional to the Square of their diſtances, revolve about the common Center of Gravity; I ſay, that the greater Axe of the Ellipſe, which either Body* P *deſcribes by this motion about the other* S, (*in the manner deſcribed in* Prop. 50.) *is to the greater Axe of the Ellipſe, which the ſame Body* P *might deſcribe about the other* S *at reſt*, (*as in* Prop. 51.) *in the ſame Periodic Time, in the ſubtriplicate ratio of the ſum of the Bodies* S *and* P, *to the quieſcent Body* S.

If the Ellipſes deſcribed about the movable and quieſcent Body *S*, by the Body *P*, were equal, the Periodic Times in them (by *Prop.* 51, 53.) would be in the ſubduplicate ratio of the Body *S*, to the aggregate of the Bodies *S* and *P*. Imagine the laſt Ellipſe to be changed, till the Periodic Time of the Body *P* (deſcribing this Ellipſe about the quieſcent Body *S*,) become equal to the Periodic Time of the Body *P* revolving about the Body *S*; (as we ſuppoſe in this Propoſition;) that is, till the Periodic Time in this

this being leſſen'd may be in the ſaid ſubduplicate ratio of the Body S, to the ſum of the Bodies S and P. But becauſe (by *Prop.* 40.) in the preſent caſe, the Squares of the Periodic Times are as the Cubes of the Tranſverſe Axes, the Periodic Times themſelves will be in the ſeſquiplicate ratio of the ſaid greater Axes. The greater Axes therefore of the latter Ellipſe, being leſſen'd, is in a ratio, whoſe ſeſquiplicate is the ſubduplicate ratio of P to $S+P$; that is, in the ſubtriplicate ratio of S to $S+P$: (For $\frac{3}{2}$ of $\frac{1}{3}=\frac{1}{2}$:) and conſequently is to the unvaried Axe (equal to this before it was leſſen'd) of the other Ellipſe, deſcribed about the Body S, in the ſubtriplicate ratio of S, to $S+P$. And therefore inverſly, the greater Axe of the Ellipſe deſcribed about the movable Body S, is to the greater Axe of the Ellipſe deſcribed about the quieſcent Body S, in the ſame or an equal Time, in the ſubtriplicate ratio of the ſum of the Bodies $S+P$, to the quieſcent Body S. Which was to be demonſtrated.

PROPOSITION LV.

THE *Law of Attraction being the ſame as before, I ſay the Orbit* P R, (*Fig.* 50.) *of the Body* P, *comes nearer to an Ellipſe, whoſe Focus is* C, *the common Center of Gravity of the Bodies* S *and* M, *than to an Ellipſe, whoſe Focus is the Center of it* S; *and that the Areas deſcribed by Radii drawn to* C, *are more proportional to the Times, than the Areas deſcribed by Radii drawn to the Centre of it* S.

For the attractions of the Body P toward S and M, compoſe its abſolute Gravity, which is directed more towards C, the common Center of their Gravity, than towards the greater Body it ſelf S, and which is more nearly reciprocally proportional to the ſquare of the diſtance

$P C$,

PC, than to the ſquare of the diſtance *PS*: For this laſt would then hold preciſely, when *M* becomes nothing; and the ſame thing would be true of *M*, if it ſhould increaſe till it immenſly exceed *S*. 'Tis certain therefore, that the Forces, in the preſent Caſe, are directed to ſome Point lying betwixt *S* and *M*, and nearer the greater of them, as is the common Center of their Gravity *C*; which alſo is that Point where the Natural Forces of any kind of heavy Bodies placed round about it, are united and concenter'd: for the aggregate of ſeveral heavy Bodies is moſt juſtly looked upon to be there, where the common Center of their Gravity is; as it is well known by Philoſophers. What therefore was propoſed is evident and certain. Which was to be demonſtrated.

PROPOSITION LVI.

Suppoſing the ſame Laws of the Attraction, I ſay the external Body P *deſcribes Areas more nearly proportional to the Times, about the common Center of Gravity* C, *of the internal Bodies* S *and* M, *by Radii drawn to that Center, and an Orbit coming up nearer to the form of an Ellipſe, having a Focus in that Center, if the moſt internal and biggeſt Body* S *be agitated by theſe Attractions, juſt as the others are, than if ſo be it ſhould be at reſt, not being attracted at all, or much more or much leſs attracted, or much more or much leſs agitated.*

By the *Prop.* preced. it is evident, that the Center to which the external Body *P* is urged, is neareſt the Center of Gravity of the internal Bodies *S* and *M*. If that Center coincided with this common Center, and the common Center of Gravity of the three Bodies *S*, *M*, and *P*, were at reſt; *P* on one ſide, and the common

Center of *S* and *M* on the other, would describe accurate Ellipses about the common Center of all three, being at rest, and as it were the common Focus, by *Corol.* 2. *Prop.* 51. And therefore the perturbation of the said Motions (in the Ellipses) will be very small, when the common Center of Gravity of three Bodies, *S*, *M*, and *P*, is at rest: And it is at rest, if the Body *S* be agitated after the same manner as the others are; as was said before. And therefore the perturbation will be the least, when *S* is attracted by the same Law as the others are. Which was to be demonstrated.

COROLLARY.

After the same manner, if more lesser Bodies revolve about the greatest, it may be inferred, that the Orbits described come nearer to Ellipses, and the description of the Areas more equable, if all the Bodies mutually attract one another with accelerative Forces, which are as their Masses directly and the Squares of their distances inversely, (for this is to be agitated, just as the others are, supposing the Law of the attractions to be the same with that in the Proposition;) and the Focus of each Orbit be placed in the common Center of Gravity of all the internal Bodies, than if the greatest Body were at rest, and made the common Focus of all the Orbits.

PROPOSITION LVII.

SEveral Bodies, whose Accelerative Forces are to one another reciprocally as the Squares of the distances from their Centers, may be moved in Ellipses about a great one, and describe Area's nearly proportional to the Times, by Radii drawn to that Body.

Let

Let the ſeveral leſſer Bodies *P*, *p*, π, [*Fig.* 51.] be ſuppoſed to revolve about a great Body, as *S*: And becauſe the common Center of Gravity of them all does not change its ſtate of Motion or Reſt, by the mutual actions of the Bodies upon one another; (as was explain'd before,) and we ſuppoſe them free from all other external agitation or impediment; that Center may be at reſt. If now we ſuppoſe the leſſer Bodies *P*, *p*, π, to be ſo little in compariſon to the great one *S*, as that it is never ſenſibly diſtant from the ſaid common Center of Gravity, then that great one *S* will be likewiſe very nearly at reſt; but the leſſer Bodies will revolve about this greater in Ellipſes, having the great Body in the common Focus; and therefore they will deſcribe Areas, by Radii drawn to it, proportional to the Times; bating the Errors ariſing either from the digreſſion of the greateſt Body from the common Center of Gravity, or from the mutual attractions of the leſſer Bodies upon one another. But theſe leſſer Bodies may be leſſen'd till that digreſſion or thoſe mutual attractions be leſs than any given ones. And therefore ſeveral Bodies may move about a great one in Ellipſes, and by Radii drawn to its Center, deſcribe Areas proportional to the Times, ſo as that the Errors committed (or what hinders them from being exact) may be leſs than any aſſignable; that is, indefinitely near. Which was to be demonſtrated.

PROPOSITION LVIII.

BY the mutual action of the Sun and any primary Planet, the Planet deſcribe an Ellipſe, whoſe Focus is the common Center of Gravity of the Sun and the ſaid Planet. But by the action of the primary Planets upon one another, the Orbit of each is nearly

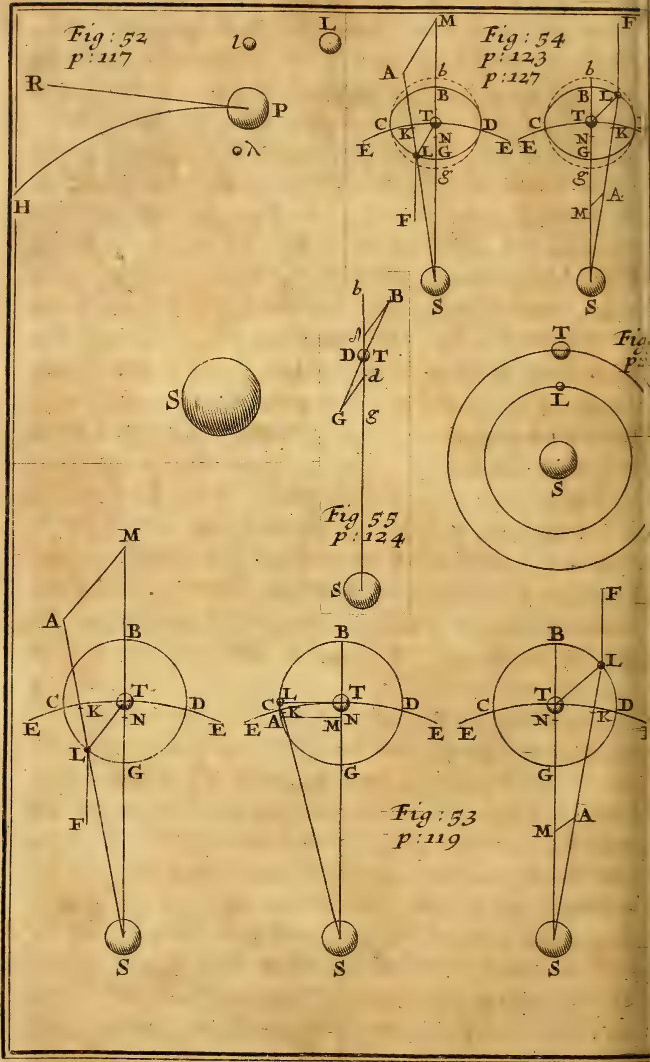
an Ellipſe, whoſe Focus is the common Center of Gravity of the Sun, and all the inferior Planets.

If the primary Planets gravitated towards the Sun, and not the Sun towards the Planets, each primary Planet would deſcribe an Ellipſe about the Sun's Center at reſt, as a Focus; as was demonſtrated in *Prop.* 42. And if the Maſſes of the Planets were inſenſible in compariſon to the Maſs of the Sun, the ſame would hold very nearly, by the preceding: And that this is almoſt the caſe of the Solar Syſtem, is evident from *Prop.*34. But becauſe (by *Corol.Prop.*47.) not only the Sun gravitates towards a Planet, but a Planet alſo towards the Sun, and beſides a Planet is not a Maſs intirely inſenſible in regard of the Sun; each Planet (by *Corol.* 2. *Prop.* 51.) deſcribes an Ellipſe, whoſe Focus is the common Center of Gravity of the Sun and the ſaid Planet: And (by *Prop.* 54.) the greater Axe of this Ellipſe, exceeds the greater Axe of the Ellipſe which the Planet would deſcribe about the Sun being at reſt, in the ſame or an equal Time, in the ſubtriplicate ratio of the ſums of the Maſſes of the Sun and Planet, to the Maſs of the Sun.

Beſides, becauſe (by the abovementioned *Corol. Prop.* 47.) the Planets gravitate towards one another, and the Sun is agitated by the ſame attractions as the Planets; Mercury, (by *Prop.*50.) will deſcribe very nearly an Ellipſe having the Focus in the Center of the Sun; Venus, an Ellipſe whoſe Focus is in the common Center of Gravity of the Sun and Mercury, and ſo on; each primary Planet will deſcribe an Ellipſe, whoſe Focus is in the common Center of Gravity of the Sun and all the inferior Planets. And after this manner, the actions of the primary Planets upon one another, are conſidered, and reduced to a ſtandard.

SECT.

Plate 10 Book 1.



SECTION IX.

Of the Motion of a Syſtem of Bodies, which revolves about another Body; all which is applied to the Syſtem of the Sun and Planets Primary and Secondary.

PROPOSITION LIX.

THE common Center of Gravity of a Syſtem of two or more heavy Bodies, revolving about one another in Ellipſes, may move in an Ellipſe, or any other Conic Section, about another greater placed in the Focus of the Ellipſe, and may deſcribe, by Radii drawn to the Focus, Areas proportional to the Times, indefinitely near.

Let us ſuppoſe a Syſtem of two Bodies revolving mutually about one another (as in *Cor.* 2. *Prop.* 51.) and another Syſtem of leſſer Bodies *L*, *l*, λ, revolving (after the manner deſcribed in *Prop.* 57.) about a great one *P*, [*Fig.* 52.] to go on uniformly in a Right line, which is evidently poſſible: (that is, for its common Center of Gravity to move along Right lines, for inſtance, *PR*;) and in the mean while to be urged laterally, by the Force of another great Body as *S*, placed at a great diſtance, and forced to move in the Curve *PH*, being drawn off from the Right line *PR*. And becauſe equal Accelerative Forces impreſſed along parallel Right lines don't alter the ſituation of the Bodies to one another; but cauſe the whole Syſtem, preſerving their Motions in regard of one another, to be transfer'd at the ſame time together: 'Tis evident that there will ariſe no alteration in the Motion of the Bodies mutually attracted, from

the attractions, towards the greatest Body S, but from the inequality of the Accelerative attractions, or from the Inclination of the Lines to one another, according to which the Attractions are made. Now the distance of the greatest Body may be increased, till the differences of the Right lines drawn from it to the Bodies of the System in Motion (namely, the great one P, and the lesser L, l, λ) in respect to the Lines themselves thus drawn, and the Inclinations to one another, become less than any given ones. And since all the Accelerative attractions towards S, by the Hypothesis of Gravity, are reciprocally as the Squares of the Distances; these (in this case) differ from equal Forces, by a difference which is less than any given one: And therefore the Motions of the parts of the System P, with its Bodies L, l, λ, will persevere after the same manner as before, when the greatest Body S did not attract that System; and the whole System attracted after the manner of one Body, will describe by its Center of Gravity, (in which the whole System of the heavy Bodies P and L, l, λ, is supposed to be contracted and united, as in other Philosophical cases,) a Conic Section whose Focus is S; and by a Radius drawn to that greatest Body, will describe Areas proportional to the Times: And the errors which are over and above, may be lessen'd at pleasure, by increasing the distance of the Body S. Therefore the Center of Gravity of a System of Bodies P, and L, l, λ, may move in an Ellipse very nearly. *Q. E. D.*

COROLLARY I.

When a System of two Bodies revolving about one another, or of a great Body with lesser Bodies revolving about it, is moved about a greater; the Motion

Motion of the parts of the Syſtem among themſelves will be ſo much the more diſturbed, as the greater Body approaches nearer the ſaid Syſtem: Becauſe the inclination of the Lines drawn from the greater Body to thoſe parts, is greater, and the inequality of the ratio likewiſe greater; and, in one word, becauſe there is a greater receſs made from that caſe, wherein there is no perturbation at all; namely, when the Body *S* is at an infinite diſtance.

COROLLARY 2.

From whence, on the contrary, if the parts of a Syſtem of the Body *P* and the leſſer revolving Bodies *L*, *l*, λ, move in Ellipſes or Circles), without any conſiderable perturbation; 'tis manifeſt, that the ſame are only urged very ſlightly by any Accelerative Forces tending towards other Bodies, or that they are urged equally, and along parallel Lines, indefinitely near.

PROPOSITION LX.

IF *a leſs Body* L [*Fig.* 53.] *revolve about a great one* T, (*as in Corol.* 2. *Prop.* 51.) *and the Syſtem of theſe two revolve about a greater* S, (*as in the preceding,*) *and in theſe three Bodies, if the Accelerative attractions of any two, towards the third, be reciprocally as the Squares of the Diſtances from it; let it be propoſed to ſhow in general the Errors committed by the Body* L, *together with the cauſes and reaſons of them; namely, why the Body* L *does not, by a Radius drawn to* T, *deſcribe Areas exactly proportional to the Times, and run in the Perimeter of an Ellipſe, one of whoſe Foci is* T.

Becauſe the Body *L* is ſometimes more, ſometimes leſs diſtant from the Body *S*, let *SK* be its mean diſtance; which may likewiſe expreſs the Accelerative attraction of the Body *L* towards *S*, in that mean diſtance. Let the place *L*

of the leaſt Body in its Orbit *B C G D*, be taken at pleaſure: And in *S L*, produced if need be, let the Right line *S A* be taken, ſo as that it be to *S K* in the duplicate ratio of *S K* to *S L*, which conſequently will expreſs the Accelerative attraction of the Body *L* towards *S*, when it is in the point *L*, thus aſſumed; becauſe, (by ſuppoſition,) the Accelerative Force in *L* is to the Accelerative Force in *K*, in the duplicate ratio of *S K* to *S L*; that is, (by conſtruction,) as *S A* to *S K*. Connect *T L*, and through *A* draw *A M* parallel to it, meeting *S T* in *M*.

The Attraction *S A* of the Body in *L* towards *S*, may be reſolved (as is well known,) into the Attractions *A M*, *M S*. Wherefore the Body *L* will be urged by a threefold Accelerative Force; one tending to *T*, and ariſing from the mutual attraction of the Bodies *L* and *T*; by this Force alone, the Body *L*, by a radius *L T* drawn to the Center of the Body *T*, would deſcribe Areas proportional to the Times, and an Ellipſe, whoſe other Focus is in the ſaid Center. The other Force, whereby *L* is urged, is expreſſive by *A M*, and is parallel to the direction of it; which becauſe it tends from *L* to *T*, adding the former Force, will with it compoſe another, whereby the Areas deſcribed, will ſtill be proportional to the Times, (by *Corol.* 1. *Prop.* 51.) But becauſe this Force, as *A M*, is not reciprocally proportional to the Squares of the diſtance *L T*, the Force compounded of this and the former, will not be ſuch; but will recede from that proportion, and that ſo much the more, as (other things being alike) this new Force added to the former, and repreſented by *A M*, is greater. Conſequently the Body *L*, which is urged by this compound Force, will not deſcribe the Perimeter of an Ellipſe, whoſe

Focus

Focus is *T*; (for to this a Force tending to *T*, and reciprocally proportional to the Square of the diſtance from it, is required by *Prop.* 39.) but will cauſe its Orbit to have ſome digreſſion from an Ellipſe, whoſe Focus is *T*; and that digreſſion will be ſo much the greater, as this compound Force (whereby *L* is urged,) recedes from a Force reciprocally proportional to the Square of the diſtance from *T*; that is, (as has been ſhewn,) the greater the ſuperadded Force repreſented by *AM*, is, in reſpect to the Force, whereby *L* is attracted to *T*, other circumſtances being alike. Again, the third Force, whereby the Body *L* is urged, which the Right line *MS* repreſents, acting according to the direction of it, (namely according to the direction of *LF* parallel to *MS*) ſuperadded to the former Forces, will compound another, whoſe direction is not from *L* towards *T*; but along a Right line declining from thence towards *F*, more or leſs, according to the ratio of this third Force to the ſum of the former Forces. And therefore the whole Force, compounded of all three, whereby *L* is urged, ſince it does not tend to *T*, (by *Prop.* 40.) will make the Body *L* deſcribe Areas not proportional to the Times, by the Radius *LT*. And the digreſſion from that ſimilitude of the ratios between the Areas deſcribed and the Times in which they were deſcribed, will be ſo much the greater, as the ratio of this third Force acting according to the direction of *LF*, to the former Forces, is greater. Beſides, the ſuperaddition of this third Force will induce a deformity in the Elliptic Figure of the Orbit, on a double account, both becauſe it is not directed from *L* to *T*, and becauſe it is not reciprocally proportional to the Square of the diſtance; and the deformity is ſo much the great-

er

er, as, other things being alike, this Force *MS* is greater. In the Right line *ST*, (from *S* towards *T*) take *SN*, to *SK* in the duplicate ratio of *SK* to *ST*; the Accelerative attraction of the Body *T* towards *S*, will be expreſs'd by *SN*. If the Accelerative attractions *SM*, *SN*, be equal, (as in the 2d of theſe *Fig.*) they will not alter the ſituation of the Bodies *T* and *L* to one another, by acting upon them equally, and according to parallel Right lines: But the Bodies *S* and *T* approaching nearer to one another, the motions of *T* and *L*, in regard of one another, will be juſt the ſame as if theſe attractions were intirely abſent. But if the Accelerative attraction *SN*, be leſs than the Accelerative attraction *SM*, (as in the firſt of theſe *Fig.*) the former, and a part of the latter equal to it, will, (as was ſhewn in the preceding caſe,) reduce themſelves mutually to that State, as if both of them were abſent, and the difference of them, on the part of the Accelerative Force *SM*, expreſſed by *MN*, remain; which therefore diſturbs the proportion of the Areas and Times, and the Elliptic Figure of the Orbit, *GCBD*. But if *SN* exceed *SM*, (as in the 3d of theſe *Figures*) their interval *MN*, will denote that difference of the Accelerative Forces, whereby the proportion of the ſaid Areas deſcribed, and Times of deſcription, together with the Elliptic form of the Orbit *GCBD*, are diſturbed. And therefore by the attraction *NS*, is the third attraction *SM* always reduced to *MN*, which exerts its Force from *M* to *N*, or from *L* towards *F*, the former Forces being intirely unchanged. And theſe are the Forces that generate the Errors of the Body *L*. *Q. E. I.*

PRO-

PROPOSITION LXI.

THE *ſame things being ſuppoſed, let it be propoſed to give an account of ſuch Errors of a leſſer Body* L. [Fig. 54.] *revolving about a great one* T, *not in the ſame plane wherein* T *revolves about a greater Body* S, *ariſing from the inclination of the ſaid Planes; as alſo of the Cauſes and Reaſons of them.*

From the demonſtration of the preceding *Prop.* 'tis ſhewn, that all the errors of the Body *L*, ariſe from the Forces repreſented by the Right lines *AM*, *MN*. But becauſe one of them *AM*, acts according to the direction *LT*, which lies in the plane of the Orbit of the Body *L*, the Force acting according to its direction, will not diſturb the Motion of the Body *L*, as to its Latitude.

If the ſituation of the Orbit of the Body *L*, be ſuch, as that the Body *S* is found in its common Section with the plane of the Orbit *ETE*, which *T* deſcribes about *S*: (that is, to ſpeak in the Language of an Aſtronomer, if the Nodes of the Orbit of the Body *L*, be in the Syzygies of the Body *S*;) *MN* alſo lies in the plane of the Orbit of the Body *L*; becauſe in the common Section: And therefore the Force acting according to the direction of it, does not diſturb theſe Motions.

The other things remaining the ſame, let the Plane *GCBD* of the Orbit of the Body *L*, be ſuppoſed to be inclined to the Plane *SET*, ſo that part of it *CGD*, may be elevated above it, and part *CBD*, depreſſed below it: Upon the ſame Center *T* and equal Semidiameter, let the Circle *CbDg*, be ſuppoſed to be deſcribed in the Plane *SET*, and the Nodes will be *C* and *D*: Suppoſe them when ſeen from *T*, to be in a Quartile aſpect with the Body *S*; that is,

ſup-

ſuppoſe the Angles *STD*, *STC* right. In the caſe of the motion of the Body *L*, along the Semicircle *C G D*, (*ſee the firſt of theſe Fig.*) the Force as *MN* of the Body *L*, acting with the direction *M N*, (that is, from *L* to *F*) attracts the Body from its Orbit *C G D* towards *C g D*; that is, leſſens the Latitude of the Body *L*, (reckoned from the Plane *C g D*): And the Body *L*, in its continual receding from the Plane *C G D* towards *S*, till it has arrived to the next Node, does not paſs the Plane *E T S* or *C g D* in the point wherein it paſſed before, but a little on this ſide of it; and makes the Node of the Orbit, now deſcribed by it, a little more *in antecedentia*, in regard of the Node *D* of the Orbit *C G D*, which it would have deſcribed without this Force *M N*. In like manner in the paſſage from the Node *D* to the Node *C*, through the Semicircle *D B C*, (to which caſe the ſecond of theſe *Figures* is accommodated,) the Force as *M N*, acting according to the direction of *M N*, or from *L* towards *F*, makes the Body *L* to go backwards perpetually from the Orbit *D B C*, and deſcribe another inclin'd to *D b C*, in a leſſer Angle, and to paſs the Plane *D b C* ſooner than in *C*; that is, than it would have done if the Force *M N* had not been preſent; and by this means there will be made a new place for the Node tending *in antecedentia*; and again, during its paſſage, the part *C G D* of its Orbit towards the conjunction with the Body *S*, the neareſt Node will recede *in antecedentia*, as before.

Beſides, (the Eye being placed in the Line of the Nodes produced,) let *S g T b* [*Fig.* 55) repreſent the Plane of the Orbit of the Body *T* revolving about *S*, *GDB* the Plane of the Orbit of the Body *L*, while the latter is inclined to the

the former, by the ſmalleſt Angle, namely, *GDg* or *GDb*; which happens (as has been ſhewn) when the Body *L* is ſeen from *T* in the Syzygies of the Body *S*. And ſince it has already been ſhewn, that the Nodes of the Orbit of the leaſt Body *L* tends *in antecedentia*; the Orbit which the Body *L*, departing from *G*, where it was ſeen in conjunction with *S*, further deſcribes, will meet *g Db* on this ſide *D* in reſpect of *S*, and will be the Line *Gd*. And the Orbit which the ſame *L* deſcribes, departing from *B*, where it was in oppoſition to *S*, namely, the Line *B* δ, will meet the Plane *g D b* in δ, *viz.* beyond *D* in regard of *S*. And it is evident that the Line *Gd* or *B*δ contains a greater Angle with *g D b* at *d* or δ, than the Angle made by the Line *G D* or *B D* with the ſame *g D b*; being about equal to that which the Orbit of the Body *L* made with *g D b*, when *L* was very near the Node. The Nodes therefore being in the Quadratures, go backwards; but in the Syzygies are at reſt; and therefore always whether they are Retrograde or Stationary, upon the whole go backwards; but in the intermediate places, partaking of both conditions, go backwards ſlower; and ſo much the ſlower as they are nearer the Syzygies. But when the Nodes are in the Quadratures, the inclination of the Plane of the Orbit of the Body *L*, is leſſened in the paſſage of the Body *L* from the Quadratures to the Syzygies, and augmented in the paſſage of the ſame from the Syzygies to the Quadratures: (For the parts of the Lines *Gd*, *B*δ, towards *d* and δ, are inclined in a greater Angle to *g D b* than their parts towards *G* and *B*; and the Orbit of the Body at every moment of time is to be looked upon as a lineola, deſcribed by it in that moment; and that is the Plane of the Orbit, which

paſſes

paſſes thro' the ſaid lineola, and the Center of the Body *T*.) And therefore the Body *L* being in the Syzygies, the inclination becomes the leaſt poſſible, and returns to its former magnitude, or thereabouts, when the Body comes to the neareſt Node. And theſe are the principal Errors ariſing from the inclination of the Planes. *Q.E.I.*

SCHOLIUM.

The ſame holds, tho' the Body *L* [*Fig.* 56.] revolves not about *T*; but both *T* and *L* about *S*, as a Center. For in this caſe the outmoſt *T*, produces the Effects of the Body *S*; and *S*, together with *L* revolving about it, ſupplies the places of *T* and *L* revolving about it, in the foregoing caſe. For the Demonſtration is the ſame, whether the diſturbing Body moves or not.

COROLLARY.

By the ſame Laws, as the Body *L* [*Fig.* 54.] revolves about *T*, let us imagine ſeveral Bodies to move about the ſame *T*, at equal diſtances from it; and a Ring *CBDG* rigid and concentric with the Body *T*, made up of them, being multiplied, and at length becoming contiguous: And the ſeveral parts of the Ring will perform all their Motions (which are not impeded by the rigidity,) according to the Law of the Body *L*. And therefore the Nodes *C* and *D* of the Ring *CBDG* are at reſt, when they are in the Syzygies of the Body *S*: and will move from them, *in antecedentia*, ſwiftly in the Quadratures, and ſlowly in all other places: And the inclination of the Ring will be changed, (the Axe of it Oſcillating,) as has been ſhewn concerning the Orbit *GCBD*, in the Propoſition.

If now the Globe *T* (having the ſame Axe with the Ring *CBDG*, and compleating its Revolutions about that Axe in the ſame times,) be conceived to be increaſed ſo big, or the Ring

(other

(other things remaining as before) leſſen'd, ſo as that the Globe touch the Surface of the Ring internally, and cling to it, becauſe cloſely encompaſs'd by it: The Globe *T* (which we ſuppoſe intirely indifferent to all impreſſions,) will partake of the Motion of the Ring, and the aggregate of them both, will oſcillate, and the Nodes of the Ring (or the interſections of it with the Plane of the Orbit of the Body *T* about *S*,) will go backwards.

PROPOSITION LXII.

THE *Body* L,[Fig.54.] *by a Radius drawn to* T, *will deſcribe Areas more nearly proportional to the Times, and a Figure coming nearer to the Form of an Ellipſe, whoſe Focus is* T; *if* T *be attracted towards* S, *by the ſame Law as* L *is, than if it were attracted a great deal more or a great deal leſs.*

For, by the preced. *Prop.* the pertubation of the Proportionality of the Areas and Times, and the deformity of the Figure of the Orbit, ariſe principally from the Force repreſented by *M N*. And therefore the Areas and Times will come neareſt to the Proportionality, and the Orbit *G C B D* to an Elliptic Figure, when *M N* is nothing, or the ſmalleſt poſſible: That is, when the Accelerative attractions *S M*, *S N*, (whoſe difference is *M N*) of the Bodies *L* and *T*, are nearly equal; or, which is much the ſame, when the Accelerative attractions of the Bodies *L* and *T* towards *S*, approach as near as poſſible to an equality: (For the other part of the Accelerative attraction of the Body *L* to *S*, expreſſed by *S A*, namely *A M*, tho' it does not diſturb the ſaid Proportionality, yet it renders the Elliptic Figure deformed; that is, when *S N*, the Accelerative attraction of the Body *T* towards *S*, is not none; (for from what has been ſhewn above,

above, the Attraction *SM* is reduced to a leſſer *MN* by the Attraction *SN*,) nor leſs than the ſmalleſt of all the Attractions *SM*, (otherwiſe *MN* would be too great in every ſituation of the Body *L*,) but as it were a mean between the greateſt and leaſt of all the *SM*; that is, neither much greater nor much leſs than the Attraction *SK*. And therefore the Body *L*, by a Radius drawn to *T*, will deſcribe Areas more proportional to the Times, and a Figure approaching nearer to an Ellipſe, if *T* be attracted towards *S*, by the ſame Law as *L*, than if it were attracted much more or leſs. Which was to be demonſtrated.

The ſame holds, if (other things remaining as before) the Body *L* did not revolve about *T*; but both *T* and *L* about *S*. For, as was ſaid in the preceding, the Demonſtration is the ſame whether the diſturbing Body move or not.

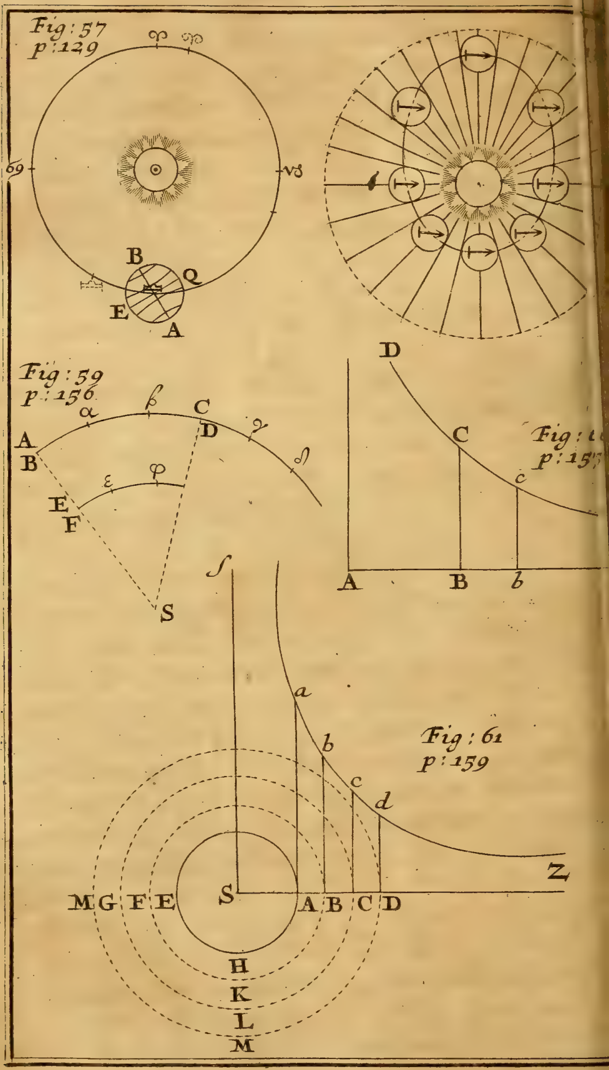
SCHOLIUM.

It may in like manner be infer'd, that if a Syſtem of Bodies *P* and *L*, *l*, λ, [*Fig.* 52.] revolving about *S*; (as in *Prop.* 59.) the Orbits deſcribed will come nearer to Elliptic ones, if all the Bodies acted upon by the ſame Laws of Attractions, than if on any account, the ſame Laws did not hold; and the common Center of Gravity of the Bodies *P*, *L*, *l*, λ, will deſcribe an Orbit whoſe Focus is in the common Center of Gravity of all the inferior Bodies; as in *Cor. Prop.* 56.

PROPOSITION LXIII.

BY reaſon of the mutual action of the Earth and Moon, their common Center of Gravity moves in the Orbis Magnus *about the Sun. After the ſame manner, the common Center of Gravity of Jupiter and its Satellites, and of Saturn and its Attendants,* *are*

Plate 11 Book 1.



are carried in the Orbits of Jupiter and Saturn; determined in Prop. 58.

Becauſe (by *Prop.* 46. and 47.) the Moon gravitates towards the Earth, and the Earth towards the Moon; the Satellites of Jupiter towards Jupiter, and Jupiter in like manner towards its Satellites; Saturn's Attendants towards Saturn, and Saturn towards its Attendants; what was propoſed is evident from *Prop.* 59. And becauſe (by *Corol. Prop.* 47.) the Sun, every Primary Planet, and its Secondary, are acted upon by the ſame Laws of attractions, the Orbit of each Secondary is very nearly an Ellipſe, having a Focus in the Center of the Primary, by the foregoing. But the motions of the Satellites are diſturbed, as has been ſhewn in *Prop.* 60 and 61. and are affected with various inequalities, that ſhall be treated of in their proper places hereafter.

PROPOSITION LXIV.

BY reaſon of the Figure of the Earth, the Equinoctial Points go backwards, and the Axis of the Earth, in every Annual Revolution, twice changes its inclination to the Ecliptic, and twice returns to its former inclination.

In *Prop.* 33. we conſidered the Axis of the Earth, as remaining always exactly parallel to it ſelf; becauſe then we had only the Phænomena of one Revolution to explain, in which ſpace of time it preſerves its Paralleliſm very nearly. But becauſe of the Earth's Figure, which is an Oblate Sphæroid, ariſing from the Cauſes pointed at in *Prop.* 31. the caſe is now ſomething different.

Let ♈ ♋ ♎ ♑ (*Fig.* 57.) be the Orbit of the Earth about the Sun ☉, *A E B* ☊ the Earth it ſelf, whoſe Poles are *A* and *B*, Equator *E* ☊. Becauſe (by *Prop.* 31.) the Earth is an oblate Sphæroid,

K (de-

(depreſſed towards the Poles *A* and *B*, and elevated towards the Equator *E* ♌,) it is like a Globe faſten'd to a Ring; for that exceſs of Matter, which is about the Equator, ſupplies the place of a Ring. And therefore, by *Corol. Prop.* 61, the Nodes of this Ring will go backwards: That is, the Earth departing from ♎, (where the common interſection of the Ecliptic and the Equator of the Earth, is directed towards the Sun ☉, and conſequently the Sun ſeen in the Equator in the Heavens, makes the Equinox,) thro' ♑ towards ♈, will arrive at the Node ♈ ſooner than at ♈ the point oppoſite to ♎: And the Earth going forward from ♈ thro' ♋, towards ♎, will arrive at the new Node ♎, ſooner that at ♎, where the Node was was in the former revolution; that is, the plane of the Equator of the Earth produced, will paſs through the Sun ſooner than the Earth's Center arrives at ♎. But then the Equinox is celebrated, when the Sun is found in the plane of the Earth's Equator; (as has been ſhewn at large, *Prop.* 33.) and they are to be looked upon as the Equinoxial Points, in which the Sun appears at the time of the Equinoxes. Wherefore it is evident, that the Equinoctial Points, and together with them, all the points of the Ecliptic, that have received any names as ſuch, (becauſe depending upon the Equinoctial Points and reckoned from them,) will ſeem to go backwards, or move *in antecedentia*; if the Fix'd Stars be taken as immovable. But if the points of the Ecliptic be looked upon as unmoved, then the Fixed Stars will be reckoned to have moved as much forwards or *in conſequentia*.

The Regreſſion of the Equinoctial Points, as has been hitherto explained, proceeds from the action of the Sun upon the Ring compaſſing the Earth

Earth at the Equator, or the redundant Matter at the Equator, as has been ſhewn, *Corol. Prop.* 61. But the Force of the Moon alſo upon this ſame Ring is conſiderably great, as is evident from the *Schol.* to *Prop.* 61, and its *Corollary*: For ſince ſhe is always in the Plane of the Ecliptic, or not far from it, ſhe will conſpire with the Sun in the ſame Effect.

Beſides, in a Semi-revolution of the Earth about the Sun from ♎ thro' ♑ to ♈, the inclination of the Equator to the Ecliptic thro' the action of the Sun, is leſs than when the Earth was in ♎; and it is leaſt when the Earth is in ♑, (by *Prop.* 60.) the Nodes of the Ring being in the Sun's Quadrature. And when the Earth is arrived at ♈, and the Nodes at the Sun's Syzigies, the former Inclination is reſtored again: In paſſing from ♈ thro' ♋ to ♎, the like leſſening of the ſaid Inclination happens again; and the Axis of the Earth Oſcillates together with the Equator. The Axis therefore of the Earth, in every Revolution, changes its inclination to the Ecliptic twice, and twice returns to the former inclination; which will happen twice, alſo in every Periodic Month, upon the account of the action of the Moon. But this Oſcillation will ſcarcely be ſenſible, whereas the regreſs of the Points of the Ecliptic is known to every Body, tho' it ariſe from the ſame Forces. For in each Equinox, the inclination of the Axis of the Earth towards the Ecliptic, returns to its former bigneſs, and in a ſeries of Years becomes not more ſenſible: But the regreſs of the Points of the Ecliptic is continually onwards, and the Equinoctial Points never return to their ancient Place, till they have compleated an intire Circle. And ſuch a change as is intirely inſen-

ſible in one Year, or half of a Year, (in which ſpace of time the whole change of the inclination of the Axis of the Earth is finiſhed,) will in a ſeries of Years become ſenſible; being made perpetually the ſame way. On the account therefore of the Figure of the Earth, the Equinoctial Points go backwards, and the Axis of the Earth every Year changes its inclination to the Ecliptic twice, and twice returns to its former ſituation *Q.E. D.*

PROPOSITION LXV.

THE common Center of Gravity of the Sun and all the Planets and Comets is at reſt; and it therefore may be taken for the Center of the Solar Syſtem, and even of the World it ſelf.

It has been demonſtrated by the Philoſophers mention'd in *Prop.*50. that the common Center of Gravity does not change its ſtate of motion or reſt, by the actions of the Bodies upon one another; but that the Law of a Syſtem of Bodies is the ſame with that of a ſingle Body, as to their continuance in the ſaid ſtate; ſince the progreſſive motion of both is to be eſtimated by the motion of the Center of Gravity. The common Center therefore of the Solar Syſtem is either at reſt, or moves uniformly forward; for excluding all external Force, this is the ſtate of a ſolitary Body. Now, the latter can't be aſſerted, ſince of the ſurrounding Fix'd Stars ſome would become nearer than formerly, and others more remote; and conſequently their ſituation and order in length of time would be ſenſibly changed, contrary to Obſervation. And ſince it has already been ſhewn, that the Sun, Planets, and Comets gravitate towards one another, and are therefore in continual motion, (only the common Center of Gravity remaining quiet,) 'tis

'tis evident that the movable Center of any one of them, can't be look'd upon as the Center of the Solar Syſtem, but the above-mention'd immovable common Center of Gravity.

But if not an inviſible Point, as the above-mention'd, but ſome conſiderable Body very near it, is to be looked upon for the Center of that Syſtem, it will be the Sun; becauſe that, by reaſon of the vaſt Quantity of Matter it contains in compariſon to the others, will ſcarce ever be ſenſibly diſtant from the common Center of Gravity above-mentioned.

And further, if any Figure agree, and bounds be ſet, to the univerſal compages of Bodies (or to the World,) no other Syſtem more juſtly claims the middle of it, than our Solar; and therefore no other Center than that above determined, is agreeable to the Univerſe. Which was to be demonſtrated.

SECTION X.

Concerning the Cauſes and Reaſon of the Motions of the Planets aſſigned by Philoſophers.

HAving been deſcribing hitherto the Syſtem of the World, and that only in general, ſo far as was ſufficient for our purpoſe, and having explained the analogous cauſe of the Motions of the greater Bodies of the World, or rather the ſame, (as is moſt fitting,) with that which we daily experience about us; namely the Gravity of all Bodies towards all Bodies, whoſe Law is the ſame with that of our Gravity, or of any other Natural Virtue propagated along Right lines

concurring in one Point; that is, having laid down the principal foundations of a Physical Aſtronomy, regularity of Method requires us to conſider the Cauſes and Phyſical Reaſons of theſe ſame Effects as are aſſigned by others; and at the ſame time ſee, whether they ſolve the Phænomena, ſo well as thoſe explained before have done. But by the Cauſes we ſpeak of here, we do not underſtand the ultimate Cauſes, or ſuch as have no Cauſes of themſelves; but ſuch as we experience about us, and can calculate their Forces, and Geometrically inveſtigate their Properties, as we do of any other Quantities; ſuch as Light, to the conſideration of which, various Sciences, as Optics, Catoptrics, Dioptrics, owe their Original; Gravity, the affections of which are conſidered in Statics, and in almoſt all Mechanics: tho' the Cultivators of thoſe Sciences neither underſtood the ultimate Cauſes of Light or Gravity, nor will they do ſo hereafter.

There are two other famous Sects of this Cœleſtial Philoſophy, that attempt to explain the Cauſes of the Motion of the Bodies of the Univerſe, the *Keplerian*, and the *Carteſian*: As for that which brought in Solid Orbs, and faſten'd Intelligences to them to move them, there is no occaſion to take notice of it, ſince there is little of Philoſophy in it, and it has been ſufficiently confuted by the Aſtronomers of the laſt Age. We ſhall give an account of both theſe, and annex the Reaſons why neither of them belongs to the Heavens, by ſhewing that the Laws of Bodies moved as theſe Philoſophers would have them, are not ſuch as are found to be obſerved by the Heavenly Bodies. And firſt we ſhall explain the *Keplerian Phyſics* deliver'd in

Book

Book 4. *Epitome of the* Copernican *Aſtronomy*, and that for the moſt part in his own Words.

PROPOSITION LXVI.

TO *explain briefly the Subſtance of the Cæleſtial Phyſics of* Kepler, *or the Cauſe and Reaſon, which* Jo. Kepler *has aſſigned, why the Planets are carried in Orbits about the Sun.*

This great Aſtronomer would have the Sun to be a vaſt Body, made to move about an Axe given by Poſition, by the Creating Power of the Divine Being in the beginning of Things; and leſt this Motion ſhould languiſh by degrees on the account of the ſluggiſhneſs of Matter, that it is conſtantly maintained by the care either of the Creator, or of ſome Mind deſtin'd to it: The ſame likewiſe he would have to be obſerved in the Earth, and the reſt of the Globes of the Univerſe. The Sun by this gyration of his Body, carrying about the immateriated Species of its own Body thro' the vaſt Spaces of the World, carries about like a Leaver, at the ſame time, the Planets which are apprehended by this Species. This Species is the Virtue of the Sun emitted in Right lines thro' the whole Mundane Space, which, becauſe it is the Species of a Body, is whirled about together with the Sun, like a rapid Vortex, running thro' the whole Mundane Space, with the ſame ſwiftneſs as the Sun turns it ſelf about its Axis, and therefore carries along with it the Planet, ſwimming as it were in this River. We are farther to take notice, that beſides this Vectory Force in the Sun, there is a natural ſluggiſhneſs in the Planets to all Motion, by which they are inclined, by reaſon of their Matter, to continue in the place where they are. So that there is a conflict between the Vectory Power of the Sun, and the ſluggiſh-

neſs of the Planet, ariſing from its Matter. Both have their ſhare in the Victory; the former moves the Planet out of the place it was in; the latter in ſome meaſure diſengages its own, that is, the Planet's Body from the ſhackles it was laid under by the Sun, when apprehended by ſome part or other of the circular Virtue; namely by that, which immediately ſucceeds the part from which the Planet had juſt diſengaged it ſelf. And becauſe this Virtue that carries along the Planet, and flows from the Sun's Body, has the ſame degrees of weakneſs at different diſtances, as Light has; this Vectory Power, like the Sun's Illumination, decreaſes as the diſtance from the Sun increaſes: Wherefore it will carry a Planet placed at a greater diſtance from the Sun more ſlowly, and conſequently will make its Periodic Time, on this ſcore, greater. Beſides, ſince a like Vectory Power is propagated from the Body of each Planet moving about its own Axis; this alſo will carry the ſecondary Planets, which it finds in this River of its immateriated Vortex, after the ſame manner; and they will never leave their reſpective Primary, becauſe they are carried by the Species of the Sun's Body about the Sun, as well as their Primary, and together with their Primary. And upon the ſimple ſuppoſition of theſe things, 'tis evident that the Primary Planets would be carried about the Sun in Circles concentric to the Sun, in the Plane of a great Circle of the Sun lying exactly betwixt its Poles; and that the Secondary Planets would be carried after the ſame manner about their Primary.

PRO-

Proposition LXVII.

To explain how Kepler *deduces from these Principles, that the Periodic Times of the Planets will be exactly in the Sesquialter Proportion of their Orbits or Circles.*

Kepler was the first who observed that this Proportion held in the Cœlestial Bodies, to the no small advantage of Astronomy and true Physics; namely, that in the Revolution of the Primary Planets about the Sun, and of the Secondary about their respective Primary, the Squares of the Periodic Times had the same Proportion as the Cubes of the Semidiameters of their Orbits or Distances from the Sun; taking the Mean distances, and abstracting from the Excentricity. And this he inferred from his own Principles after the following manner. Since the Vectory Power of the Sun, (whereby the Planets are carried about by the immateriated Vortex or River of the Solar Species,) decreases like the Illumination, the distance from its fountain or the Sun increasing; (that is, in the duplicate ratio of the distances,) and only exercises its Virtue according to one dimension, namely, in Length. This Vectory Power in producing this Effect, (namely, carrying the Planet round) will be lessen'd only in the simple ratio of the distance from the Sun increased: And since (by his Archetypical Proportions) he makes the bulks of the Planetary Bodies to have the same ratio as their distances from the Sun; the Vectory Power of the Sun, will employ equal Forces in the carrying round each Planet; namely, so much greater on the account of the greater size or magnitude of the Planet, that the Moving Species lights upon, as the Moving Virtue in that distance is weaker, and in the same ratio; namely, of the distance

diſtance from the Sun. Again, induced by the Geometrical concinnity, he ſettles the Quantity of Matter in the Planets compared with one another, in the ſubduplicate ratio of the Bulk or Magnitude; (and ſo the Denſity will be in the ſame ſubduplicate ratio of the Bulk or Magnitude reciprocally,) that is, as was ſaid before, in the ſubduplicate ratio of the diſtance from the Sun.

Since therefore the Periodic Time in a Planet, that is more diſtant from the Sun, is greater than in a Planet that is nearer, becauſe the Way, (that is, the Orbit) is greater, and becauſe the Quantity of Matter to be tranſported is greater, (but in other reſpects equal, becauſe equal moving Forces are employed upon them, as was ſhewn above,) and ſince the greatneſs of the Way is as the diſtance from the Sun, and the Quantity of Matter as the ſubduplicate of that diſtance; on both cauſes conjoined, the Periodic Time of the remoter Planet will be greater, in the ratio compounded of theſe: But the ratio compounded of the ſimple and ſubduplicate, is the ſeſquialter or ſeſquiplicate: Wherefore the periodic Time of the remoter Planet will be greater than that of the nearer, in the ſeſquiplicate ratio of the diſtances from the Sun; that is, the Squares of the Periodic Times are as the Cubes of the diſtances from the Sun. Which things are to be underſtood after the like manner concerning the Secondary Planets revolving about their Primary.

PROPOSITION LXVIII.

TO *explain the Cauſes aſſigned by* Kepler, *why the Orbits of the Planets are Excentric.*

Since each Planet is a Body cognate with the Body of the Sun, having one part (like a Loadſtone)

ſtone) that is, friendly to the Sun, and the other the quite contrary. In the converſion of the Body of the Sun, this Virtue thereof, whereby it attracts or repels the Planet, according as its friendly or contrary part is turn'd to the Sun, is alſo turned about; juſt as in the converſion or turning a Load-ſtone round, the attractive or repelling Force is transferred toward different parts of the World. And when the Sun lays hold of a Planet by this Virtue of its Body, whether attracting or repelling, or acting dubiouſly between both, it carries it round along with it; for it retains it both by attracting and by repelling it, and by retaining it, it carries it round. Each of our Load-ſtones indeed has two Poles or Places of different Virtues, by the one of which, one Load-ſtone attracts another after a certain direction; but by the other it repels the ſame. But in the Heavens things are ſomething otherwiſe. For the Sun poſſeſſes this active or energetic Faculty of attracting or repelling a Planet, not in this or that place or part, but in all the parts of its Body. Therefore it is probable, that the Center of the Body of the Sun anſwers to one extremity, place, or pole of the Load-ſtone; and the whole ſurface to the other; ſince it attracts or repels a Planet according to a certain direction, not by this or that place on its ſurface or part of its Body, but by the whole.

Theſe things being thus ſettled, 'tis evident, that the Body of the Planet will not be carried about the Sun at the ſame diſtance from the Sun, every where; but while it turns [*Fig.* 58.] its friendly part to the Sun, it will come nearer to the Sun, till the Right lines drawn according to the direction of this part, (that is, the Fibres along which this attractive Virtue is pro-

propagated

pagated from the Sun,)being almoſt every where parallel to themſelves, are no more inclined to the Sun, but a Right line drawn from the Sun to the Planet becomes a perpendicular to theſe Fibres; in which caſe, ſince the Sun does attract no longer, the effect of the attraction, (namely, the acceſs of the Planet towards the Sun) will ceaſe, and the Planet will then be in its neareſt approach to the Sun or in its Perihelion. The Planet being ſtill carried about, by the above-deſcribed Vectory Power of the Sun, its unfriendly part, or that which ſhuns the Sun, will by little and little be directed towards the Sun, and hereupon the Planet will be continually repelled from the Sun; and that will be continued till neither of theſe parts or places be inclined towards the Sun, but a Right line connecting the Sun and Planet, would again become perpendicular to the Fibres of the Virtue; and then the Planet will be found in its Aphelion, becauſe at the greateſt diſtance that the repelling force of the Sun can remove the Planet to, while it is carried about. And the Planet being librated by theſe Fibres, ſo as to be attracted by the Sun in one part of its Orbit, and repelled by it in the other, deſcribes an Orbit about the Sun indeed, but not about it as its Center; but Excentric to the Sun. Again, becauſe the Species of the Sun's Body communicating a Motion to a Planet, is thinner and weaker in a larger Circle in the ratio of the diſtance from the Sun, as it produces this effect, (as was ſhewn in the preceding Propoſition) and the Planet in the mean while continues of the ſame Bulk and Denſity, becauſe the ſame Planet: 'Tis evident, that, other circumſtances continuing the ſame, the Celerity of the Planet is leſſen'd, in the ſame ratio with the conſtipation

tion and density of the Vectory Power of the Sun; that is, reciprocally as the distance from the Sun. Farther, the Virtue of the Sun that attracts and propels the Planet issuing along virtuous lines proceeding from the Center of the Sun, and going round together with the Sun, has its strength proportional to theCo-sine of the Angle, made by the Radius of the Sun and the Fibre that admits the Solar Virtue into the Planet: And therefore this Excentric Orbit will not be a perfect Circle, but an Ellipse, narrower and more depress'd on the sides, and having the Sun for one of the Foci.

Now, from the same Principles he shews, that the Ellipse is described about the Sun in the Focus, in such a manner as that equal Areas are described in equal Times, by a Radius drawn to the Sun, which he calls the *Vector*; and universally, that the Areas, which this same Radius sweeps over, are proportional to the Times wherein they are described.

But since these libratory Fibres, by the help of which the Planets are librated upwards, always continue as it were parallel to themselves while the Planet is carried intirely round, and (by what was shewn before) is so inclined in respect of the whole Universe, as that the line of the Apsides of each Planet is perpendicular to those Fibres, which are placed either in the plane of the Orbit, or of the Planet, or in a situation parallel to it; and since the Axis of no Primary Planet, about which it is turn'd as it were by a diurnal Motion, has this Position, and no other Line but this Axis, and Right lines parallel to it, can keep the same situation in regard of the Universe, if the intire Globe of the Planet be solid and united; 'tis evident that *Kepler* was obliged to suppose within this exterior Crust of the Planet,

net, (whose Axis keeps a given Position, namely, that about which the Planet is turn'd in its Diurnal Motion,) a Globe intirely separate and divided from this Crust, and no ways yielding to its Diurnal Motion, whose Axis and Right lines parallel to it may sustain the Places of the said *Libratory* Fibres, and conserve the Position necessary to this purpose described above; while the exterior Crust, according to the nature of each Planet, revolves by a Diurnal Motion peculiar to it self.

PROPOSITION LXIX.

T*O explain the Causes assigned by* Kepler, *why the Orbits of the Planets are inclined to the Ecliptic.*

In this *Keplerian* Physics, which we are now upon, if there were no Fibres (such as we described in the preceding Proposition) in the Planets, by which they are librated upwards; or if these Fibres turned their side to the Sun; that is, neither that part which was friendly to the Sun, nor that part which was the contrary; each Planet remaining at the same distance from the Sun, would describe a perfect Circle concentric with the Sun. After the like manner, all the Planets (being acted upon only by Forces arising from hence) would go round in the same Plane in an uniform and direct Stream as it were, in the middle of the Channel, (or in a Circle lying exactly between the Poles of the Revolution of the Sun, called by him the *Via Regia*, or *Royal Way.*) But because they wander variously on this or that side of the *Via Regia*, the Cause of this Effect must be somewhat like that, whereby a Ship under sail is thrust to one side of the line, according to which it would have been moved by the Wind alone, namely, by an Oar held

held obliquely to the Stern, or by the Rudder kept in an oblique poſition. In each Planet there are Fibres (which he calls from their Office, the *Fibres of Latitude*) always placed in the Plane of its Orbit, or parallel to it, and which remain always parallel to themſelves in every converſion or motion of the Planet; ſo that when a Planet is in a Node or common interſection of the Plane of the *Via Regia*, with the Plane of the Orbit, the middle of the Fibre of Latitude that paſſes thro' the Center of the Planet, touches the Orbit; and therefore at every one of theſe Fibres, one part towards one ſide of the *Via Regia*, the other part towards the other ſide, ſtands without that Plane, inclined to it at the ſame Angle, as the Orbit of the Planet is to the *Via Regia*. When therefore the Species of the Sun's Body meets with that half of the Fibre, which is behind and ſtands upwards, it will impel the Planet that way, as the half, which is foremoſt, tends towards; that is, according to the Plane of the Orbit proper to that Planet: But ſince theſe Fibres of Latitude continue in a poſition parallel to themſelves thro' the whole Revolution, hence it comes to paſs, that in a Planet placed in its Limit or greateſt diſtance from the *Via Regia*, neither extremity of the Fibres go foremoſt, but the Fibres directed as it were towards the bottom of this River, (that is, towards the Sun,) and receiving the impulſe on it's ſide directly oppoſed to it, give no cauſe for an ejection farther any way; 'till a change be made in theſe points of the Limits, ſo as that the extremity of the Fibres, which went foremoſt before the arrival of the Planet to the Limit, may now go behind, the other going before; and for that reaſon the Planet may begin to approach again towards the

the *Via Regia.* For inſtead of the Power of the Rudder in a Ship, there is in the Planet another Force of the Fibres much more ſuitable; becauſe, as the Libratory Fibres have a natural Power to retain a parallel ſituation in the tranſportation of the Body, ſo alſo there is in the Fibres of Latitude, beſides a like Power of retaining a parallel ſituation, another natural Power of agility, or of preſerving the ſame Line, and of deriving according to the direction of it, a Motion impreſs'd upon themſelves, ſo far as the Motion tends the ſame way with that other extreme of the Fibre. Becauſe theſe Fibres of Latitude are entirely different from the Fibres deſcribed above, by which the Sun attracts or repels a Planet, as well in regard of their nature and office, as in reſpect to their ſituation ; for the Fibres of Latitude are every where parallel to a Right line touching the Orbit at the Node in the *Via Regia* : But thoſe others (by preced. *Prop.*) are parallel to a Right Line, which is a Tangent to the Orbit in the Aphelium : Theſe Fibres, according to *Kepler*, ought to be found in different Bodies, ſince (in *Kepler*'s judgment) the Apſides go forward, and the Nodes go backward, tho' it be but ſlowly in the Primary Planets ; and 'tis certain that the ſame happens in the Secondary. Wherefore, beſides an exterior Cruſt furniſhed with Fibres, by which the Axis, about which the Planet revolves by its Diurnal Motion, is kept parallel to it ſelf in its whole circuit about the Sun, he found it neceſſary to ſuppoſe an interior Cruſt (which might be like a Nucleus,in regard of the exterior,) intirely ſeparate from the former,and agitated by other Forces ; in which there are Fibres, by which the Planet may be librated upward and downwards : And within this Nucleus,

cleus, or rather Cruſt, he is forced to ſuppoſe again another interior Nucleus ſeparate from the former middle one, and having a different Period, in which the Fibres of Latitude are expanded; ſince it is impoſſible that theſe different Fibres having different Periods, ſhould be ſpread in one and the ſame Solid.

Proposition LXX.

To point out and refute very briefly the miſtakes that are to be found in the Keplerian *Celeſtial Phyſics explain'd above.*

Tho' we undertake to oppoſe *Kepler's* Celeſtial Phyſics, yet we don't do it with ſuch a temper, as if we rank'd him a Perſon, whoſe Fame will be Immortal among the common Syſtem-makers or hunters after Phyſical Cauſes. For he was ſo far from this, that on the contrary, he is the only Man (excepting perhaps ſome of the Ancients, as *Pythagoras*, &c.) who has treated of the Celeſtial Phyſics in a Mathematical manner. His *Archetypical Ratios, Geometrical Concinnities*, and *Harmonic Proportions*, ſhew ſuch a force of Genius as is not to be found in any of the Writers of Phyſical Aſtronomy before him: So that *Jeremiah Horox*, a very competent judge of theſe matters, tho' a little averſe to *Kepler* in the beginning of his Aſtronomical Studies, after having in vain tried others, entirely falling in with *Kepler's* Doctrine and Phyſical Reaſons, thus addreſſes his Reader, *Kepler is a Perſon whom I may juſtly admire above all Mortals beſide; I may call him Great, Divine, or even ſomething more; ſince* Kepler *is to be valu'd above the whole Tribe of Philoſophers. Him alone let the Bards Sing of----. Him alone let the Philoſophers Read; being ſatisfy'd of this, that he who has* Kepler, *has all Things.*

For what greater, or more, like something Divine, could happen to a Mortal in this matter than by these Harmonic Speculations, exactly to determine the proportion of the Orbits of the Planets; than to be the first that should find out the true way of the Planet about the Sun (namely the Elliptic;) and lastly, than to shew the true manner of a Planet's Motion in this way; *viz.* that by a Radius drawn to the Center of the Sun, placed in the Focus of the Ellipse, it describes Areas proportional to the Times? But there are some things which I can't approve of in *Kepler*'s Mechanical cause of these Motions and they are these following.

1. If neither the Friendly nor the Opposite part of the Planet be turned to the Sun, or if the Planet (on the account of the weakness of the Fibres, whereby it is librated upwards and downwards) be so disposed, as neither to be attracted or repelled by the Sun, (as it happens in Venus, according to *Kepler*'s judgment;) then he would have it, that the Planet describes a perfect Circle, by the Species of the Sun which is carried about; whereas 'tis certain, that a Body which is not urged by a Force tending to a Center, will not describe a Circle, but a Right line. Since therefore the Orbits of Planets are Circular even in this case, 'tis evident that they are impelled towards a Center by some other Force, besides that which arises from the obverting the Friendly part.

2. During the Planet's ascent from the Perihelium to the Aphelium, that part which is Unfriendly is turned to the Sun, and consequently is repelled from the Sun, and in this case it is urged only by that Force, which is directed along the Line that connects the Sun and Planet. But 'tis well known in Mechanics, that a Plane

Planet acted upon by this Force alone, would be ſo far from deſcribing an Ellipſe, having the Sun for a Focus, that on the contrary, it would deſcribe a Curve convex towards the Sun. Wherefore a Planet, whoſe Orbit is an Ellipſe, having the Sun for a Focus, muſt neceſſarily be urged, even in its aſcent from the Sun, from the Perihelium to the Aphelium, by a Force tending to the Sun, ſince an Ellipſe is concave towards both Foci. *Kepler* therefore, muſt be very much out of the way, when he would have a Planet repell'd by the Sun, in its aſcent to the Aphelium, or when he would have a Planet neither attracted nor repelled, when it deſcribes a perfect Circle about the Sun concentric to it: For unleſs in both caſes it be attracted or otherwiſe impelled towards the Sun, it will not deſcribe a Curve concave towards it: For it can't be retain'd thus, without ſome kind of attraction.

3. In his enquiry into the proportion of the Planetary Orbits, he aſſumes ſome things plainly contrary to Obſervation, how much ſoever it is that they agree with his imagined Architypical Ratios: As that the Magnitudes of the Planetary Bodies are as their Diſtances from the Sun; and that their Denſities are reciprocally as the Square Roots of their Magnitudes. From whence it follows, that the Quantity of Matter in the Planets is directly as the Square Root of their diſtances from the Sun: Becauſe the quantity of Matter is as the Magnitude and Denſity conjunctly. But every Body knows that Mars is leſs in bulk than Venus, which notwithſtanding ought to be as big again as Venus, according to *Kepler*'s reaſoning; becauſe diſtant as far again from the Sun. Obſervation informs us, that the Globe of Jupiter is bigger than Saturn, tho' it ought by the aforeſaid Rule to be as little a-

gain. Notwithſtanding he might deduce from the things that he knew, and firſt diſcovered, (namely, that the Path of a Planet is the Perimeter of an Ellipſe, one of whoſe Focus's is in the Center of the Sun, which it ſo deſcribes, as that its Radius that carries it, ſweeps Areas proportional to the Times,) without the conſideration of the Magnitude and Denſity of the Planet, that the Periodic Times of the Planets revolution about a common Center, are in the ſeſquiplicate ratio of their greater Axes, or mean diſtances from the Sun. But it is probable that *Kepler* deduced the Proportion of the Planetary Orbits, and their Number, from the intervals neceſſary for the interpoſition of five regular Bodies; and that he afterwards aſſigned to them ſuch Magnitudes and Denſities, as he could draw this ſame proportion from by another Method.

4. Not to ſpeak of his Virtue, which has too little of Mechaniſm, and too great an affinity to an Animal Power, which he gives to the Fibres of Libration and Latitude in the Planets; (to the latter, natural power of Agility, but to the former, in one part, an inclination towards the whole Body of the Sun; in the other, a flight from it;) and the diviſion ariſing from hence, of every Planet into a Shell, and a Kernel ſeparate from the Shell, and librated in its cavity, in a different Period, and in ſome another Nucleus within the former.

Not to mention, I ſay, theſe things, which have but little of Mechanics in them, and foreign to the Method which Nature uſes, *Kepler*'s Syſtem about the manner of the Sun's carrying round, attracting, or repelling the Planets, is founded upon what is contrary to one of the chief Laws of Nature and Motion, whereby

care

care is taken that there be a Reaction always contrary and equal to any Action. For according to *Kepler*, in Epit. Book IV. *This Virtue of Expulsion and Attraction is not in the Planets themselves, but in the Sun only. And tho' the attractory and repulsory Power of the Sun be extended to, and affect the Planets, yet like power of a Planet does not reach to the Sun and affect it.* For were it so, he confesses, that *the Sun would be moved, out of the place it has in the Center of the World, by the Planets, in the reciprocal ratio of their Bodies; or at least it ought to titubate or reel as it were, being sometimes attracted more this way, sometimes more that way, according as more Planets happen to come together on the same side of the Sun, having similar Powers.* But this he entirely gets rid of, by denying a mutual attraction. *Kepler* here had forgot the true Doctrine of Gravity, that he had laid down about 11 Years before, in his *Introduction to his Commentaries upon Mars*, built upon these Axioms: *That Gravity is a mutual affection of union or conjunction in Bodies that are a-kin to one another. That two Bodies, if they are not hindered, will meet in the intermediate Space, each having moved one towards the other a length reciprocally proportional to the magnitude of the other: So that if the Moon and Earth were not retain'd each in their Circuit, the Earth would ascend towards the Moon the Fifty fourth part of the distance betweeen them; and the Moon would descend towards the Earth about Fifty three parts of the interval, and they would both meet there; supposing the substance of both to be of the same density. That the Moon draws up the Waters of the Terrestrial Globe, from whence arises a flux, where the Channels of the Ocean are broadest, and afford room for a reciprocation. And if the Earth should cease to attract towards it self its own fluids, the Water of the Sea would be raised and flow towards the*

Moon. Nor did he fear himſelf, anſwering the difficulties above-mentioned, about the titubation of the Sun, and its removal out of the Center of the World; where he ſpeaks after the following manner: *For ſo great is the bulk, ſo great the denſity of the Matter of the Body of the Sun, and ſo great the attractive and repulſive Virtue of it; on the contrary, ſuch is the ſmallneſs both of the Planet and its renitence, that the Sun is in no danger of being removed out of its place.*

From what has been deliver'd, 'tis evident, how near this great Philoſopher and Aſtronomer was, to the finding out the true Cœleſtial Phyſics, (who even in *Epit. Aſtron.* Book 5. § 1. had ſaid, *As the Sun attracts the Planet, ſo the Earth attracts Bodies; on the account of which attraction, the Bodies are ſaid to be heavy,)* and how much aſſiſtance he has given in this matter, and how rightly he Propheſied, when he ſaid, *The true Cauſes, Quantities, and Directions of the Motions of the Apſides and Nodes of the Planets, and other matters of this kind, remain in the Pandects of the following Age, not to be underſtood before the Divine Being, the diſpoſer of Times and Seaſons, has open'd this Book to Mortals.*

PROPOSITION LXXI.

TO *explain briefly and ſummarily the* Carteſian *Cœleſtial Phyſics, or the Cauſe and Reaſon, why the Planets are carried round the Sun, which* Renatus des Cartes *invented.*

Such as have endeavored to find out the Phyſical Cauſes of the Cœleſtial Motions, have all of them ſought them in the Sun. For the Rotation of the Sun in his Space about an immovable Axis, the ſame way that all the Planets take and follow in, and that in a ſhorter Period than that of Mercury,which is the neareſt to him,and every other Planet in a greater period

as it is farther from the Sun ; and the Rotation of the Primary Planets about their proper Axes ; and of the Secondary again about theſe Primary, juſt as thoſe Primary revolve about the Sun ; all towards the ſame coaſt of the Heavens, and almoſt in the ſame Plane, have the higheſt degree of probability. And theſe Phænomena do plainly beſpeak, either that all this motion of the Primary Planets ariſes from the Sun's motion, or at leaſt, that both of them depend upon one common Cauſe. Hence it came about, that ſeveral Philoſophers thought the Planets were carried by the Æther, whirl'd about the Sun by a Vortex or Whirl-pool ; and that long before *Cartes*, or even *Epicurus*'s Time. But *Kepler*, who was better acquainted with the motions of the Stars, ſaw well enough that they could not be carried about by any material Vortices: For if the Planets ſwimming in the Matter of the Vortex, and as it were at reſt in it, were carried about together with the parts of the Vortex, he ſaw that they would deſcribe perfect Circles, and have other periods in regard of their diſtances from the Center, than we find by Obſervation agree to them. He therefore rather choſe to ſubſtitute the immateriated Species of the Sun's Body, inſtead of Matter whirl'd about like a Vortex, yet calling it by the name of the *Cæleſtial Vortex and River*. But afterwards *Cartes*, making light and taking no notice of theſe niceties and Aſtronomical Obſervations, and being reſolved to frame a World, brought in Vortices again ; together with a popular way of repreſenting the motions of the Stars. And tho' Vortices were introduced into the Heavens by others before *Cartes*, and have been made uſe of by others ſince him, and preſſed into other ſervice, yet they retain

Cartes's Name. We ſhall therefore briefly deſcribe this Syſtem, ſo far as ſerves our purpoſe, in *Cartes's* Words, out of the *Third Part of his Principles of Philoſophy.*

Cartes ſuppoſes then, That all the Matter of which the viſible World is made, was, in the beginning of Things, divided by God into Particles very nearly equal, of a moderate ſize, and equally moved, not only each about its proper Center, and ſeparately from one another, ſo as to compoſe a Fluid; but ſeveral alſo about ſome other points, in like manner remote from one another, and ſo they compoſed various Vortices. Afterwards he reſolves theſe equal Particles, worn a little by their inteſtine motion, into Globules of different Sizes, all which he calls Particles of the ſecond Element, and a part of a Fluid form'd out of the irregular minute parts rubbed off from the corners of the Globules, and moved always with a very great rapidity, he calls the Matter of the firſt Element. Now, there being more of this ſort of Matter than was neceſſary to fill up the interſtices of the Globules, what remained was thruſt down towards the Center by theſe Globules receding from their Center, by reaſon of their circular motion, and being there collected into a Spherical Body, conſtitutes the Body of the Sun in the Center of each Vortex. Beſides this equal Motion impreſſed originally upon theſe Particles, about the Center of the Vortex, the Sun, juſt form'd in the center of the Vortex, and revolving about its own Axis the ſame way with the reſt of the Matter of the Vortex, and always emitting ſome of it ſelf along the narrow paſſages that are within the Globules of the ſecond Element towards the Ecliptic, or Circle lying juſt in the middle between the Poles, (receiving as much at its Poles

Poles from the neighbouring Vortices, as it emits,) has a power of carrying thoſe Globules along with it; thoſe nearer, quicker, but thoſe farther off, ſlower. But ſince theſe lower Globules of Cœleſtial Matter move quicker than the ſuperior ones, they ought alſo to be leſs. For if they were greater or equal, they would upon that account have more Forces, and therefore would become uppermoſt by their centrifugal Force. And all theſe particulars will be the ſame in any Vortex whatever, within certain limits, beyond which the upper Globules are moved quicker than the lower, being as to their Sizes equal. Theſe limits in the Solar Vortex he places about the Orb of Saturn, or a little beyond.

Theſe things being ſuppoſed, if any Star, which being placed in the center of its Vortex, does the office of a Sun, ſhould happen to be covered with Spots, and by that means weakened ſo as to be overcome and carried along by the neighbouring Vortex of the Sun; if this Star be but capable of a leſs degree of agitation, or have leſs ſolidity than the Globules of the ſecond Element that are near the circumference of our Vortex, yet ſomething more than ſome of thoſe which are nearer the Sun; we are to underſtand that this Star, ſo ſoon as 'tis arrived into the Vortex of the Sun, ought to deſcend continually towards its Center, till it arrive to ſuch Cœleſtial Globules as it is equal to in ſolidity or aptitude to perſevere in its motion along Right lines. And when it has at laſt got thither, it will neither approach nearer to the Sun, nor yet farther off from it, unleſs it be ſomething propelled from thence, by ſome other Cauſes afterwards to be related: But being in *equilibrio* with thoſe Cœleſtial Globules, will

revolve

revolve about the Sun with them, and become a Planet. Every one of them therefore is at reſt, in that part of the Heaven where it is; and all the variation of ſituation obſerved in them, only ariſe from the motion of the matter of Heavens or Vortex that contains them. According to this Syſtem then, nothing hinders us from imagining the Space, in which at preſent the ſingle Vortex of the Sun is contained, to be at firſt divided into ſeventeen or more Vortices; and that they were ſo diſpoſed, that the Stars, which they had in their Centers, were by little and little at length covered with ſeveral Spots, and then theſe deſtroy'd by other Vortices, the one quicker, the other ſlower, according to their different ſituation. So that ſince theſe four, in whoſe Centers were the Sun, the Earth, Jupiter, and Saturn, were bigger than the reſt, the Stars which were in the Centers of the five leſſer that ſtood about Saturn, deſcended towards Saturn; and thoſe in the Centers of four others about Jupiter, towards Jupiter; in like manner the Moon near the Earth, towards the Earth: Afterwards Mercury, Venus, the Earth, with the Moon, and Mars, (each of which Stars alſo formerly, had a Vortex,) towards the Sun; and at length Jupiter and Saturn, together with the leſſer Stars about them, ſunk together towards the Sun, much greater than themſelves, after their own Vortices were deſtroyed: But the Stars of the other Vortices, if they were ever more than ſeventeen in this Space, paſs'd away into Comets, moving almoſt in a Right line out of one Vortex into another, and never returning. So that now, ſince we ſee the Primary Planets carried at different diſtances from the Sun, we are to judge, that this happened, becauſe the ſolidity of ſuch of them as

are

are nearer the Sun, is leſs than that of thoſe that are farther off: And that from hence it came to paſs, that the ſame part of the Moon is turned towards the Earth, or at leaſt does not defect much from it, becauſe its other part, which is turned from the Earth, is ſomething more ſolid, therefore in moving round the Earth ought to deſcribe a greater compaſs. And ſince we ſee the inferior of thoſe higher Planets to move round quicker, we ſtill judge it to ariſe from hence, that the Matter of the firſt Element, which compoſes the Sun, by whirling round very rapidly, carries along with it the nearer parts of the Heaven, and the Planets ſwimming in them, which are there relatively at reſt, better than the more remote. Jupiter, the Earth, *&c.* revolve about their own Axes, becauſe they were formerly lucid Stars placed in the Centers of ſome Vortices, and no doubt gyrating there like our Sun, and even now the Matter of the firſt Element collated in the Centers of them, ſtill retains the ſame Motion, and impels them.

Laſtly, we are not to think that all the Centers of the Planets are always accurately in the ſame Plane, nor the Circles they deſcribe perfect and compleat: But, as we ſee it happens in all other natural things, only nearly ſo, and in time perpetually changing. For ſince all the Bodies that are in the Univerſe, are contiguous, and act upon one another, the motion of each depends upon the motion of all the reſt, and is therefore varied innumerable ways. On which account we find, that tho' all the Planets affect a Circular Motion, yet none of them ever deſcribe Circular ones, but always err ſomething from them, both in Longitude and Latitude.

PROPOSITION LXXII. LEMMA.

IF two Bodies revolve about the ſame Axe, in the ſame diſtance from the Axe, and two other Bodies likewiſe in any other diſtance; I ſay, the ratio of the difference of the Angular Motion of the former Bodies, to the difference of the Angular Motion of the latter, is compounded of the ratio of the tranſlation of the former Bodies from one another, to the tranſlation of the latter Bodies from one another, and of the ratio of the diſtance of the latter from the Axe, to the diſtance of the former from the ſame.

The *Angular Motion* is that by which the Angle is formed at the Axe about which a motion is: Hereupon they are ſaid to have the ſame or equal Angular motion, that being placed at any unequal diſtance from the Axe, make equal Angles at the Axe, in the ſame or equal Times: And the difference of the Angular Motions is the difference of the Angles formed at the Axe in the ſame Time by the Bodies.

At the ſame diſtance from *S* [*Fig.* 59.] the Axis of Motion, let there be two contiguous Bodies *A* and *B*; likewiſe two others *C* and *D*. Let them afterwards be moved any how, and ſeparated from one another; namely, *A* in α, and in the ſame time *B* in β, *C* in γ, and *D* in δ. 'Tis evident (from *Prop.* 33. *Elem.* 6.) that the difference of the Angular Motions of the two Bodies *A* and *B*, (namely the Angle $\alpha S \beta$) is to the difference of the Angular Motion of the two other *C* and *D* (that is, the Angle $\delta S \gamma$) as the tranſlation of the two former from one another, (or the way, whereby the one is transferred from the other, namely $\alpha \beta$,) to $\delta \gamma$, the tranſlation of the two latter; that is, if the diſtances are equal, the tranſlations are as the differences of the Angular Motions. Again, if at any unequal diſtances from the Axe *S*, the two contiguous

guous Bodies A and B, and the two others E and F, be so moved into the places α, β, and ϵ, φ, that the differences of the Angular Motions (*viz.* the Angles $\alpha S \beta$, $\epsilon S \varphi$) be equal; then will (by *Prop.* 33. *El.* 6.) the Translations $\alpha \beta$, $\epsilon \varphi$, be as the distances $S A$, $S E$. And therefore (by *Prop.* 23. *Elem.* 6.) if neither the distances from the Axis, nor the differences of Angular Motion be equal, the ratio of the Translations will be compounded of the ratio of the differences of the Angular Motions, and of the ratio of the distances from the Axis: And therefore (taking away from both sides the ratio of the distances from the Axe,) the ratio of the differences of the Angular Motion is compounded of the direct ratio of the translations of the Bodies from one another, and the reciprocal ratio of the distances from the Axis of Motion. *Q. E. D.*

PROPOSITION LXXIII. LEMMA.

If the Curve DCE (Fig. 6.) *be so related towards the Right line* A E, *produced infinitely towards* E, *that a Perpendicular* C B, *let fall from any point of the Curve to* A E, *is reciprocally as the Cube of the Right line* A B, *intercepted between* B *and* A *a point given in the Right line* E A, *the infinite space* B C E E, *contain'd between the Right lines* C B, B E, *and the Curve* C E, *will be reciprocally as the Square of the Right line* A B.

Call $A B$, x; $B C$, y: and because y is as $\frac{1}{x^3}$ then will $y = \frac{1}{x^3}$ drawn into a certain constant or invariable Quantity; let that be a^4, that the Law of Homogenity may be observed: that is, $y = \frac{a^4}{x^3}$ or $y = a^4 x^{-3}$. And therefore, the sum

of

of all the y's (or the Flowing Quantity whoſe Fluxion is y, or its value a^4x^{-2}) will be $-\frac{1}{2}a^4x^{-2}$: That is, as is known to Geometers, the Area that lies on one ſide of the Line BC, namely the Area $BCEE$, is equal to $\frac{a^4}{2x^2}$. And, becauſe $\frac{1}{2}a^4$ is a given quantity, the Area $BCEE$ will be as $\frac{1}{x^2}$; that is, reciprocally as the Square of the Right line AB: or the Area $BCEE$ is to the Area $bcEE$, as Ab^2 to AB^2. *Q. E. D.*

PROPOSITION LXXIV. LEMMA.

IF two contiguous Bodies ſlide one upon the other, and the ſame be done in two other Bodies, preſſed by the ſame Force as the former towards one another; I ſay the impreſſion made by the friction of the former upon one another, is to the like impreſſion of the latter, in a ratio compounded of the ratio of the tranſlation of the former to the tranſlation of the latter, and of the ratio of the Superficies, wherein the former touch one another, to the Superficies, wherein the latter touch one another; namely, wherein the impreſſions are made.

If the contiguous Surfaces were perfectly ſmooth, there would be no impreſſion of the Bodies upon one another, while they ſlide. 'Tis evident that this ſuppoſition is very natural, that the reſiſtance ariſing from the defect of ſmoothneſs in the parts, is (other things being alike) proportional to the Velocity whereby the parts are ſeparated from one another; that is, if the contiguous Surfaces are equal, that the reſiſtance and the impreſſion of the Bodies upon one another ariſing from thence, is proportional to the relative Velocity, or to what is in a given Time proportional to it, the tranſlation of the Bodies. But if the Tranſlations (or Velocities wherewith the

the parts are ſeparated from one another) are equal, the Reſiſtances or Impreſſions, are as the Superficies wherein the Bodies two by two touch one another.

And therefore, ſince neither the Superficies, nor Tranſlations are equal; tho' all the reſt are, (as the Force preſſing the Bodies towards one another, the want of Smoothneſs, *&c.*) the Impreſſions which the Bodies two by two make upon one another, are in a ratio compounded of the ratio of the contiguous Surfaces, and of the ratio of the tranſlations from one another. *Q. E. D.*

PROPOSITION LXXV.

If a ſolid Sphere revolves in an uniform and infinite Fluid, about an Axe given by poſition, with an uniform Motion, and by the ſole impulſe of it a Fluid be moved circularly, and each part of this Fluid perſevere uniformly in its motion; I ſay, the Periodic Times of the parts of the Fluid, are as the Squares of the Diſtances from the Center of the Sphere.

Let *A E H* [*Fig.* 61.] repreſent a Sphere revolving uniformly about the Axis *S*. Let the infinite Fluid of the Sphere ſpreading about every way, be conceived to be divided into innumerable concentric Orbs of the ſame thickneſs, every one of which conſidered alone is Solid; let theſe be expreſſed by the Circles *B F K*, *C G L*, *D M N*, *&c.* Becauſe the Fluid thus divided is ſuppoſed to be homogeneous; (that is, all its parts equally compreſs'd;) the impreſſions of the contiguous Orbs upon one another (by the preced. Lemma) are as their tranſlations from one another, and the contiguous Superficies in which the impreſſions are made conjunctly. If the impreſſion upon any Orb is greater or leſs on the Concave part, than on the Convex part, the

the ſtronger Impreſſion will prevail, and either accelerate or retard the velocity of the Orb, according as it is directed towards the ſame way with its Motion, or a contrary. Wherefore, ſince (by ſuppoſition) a Fluid perſeveres uniformly in its motion, the Impreſſions made on both parts of each Orbit towards the contrary ways, are equal; and therefore the ratio which is equal to the ratio of the Impreſſions, (namely the ratio compounded of the ratio's of the tranſlations and contiguous Surfaces) is alſo a ratio of Equality. Conſequently the Tranſlations are reciprocally as the contiguous Superficies; that is, reciprocally as the Squares of the diſtances from the Center: But (by *Prop.* 72.) the differences of the Angular Motions are as the Tranſlations directly, and the diſtances from the Center of Motion reciprocally: And therefore the ſaid differences of the Angular Motions of theſe concentric Orbs, are reciprocally as the Squares of the diſtances from the Center, and reciprocally as the diſtances from the Center; that is, reciprocally as the Cubes of the diſtances from the Center conjunctly. If, therefore, upon all the points of the Right line *S Z* drawn from the Center, where the contiguous Superficies of the concentric Orbs cut that Line (namely, *A*, *B*, *C*, *D*, *&c.*) perpendiculars to *S Z* be erected, *A a*, *B b*, *C c*, *Dd*, *&c.* reciprocally proportional to the Cubes of *S A*, *S B*, *S C*, *S D*, *&c.* theſe Ordinates will be reſpectively as the ſaid differences of the Angular Motions of the reſpective concentric Orbs, and the ſum of theſe Ordinates, as the ſum of the ſaid differences; that is, (ſince the outmoſt or infinitely diſtant Orb is not moved at all,) as the whole Angular Motions. If now the thickneſs of thoſe Orbs be leſſen'd *in infinitum* (by which means the Medium ſpread round

round about becomes ſo far uniformly fluid,) the Sums of the foreſaid Ordinates will become the Areas *A Z a*, *B Z b*, *C Z c*, *D Z d*, &c. and ſo the intire Angular Motions of the Orbs *B F K*, *C G L*, *D M N*, &c. are as the Areas *B Z b*, *C Z c*, *D Z d*, &c. reſpectively. But in the Figure *A Z a*, the Ordinates *B b*, *C c*, *D d*, &c. are reciprocally as the Cubes of the Right lines *S B*, *S C*, *S D*, &c. from whence (by *Prop.* 73.) the Areas *B Z b*, *C Z c*, *D Z d*, &c. are reciprocally as the Squares of the Right lines *S B*, *S C*, *S D*, &c. And therefore the Angular motions of the Orbs, *B F K*, *C G L*, *D M N*, &c. are reciprocally as the Squares of their Semidiameters *S B*, *S C*, *S D*, &c. But the Periodic Times are reciprocally proportional to the Angular motions: Therefore the Periodic Times of the Orbs *B F K*, *C G L*, *D M N*, &c. are in the direct (becauſe the reciprocal of the inverſe) ratio of the Squares of their diſtances from their Center *S*; namely, of the Right lines *S B*, *S C*, *S D*, &c. And therefore the Propoſition is evident, if the medium of the Sphere circumfuſed be ſuppoſed to conſiſt of innumerable thin concentric ſolid Orbs. But if there be ſuppoſed to be drawn a vaſt number of infinite Right lines making with the Axis Angles mutually exceeding one another by equal exceſſes, and theſe Right lines being revolved about the Axis, and deſcribing Conic Superficies, Orbs be imagined cut into innumerable Rings; each of theſe Rings will have four others contiguous to it, one internal, another external, and two lateral, cut out of the ſame Orb. By reaſon of the attrition of the interior and exterior, the intermediate can't be moved any otherwiſe than formerly, before the Orbs were cut: otherwiſe the parts of the fluid would not perſevere in their motion uni-

formly; but the middle one would be accelerated and retarded in their motion, (as was ſhewn before in the intire Orbs,) contrary to the ſuppoſition. And therefore any ſeries of Rings going on directly from the ſolid central Sphere *in infinitum*, and comprehended between the two next Conic Superficies, will be moved after the ſame manner as theſe were moved before the diviſion of the Orbs into Rings; bating what hindrance each Ring in this Series meets with from the attrition of the Rings laterally. But here the attrition is nothing at all, becauſe all the Rings equally diſtant from the middle ſolid Sphere, (that is, cut off from the ſame Orb) are revolved in the ſame Time: For if it were not ſo, but thoſe which are towards the Poles, finiſhed their revolution quicker or ſlower than thoſe which are towards the Circle that lies in the middle between the Poles; the ſlower, by reaſon of the mutual attrition, would be quickened in their motion, and the quicker would be retarded, contrary to the ſuppoſition, which was, that the Fluid perſevered in its motion uniformly. Since therefore the Periodic Time of all the Rings, at the ſame diſtance from the Center, is the ſame, they will revolve after the ſame manner as they would do, were they not as yet cut off from the ſolid Orb; that is without any attrition: And therefore the ſame Law holds in this caſe alſo of Orbs cut into Rings, as held before any diviſion into Orbs; that is, the Periodic Times of each of the Rings will be as the Squares of their diſtances from the Center of the middle ſolid Sphere.

Let each Ring be divided by tranſverſe Sections, into innumerable Particles conſtituting a Fluid abſolutely and uniformly ſuch: And becauſe theſe Sections have no reference to the

Law

Law of Circular Motion, but conduce only to the conſtitution of a Fluid; the circular Motion will perſevere as before. By theſe diviſions all the infinitely ſmall rings will either not change the Aſperity and Force of mutual Attrition at all, or will change them equally: and the proportion of the Cauſes continuing the ſame, the proportion of the Effects will likewiſe remain the ſame; that is, the proportion of the Motions and Periodic Times as before. And therefore the Periodic Times of each of the parts, in the above-deſcrib'd Vortex, will be as the Squares of the Diſtances from the Center of the Vortex. *Q.E.D.*

COROLLARY 1.

If the reſiſtance of the Parts of the Vortex be greater in a greater diſtance from the Center than in a leſs, either on the account of the greater thickneſs or leſs fluidity of the Particles which compoſe the Fluid, or on the account of any other Cauſe; then the more remote parts of a Vortex will move ſlower than in the ratio ſettled above in the Propoſition: that is, the Periodic Time of the parts of the Vortex more remote from the Center, will be to the Periodic Time of the parts nearer the Center, in a greater than in the duplicate ratio of the Diſtances of theſe Particles from the Center.

COROLLARY 2.

If the Vortex be not extended *in infinitum*, but the Fluid moved after the manner of a Vortex, be contained in a rigid Veſſel held by force, of a different Figure from a ſpherical one concentric to the central Globe; the Particles of the Vortex will be moved, not in Peripherics of Circles concentric with the central Globe, but in Lines almoſt conformed to the Figure of the Veſſels; and the Periodic Times will be as the Squares of the Mean Diſtances from the Center very nearly.

PROPOSITION LXXVI.

TO *point out and refute, in few words, the Errors committed in the above-described* Cartesian *Physics.*

1. Since from the preceding *Prop.* 'tis evident, that every Vortex made by a solid Sphere, revolved uniformly about an Axis given by Position, will be propagated *in infinitum*, if not hindred; if the World was made up of such kind of Vortices, (as it is according to *Cartes's* System,) there would be as many Vortices going forwards *in infinitum*, as there are Fix'd Stars, every one of which, (according to *Cartes*) like our Sun, makes a Vortex. Nor would these Vortices be bounded by certain limits, but creep on, and by little and little run into one another. And, by this means, each part of the infinite Fluid making up the Universe, would be agitated by that Motion which results from the actions of all the Central Spheres or Suns. But how foreign this uncertainty is, from that certain Order, Situation, and Motion of the Fixed Stars, which are firmly connected by a very simple Calculus, let such as are acquainted with Astronomy judge.

2. Further, because the parts of the Vortex, generated in the manner above-described, that are nearer the Center, being(by the preced. *Prop.*) moved quicker, urges the exterior, and constantly communicate Motion to them by that action; and these exterior Particles transfer at the same time the same quantity of Motion into others that are still more exterior; because (by Hypothesis) it is a condition of the Vortex, that each part of the Fluid persevere in its motion uniformly, and is not incited, sometimes more, sometimes less, in the same distance from the Center:

Center: 'tis evident that Motion is perpetually transferred from the Center to the Circumference of the Vortex, and absorbed by the infinity of the Circumference. And therefore to the conservation of a Vortex in the same state, there is required some active Principle, from which the Central Sphere (or Sun of that Vortex) may always receive the same quantity of Motion, as it impresses on the matter of the Vortex. But from whence the *Cartesians* can get such an active Principle residing in each Sun that moves the whole Vortex, I cannot see; since *Cartes* himself, in *Parag.* 146. *third Part Princ. Phil.* boldly says, *There is nothing to be met with in the Phænomena of the Planets, that can't be accounted for from what has been already said.* The penetrating *Kepler* saw well enough the necessity of such an active Principle in the central Spheres of the Vortices: *And therefore lest the Motion should languish by degrees,* which Creating Omnipotence impressed upon them in the beginning of Things, he would have it *maintained either by the perpetual care of the Creator, or by the assistance of a Spirit destin'd to that imployment.*

For without such an active Principle, the central Spheres and the interior parts of the Vortices, always propagating their Motion into the exterior, and receiving no new Motion, must necessarily move slower by degrees, and cease to move circularly. And consequently this World of *Des Cartes,* without such an active immechanical Principle naturally tends towards rest and its dissolution.

3. *Des Cartes* and the *Cartesians* acknowledge, that the Bodies which are carried in this Vortex and return again, are of the same density with the parts of the Vortex, where they are placed, and move by the same Law with them, as to

the Velocity and determination of the courſe: the deferent Fluid, and that Body that is carried by it, differ only in this, that the parts of the Bodies thus carried by the Vortex, keep the ſame ſituation in regard of one another; and that they are as it were congealed and ſolid parts of the Vortex, and can't be mingled together by an inteſtine Motion, as the fluid parts of it are; being, in other reſpects, intirely equal. Since therefore the parts of the Vortex (by the preced. *Prop.*) revolve by this Law, that their Periodic Times are in the duplicate ratio of their diſtances from the Center, the Planets alſo carried about the Sun, and at relative reſt in the fluid Matter of the Vortex, in which they ſwim, will revolve by the ſame Law. Since therefore the Earth, for inſtance, compleats its Period in a Year, the Period of Saturn, whoſe diſtance is 9 $\frac{1}{2}$ times the diſtance of the Earth, would be 90 Years, when it is not quite 30 Years: After the like manner the Period of Saturn would be 27 Years, whereas it is not 12; And all the ſuperior Planets would have Periods greater than the truth; and the inferior leſs. If the Patrons of Vortices anſwer, that the Vortices are not uniformly fluid, (as was ſuppoſed in the preceding *Propoſition,*) but the parts conſtituting them in the receſs from the Center are groſſer, as *Cartes* ſays in *Parag.* 82. *Part* 3; then (by *Corol.* 1. *Prop.* preced.) the more remote Planets from the Sun are ſtill ſlower, whereas they are too ſlow already, (according to this Syſtem;) and it is evident from this conſtitution of the Vortices, according to *Cartes*, there will be a greater diſtance from Obſervation, than if the Vortices were made up of an uniform fluid. Since therefore it has been demonſtrated, that the parts of the Vortex revolve by a very diffe-

rent

rent Law from that which is obſerved in the Planets; namely, that the Periodic Times of the parts of the Vortex are as the Squares of the diſtance from the Center, if the matter of the Vortex be homogeneal; or as ſome higher Power of the Diſtances, if the matter of the Vortices be groſſer at a greater diſtance; whereas in the mean while (by *Prop.* 22.) in the Planets, the Squares of the Periodic Times are as the Cubes of the Diſtances; (that is, the Periodic Times of the Planets are as thoſe Powers of the Diſtances, whoſe Exponents are $\frac{3}{2}$, which are much lower than thoſe whoſe Exponents are 2, or a number greater than two;) 'tis evident therefore, that the Planets are not carried by any ſuch corporeal Vortices.

4. If the Vortex were extended *in infinitum*, 'tis evident (from the preced. *Prop.*) that each part of the Fluid ought to deſcribe a perfect Circle, and (by *Corol.* 2.) that the aberration from a circular Path proceeded from the rigid Veſſel (or what is equivalent to it,) in which the Matter of the Vortex is contain'd. From whence it evidently follows, that the greater the Vortex is in reſpect of the Path deſcribed by its fluid part, (or by the Planet carried together with it,) the more ought the Path of the Planet to come up to a perfect Circle; that is, the excentricity of a Planet nearer to the Sun will be leſs than that of one more diſtant from the Sun; whereas Aſtronomical Obſervations teſtify and aſſure us, that the Orbit of Mercury is much more excentric than that of Saturn. Again, becauſe (by *Corol.* 2. preceding *Prop.*) the Particles of the Vortex, and the Planets carried along with them, will be moved in lines almoſt conformed to the figure of the Veſſel, in which the fluid is contain'd, the Aphelia of all

the Planets viewed from the Sun will be posited towards the same Fixed Stars; whereas the Aphelia of the Planets are at a great distance from one another, as Jupiter's and the Earth's, by a quarter of a Circle; Mars's and the Earth's, by a third part of a Circle; those of Mars and Venus are almost opposite. Besides, the matter of the Vortex (like a River) being within narrower bounds, (by the Laws of Mechanics) ought to move quicker than when it moves freely in a broader Channel; and that in the reciprocal ratio of the breadths of the Channels, or what is equivalent to them: But in the beginning of ♍, the distance between the Orbs of Mars and Venus is to their distance in the opposite Point, (namely in the beginning of ♓) almost as 3 to 2; and therefore the intermediate fluid of the Vortex, that carries the Earth along with it, will be carried swifter in the same ratio in the beginning of ♓ than in the beginning of ♍; because contain'd within narrower limits; (for the paths of the Fluid, which carries Mars and Venus, serve instead of boundaries or a Channel;) that is, the Earth seen from the Sun in the beginning of ♓, will be carried half as swift again as when it is seen from the Sun in the beginning of ♍, or the Sun seen from the Earth in the beginning of ♍, will seem to move half as swift again among the Fix'd Stars, as in the beginning of ♓: But just the contrary happens by the Observations of all Astronomers: and the Sun seen in the beginning of ♍ from the Earth, seems to move in its diurnal Motion only 58 Minutes; whereas in the beginning of ♓ it seems to move a whole Degree in the same time.

5. A Body carried by a Vortex, and of the same density with the parts of the Vortex in which

which it is, in its return round, if it be not otherwise hindered, describes a Circle; to whose Plane the Axis of the central Sphere, whereby the Fluid is turn'd about in the form of a Vortex, is perpendicular; and if the Path of the Body be inclined to this Plane, that inclination will be continually lessen'd, and they at last will coincide. For it is evident from the demonstration of the preced. *Prop.* that each part of the Fluid composing the Vortex, describes such a Circle; wherefore even the Planet, which is carried only by these Particles being relatively at rest between them, will also describe such an one; and if by any external Force impress'd it deflect a little from hence, the inclination will be lessened by degrees by the encounter of the Particles of the Fluid describing the above-mentioned Circles, and the Planet it self will describe such a Circle, being thus at last relatively at rest in the fluid of the Vortex, and carried about together with it. But there is not one Planet to be found having the Axis of the Sun perpendicular to the Plane of the Orbit; nor during these two thousand Years, wherein Observations have been made, has the Inclination of the Orbit of any Planet to that Plane been found lessened; nor are there any fibres of Latitude assigned by these Philosophers in the Planets, for the preserving the same Inclination. But the Apology that *Cartes* brings for the various excentricity of the Planets, and the various inclination of the Orbits to one another, and the various situation of the Aphelias, which is, in *Parag.* 34. *Part* 3. *Princip.* (namely this: *But as we see it happens in all other natural things, that they are only nearly so, and in succeeding Ages are continually changed,*) shews plainly that he was unacquainted with the Calculatory

culatory part of Aſtronomy. For tho' it can't be denied that Phyſical Cauſes have a place in the motion of the Stars, yet no Perſon uſed to theſe matters will judge that ſuch caſual and almoſt irregular inequalities, (ſuch as *Cartes* ſeems to inſinuate,) are to be admitted in the motions of the Stars; but will rather ſay, with that moſt acute and beſt fitted for Aſtronomical Speculations *Jer. Horrox*, who labour'd to defend the contrary Opinion, *We may juſtly wonder at ſome Slingers or Enginiers, that can throw Bodies out of Slings or Cannons with ſuch incredible certainty of aim. But if we ſee ſo great exactneſs in thoſe momentaneous things, that are moved thro' the thick unſettled Air; ſhall we not expect a much greater in thoſe perpetual Bodies as are revolved in a quiet and pure Æther by an eternal Law?*

6. Since the Globules compoſing the fluid of the Solar Vortex are leſſer near the Sun, and conſequently their Maſs, contained in a given Space and filling it, is leſs denſe than in a greater diſtance from the Sun; it follows, that even the Planets which are of the ſame denſity with the particles of the Vortex, in which they ſwim, are leſs ſolid the nearer they are to the Sun; which *Cartes* alſo acknowledges *in Parag.* 147. *Part* 3. *of Princip.* But the contrary to this we ſhall demonſtrate in its proper place hereafter; namely that each Planet, the nearer it is to the Sun, the ſolider or more denſe it is; that is, it contains more Matter under an equal bulk. Which alſo agrees much better with the Architypical Ratio's, Geometrical Concinnities, and Final Cauſes: From the conſideration of which, 'tis evident, that the Divine Being has placed the Planets at different diſtances from the Sun, ſo that each, according to its degree of denſity, may enjoy a greater or leſs

degree

degree of Heat; and consequently the more dense Planet is nearer the Sun, since all more dense sort of Matter require a greater heat for the performance of natural Operations. And tho' the ratio of the density of the Planet be not the same with what *Kepler* makes it, namely the reciprocal of the Roots of the distances from the Sun; but the reciprocal of the distances from the Sun drawn into the roots of the apparent distances: Yet here (where he was most of all mistaken,) *Kepler* came at this general Proposition, by his Architypical ratio's, that the denser Planets are nearer the Sun; whereas *Cartes*, who was since him, from a neglect, (or rather contempt) of the Architypical ratio's and final Causes, cherished opinions entirely contrary to these Final Causes, and true Physics or Mechanics, (which generally lead us to the same thing;) and amongst others, this opinion, that the solidity of the Planets near the Sun, is less than that of those farther off.

7. *Tycho* formerly, from the motion of the Comets, demonstrated, that the Planets were not moved in solid Orbs; because the Comets passed thro' them. The same Comets do now with equal evidence shew, that the Planets are not carried by corporeal Vortices; seeing the Comets move in Paths very oblique, and sometimes cutting the Plane of the Zodiac at Right Angles, and sometimes directly contrary to the course of the Planets, and they preserve these motions with the greatest freedom and length of time, being subject to this universal Law, that by a radius drawn to the Sun, they describe Areas proportional to the Times; which would be altogether impossible, if the intire matter of the Vortex were carried about the Sun with a force sufficient to carry along with them the

vaſt Bodies of the Planets. Nor does it help *Cartes* at all, that he has baniſhed the Comets beyond Saturn, contrary to the credit of all Obſervations, (which *Tycho* and the Aſtronomers never forc'd to ſerve their Hypotheſes, as *Cartes* would inſinuate ;) ſince the matter of the Solar Vortex beyond Saturn (according to himſelf) has a greater moment to carry them along with it, than near Venus and Mercury; both becauſe the Globules compoſing the Vortex are greater there than here; and eſpecially becauſe they are moved much ſwifter, ſince they run a Circle immenſly greater *within a few Weeks*, as *Cartes* has it, *Parag.* 62. *Part* 3. *Prin.* From all together, and each of theſe in particular 'tis evident, that this Hypotheſis of Vortices is entirely diſagreeable with the Phænomena of Aſtronomy, and conduces not ſo much to the explaining as the diſordering of the Celeſtial Motions.

PROPOSITION LXXVII.

TO *enumerate and explain the Cauſes of the Celeſtial Motions brought by Mr.* Leibnitz.

'Tis ſo natural in enquiring after the Cauſes of the Celeſtial Motions, to ſeek them in the ambient fluid Æther, that the Philoſophers have looked upon the Planets to be carried about the Sun, after the ſame manner as ſo many Straws ſwimming in Water, are carried about by the Water put into the motion of a Vortex, by a Stick whirled about its Axis in the middle of a Veſſel at reſt. This ancient opinion lay neglected for a long time: But in the laſt Age was revived by ſome great Men; for before *Cartes*, (of whom above,) it was approved of by *Torricellius* and *Gallilæo*; for the reaſons without doubt, ſummarily propoſed about the beginning of *Prop.* 71.

This

This opinion pleas'd ſo well, that that moſt acute Philoſopher Mr. *Leibnitz*, (even after the publication of *Newton's Mathemat. Princip. of Nat. Philoſ.*) adopts it; and *judges that there remains now no more, than to ſhew how the Cauſes of the Celeſtial Motions ariſe from the motions of the Æther, or (ſpeaking Aſtronomically) from deferent Orbs, but then ſuch as are not ſolid, but fluid.* A name ſo celebrated among Geometricians certainly deſerves, to have his *Eſſay upon the Cauſes of the Celeſtial Motions* (which he communicated to the Learned World in the *Acts of the Learned*, Publiſhed at *Lipſic* in *February* 1689.) exactly conſidered.

For Mr. *Leibnitz* (if any Man) is able to accommodate Vortices to the Celeſtial Motions, and *aſſign reaſons for the Laws of the Heavenly Bodies diſcovered by* Kepler. Eſpecially ſince *he has attained ſome peculiar Light in that matter, and the enquiry he made, ſeemed* to himſelf, who is ſo penetrating a Perſon, *to ſucceed very well and agreeable to Nature, ſo that hereupon he had good hopes, of having arrived at the true Cauſes of theſe Motions.* We will therefore lay down the Method this Philoſopher uſes in explaining theſe great Works of Nature, in as few Words as poſſible, and, as near as we can, in his own.

Firſt, *Leibnitz* takes for granted, that all Bodies which in a fluid deſcribe a Curve line, (and conſequently that the Planets themſelves,) are acted upon by the motion of the Fluid itſelf. For all Bodies deſcribing a Curve, endeavour to recede from it along a Right line, which is a Tangent, (from the nature of Motion;) there muſt therefore be ſomething that keeps them in. But there is nothing contiguous except the Fluid, (by ſuppoſition,) and no endeavour to move thus can be reſtrained but by a contiguous

Body

Body likewiſe in Motion, (from the nature of Matter;) the fluid itſelf therefore muſt neceſſarily be in Motion.

Since from the moſt exact Obſervations that have been made, every Planet deſcribes an Orbit about the Sun, according to this Law, that by radii drawn from the Sun to the Planet, Areas are always cut off proportional to the Times; 'tis reaſonable enough that the Æther or Fluid Orb of each Planet ſhould move about the Sun after ſuch a manner as may agree with the ſaid Law of Motion. And this will be done, if each of the innumerable circular concentric Orbs of ſmalleſt thickneſs, into which the fluid Orb of the Planet is divided by the Mind, have its proper Circulation, ſo much the ſwifter in proportion, as each of them is nearer to the Sun; that is, if the Velocities of circulating, in the parts of the fluid, are reciprocally proportional to the radii or diſtances from the Sun. By which means it will happen, that whether the Planet be more or leſs diſtant from the Sun, the Sectors that are deſcribed in equal Times by the Radius, are equal. For the ſaid Sectors are in a ratio compounded of the direct ratio of the radii or diſtances from the Sun, and of the reciprocal of the Arcs or Circulations, and therefore (from the nature of the Circulation) in the ratio of equality. The Circulation above deſcrib'd (where the Velocities of Circulating are reciprocally proportional to the diſtances from the Center) he calls *Harmonic*, becauſe ſuppoſing the Diſtances from the Center to increaſe equably or Arithmetically, the Circulations decreaſe in an Harmonic Progreſſion; being reciprocally proportional to the Quantites that are in an Arithmetic Progreſſion, which is the known property of Harmonic Proportionals.

This

This Great Man therefore ſuppoſes the Planet to be moved by a double Motion, or one compounded of the Harmonic Circulation of its deferent fluid Orb, and of a *Paracentric* acceſs to and receſs from the Sun. The Circulation of the Æther therefore makes the Planet circulate Harmonically, not by any Motion of its own, but as it were by a perfectly quiet ſwimming in a deferent fluid, whoſe Motion it obeys. Conſequently neither retains an impetus of circulating ſwifter, than it had in an inferior or nearer Orb, but one that languiſhes and leſſens, as it paſſes through the ſuperior (reſiſting a greater Velocity than their own) till at laſt it inſenſibly accomodates itſelf to the Orb it approaches; and on the contrary, while it moves from the ſuperior to the inferior Orbs, it receives their Impetus. Conſequently the Harmonic Proportion holds not only in Arcs of Circles, but in deſcribing any other Curve.

For the Area deſcribed in an Element of Time, cut off by radii drawn from the Sun to the Planet, moved in any other Curve, doth not comparably differ from the Sector of a Circle, whoſe Angle at the Center is the ſame, and radius any one of the preceding.

The other Motion compounding with the former, the intire Motion of the Planet (namely the Paracentric,) ariſes from a double Curve, namely, the *excuſſory impreſſion of Circulation and the Sun's attraction* (or what is equivalent) compounded together. For ſince every Body endeavours to recede from the Curve line, it deſcribes, along a Tangent, by this Motion alone, the Planet carried by the Vortex circulating Harmonically, will recede from the Sun. And theſe Centrifugal Forces are ſhewn by the Author, to be reciprocally as the Cubes of the radii.

radii. The other component part of the Paracentric Motion is the Sun's Attraction, or what supplies the place of it, the Planets Gravity: And tho' he call this Force Attraction, he does not question but it is derived from the impulse of the ambient Fluid, like the actions of the Magnet.

Because it is certain by Observation, that each primary Planet describes an Elliptic Orbit, in one of whose Foci is the Sun, so as that by radii drawn from the Sun to the Planet Areas are always cut off proportional to the Times; and there is no Law of Circulation in a Vortex that answers this condition, but the Harmonic: It remains that a Law of Gravity be sought after, that join'd with the Centrifugal conatus of the Body circulating Harmonically, may compose a paracentric Motion, which in conjunction with an harmonic Circulation makes the Body to move in the Perimeter of an Ellipse, which (after the premising of a Lemma shewing the ratio of a Circulation in a Conic Section to the paracentric Velocity) he defines, by demonstrating the following Theorem: *If a heavy Body be carried in an Ellipse (or any other Section of a Cone) with an harmonic Circulation, and the Center both of Attraction and Circulation be in the Focus of the Ellipse; the Attractions or Sollicitations of Gravity will be as the Squares of the Circulations directly, or as the Squares of the radii or distances from the Focus reciprocally.*

He has found therefore that there is such a thing as Gravity in the Planets, tending to the Center of the Sun, the Law of which is, that it is reciprocally proportional to the Squares of their distances from the Center. And he farther acknowledges that this Law was known before to Mr. *Newton*; his *Principles of Philosophy* being pub-

publiſhed before that time, and an account of them given in thoſe *Leipſic* Acts.

After having annexed ſome Corollaries naturally flowing from his chief Theorem, he cloſes his Phyſico-Aſtronomical *Eſſay*, with a confeſſion that there remained two things to be done on this Argument: The one is to explain what motion of the Æther it is, that makes the Planets heavy, or impels them towards the Sun, and that reciprocally as the ſquares of the Diſtances, or (as he expreſſes it) as the ſquares of the Vicinities: The other is, what is the cauſe of the compariſon of the Motions between the different Planets of the ſame Syſtem, that the Periodic Times ſhould be in the ſeſquialter ratio of the mean Diſtances. And ſince (according to the Author) they both ought to depend upon the contiguous Bodies, all which he calls by the name of the Æther, theſe two tend to explain more diſtinctly the motion of the Vortex or Æther that conſtitutes each Syſtem. But theſe things requiring a deeper ſearch, , and therefore not to be contain'd in ſo ſmall a compaſs as a Sheet, he ſays, 'tis more proper for him to publiſh his Thoughts upon that Matter ſeparately.

PROPOSITION LXXVIII.

TO point at ſuch Things as are not agreeable to Natural Philoſophy, in the above recited Cauſes of the Celeſtial Motions; aſſigned by Mr. Leibnitz *in his* Eſſay.

There are ſome of the Reaſons againſt Corporeal Vortices carrying Planets, that likewiſe make againſt the above deſcribed *Eſſay of* Leibnitz *about the Cauſes of the Celeſtial Motions*; namely, that the Paths of ſome Comets are very oblique to the Zodiac, ſometimes they cut this

Plane at Right Angles, nay, they are even di-rectly contrary ſometimes to the courſe of th Planets. And ſince the Comets (all the whil they are obſerved by us) deſcribe Areas abou the Sun proportional to the Times, by parit of reaſon, an Harmonically circulating Vorte ought to be ſuppoſed, ſuch an one being a neceſſary for the motion of a Comet as of Planet; that is, there ought to be one Vorte contrary to another.

In conſidering indeed but one Planet carrie about the Sun (or any other Center) if it b to be carried by a Vortex, without any prope motion of its own, only ſwimming quietly a long in a deferent Fluid, circulating Harmoni-cally, and obeying its Motion, (which the Au-thor ſuppoſes;) the Vortex ought to circulat Harmonically, ſo as that the Areas cut off by Radi drawn from the Planet to the Sun, be propor-tional to the Times. No other conſtitution o a Vortex can make the ratio between the Area and Times to be the ſame. If therefore th Vortex of our Author will not ſolve the Moti-ons of the Planets, certainly no other will. An this being once ſuppoſed, all the reſt, whic our Author draws from thence by Geometry proceeds very well and juſtly, as things uſually do under his Hand. To which I add, that ſuc a Vortex as this (whoſe Fluid circulating Harmo-nically in a Plane thro' the Center, to whic the Axe is perpendicular,) may be produce Mechanically, by a ſolid Sphere, if it be revol-ved in an uniform and infinite Fluid, about an Axe given by poſition, with an uniform Motion and the Fluid be made to move round by th ſole impulſe of it, and every part of the Flui perſevere in its motion uniformly. And this i a third thing, which remains for the Author t

explain

explain concerning the motion of the Solar Vortex or Æther; namely, how it might be impelled and put into an Harmonic circulation. For, since universally the Times of describing or running any Space, are directly as the Spaces run, and reciprocally as the Velocities; and in a Circular motion the spaces run in one revolution are as the radii, and in an Harmonic circulation the Velocities or Circulations are reciprocally as the radii; 'tis evident, that the Periodic Times of the parts of a Vortex circulating Harmonically, are as the Squares of the radii: But we have demonstrated, *Prop.* 75. that this is the property of a Circulation in a Plane thro' the Center, to which the Axis is perpendicular in the Vortex, put into its motion by a central Sphere.

But, if the motions of two or more Planets be compared together, it will appear, that the fluid Matter of the Solar Vortex is not moved in an Harmonic Circulation. For we have already demonstrated, that the Periodic Times of the parts of a Fluid circulating Harmonically (and consequently of Bodies that are carried by a quiet swimming in the Fluid) are as the Squares of the radii. But the Periodic Times of the Planets are not as the Squares, or in the duplicate, but in the sesquiplicate ratio of the distances from the Center. And therefore different Planets are not moved by an Harmonic circulation.

The Author no doubt will say, that he did not suppose that the whole Fluid of the Solar Vortex was moved by a continual Harmonic circulation, no where interrupted, from Mercury to Saturn inclusively; but that the Fluid carrying about Mercury moved in an Harmonic Circulation from its Perihelium to its Aphelium; then (this Harmony being interrupted,)

again Circularly harmonical from the Perihelium of Venus to its Aphelium, but not in an Harmony continued with the former circulation near Mercury; that is, ſo as that the circulation of the Fluid carrying Venus, is to the circulation of the Fluid carrying Mercury, as the diſtance of Mercury from the Sun, is to the diſtance of Venus from the ſame: And again, (the former Harmony being interrupted,) the Fluid carrying the Earth is moved Harmonically from the Perihelium of the Earth to the Aphelium, if this Harmony be conſidered ſolitarily, and not compared with the Harmony of the Fluid to Venus or Mercury; and ſo on in the other Planets: that is, that by reaſon of the thickneſs of each Orb, there holds a particular Harmony ſeparate and diſtinct from the reſt; and in like manner in the Satellites of Jupiter and Saturn, aſcending from the Center of Jupiter and Saturn. But it muſt be confeſſed, that the Author does ſeem likewiſe to extend his Harmonic circulation alſo to different Planets, in *Parag.* 17. where he ſays: *In double the diſtance, only a fourth part of the Angle ſeen from the Sun, is finiſhed in the ſame Element of Time; in triple the diſtance only the ninth part*: For certainly theſe numbers have no place in the ſame Planet. But if, (as he ought principally to do,) he ſpeaks expreſly concerning the interrupted Harmony of the circulating parts of the Fluid, he will hardly perſwade Philoſophers that things are thus; namely, that by reaſon of the thickneſs of the Orb of Mercury there is an Harmonical circulation from the Perihelium to the Aphelium, or the Velocities of the Fluid are as the diſtances from the Center reciprocally: But comparing this Circulation with the Circulation of the Æther at any other Planet, the one is to the other reciprocally

ciprocally as the Squares of the Radii: For we have demonſtrated (in *Prop.* 27 and 28.) independantly from all Phyſical Cauſes, that the Celerities of the Planets are reciprocally as the Square Roots of their diſtances from the Sun. Certainly the Vortex of the Sun would be monſtrous, and theſe deferent fluid Orbs more abſurd than the deferent ſolid ones, if by reaſon of the thickneſs of each of the ſix deferent Orbs the Harmonic circulation has a place, and yet this Harmony in the mean while is five times interrupted; namely, at the interſtices of the deferent. And it makes contrary to this interruption, that a Comet carried thro' the Zodiac *in conſequentia*, and paſſing thro' theſe interſtices of the Orbs carrying the Planets, is moved juſt as if it were carried by an Æther circulating Harmonically; that is, ſo as that the Areas deſcribed by the radius that carries it, are proportional to the Times. It may be obſerved likewiſe, how difformly difform the Solar Vortex would be, in which the thickneſs of the Orb of Saturn, where the Harmonic circulation of the Æther holds, is greater than the diſtance of Mercury from Venus, or of Venus from the Earth, or of the Earth from Mars, in every one of which the Harmony is interrupted; and where, having laid aſide the reciprocal ratio of the radii, which they keep in the Harmony, the circulation of any two of theſe compared together, are reciprocally as the ſquare roots of the Radii, being about to reſume the former Harmony. The famous Author himſelf ſeems to have diſcovered this fault of his Harmonically Circulating Vortex, when he confeſſes, *that this likewiſe was one thing remaining*, namely, *to explain what ſhould be the cauſe of the compariſon of the motions between different*

Planets of the ſame Syſtem, ſo as that the Periodic Times may be in the ſeſquiplicate ratio of the Mean Diſtances: For he ſaw plain enough, that this could not be the effect of an Harmonic Circulation.

Perhaps ſome Perſons may wonder, that Mr. *Leibnitz,* who deduced the Motions of the Planets from Cauſes (at firſt ſight) ſo different from thoſe eſtabliſhed before, ſhould notwithſtanding light upon the ſame Law of Gravity, whereby every Planet is kept in its Elliptic Orbit. The reaſon of that coincidence is thus: Tho' *Leibnitz* makes uſe of the Matter of a Vortex Circulating Harmonically for carrying the Planet about; yet becauſe in the Planet thus carried about, and deſcribing any Curve, he confeſſes an Excuſſory Conatus, whereby a Planet would be made to move in a Right line touching the Curve, if not hindred. He does the ſame as if he had made the Planet to move equably in a Right line, if not hindered: For theſe two Motions together make it ſo; that is, all one as if it were carried in free Spaces, acted upon by no external force at all. For in both caſes it will deſcribe a Right line with an equable Motion: For the Body deſcribing the Right line with an equable Motion, runs in it after ſuch a manner as (by *Prop.* 1. *Elem.* 6.) that the Areas cut off by radii drawn to any given Point, are proportional to the Times. Since therefore the Harmonic Circulation and Excuſſory Conatus together perform the ſame as an equable Motion in a Right line does, he could not but find the Law of Gravity the ſame with that found out before; namely, that which would force the Planet from the ſame Right lined Tangent which it would run after the ſame manner, into the ſame Conic Section towards the Sun, poſited after the ſame manner.

SECTION

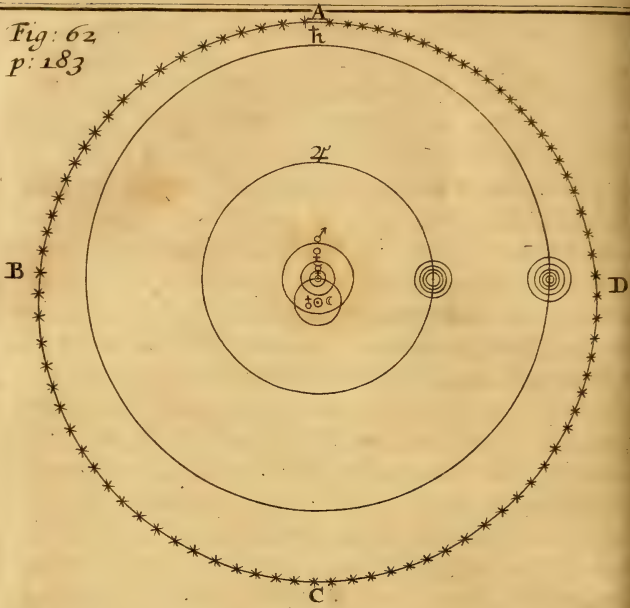


Fig: 62
p: 183

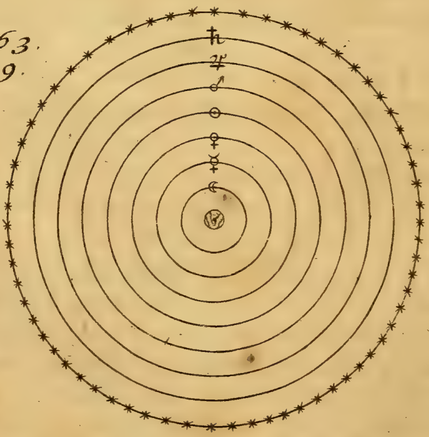


Fig: 63.
p: 189.

SECTION XI.

Of other Systemes of the World, and Forces necessary to preserve them.

HItherto we have been explaining the old *Philolaic* Syſtem of the World, by ſhewing what Phænomena would happen in it, and that they agree with Obſervation. We largely treated alſo concerning the Direction and Law of the Forces, whereby the Syſtem of the Sun is kept in that ſtate; and after what manner the Philoſophers, who were Maintainers of it, thought it was done. It remains that we diſpatch briefly the Order and Motion which other Philoſophers aſcribed to the Bodies of this Syſtem, which Opinions are now-a-days called the *Syſtems* of their Aſſertors; and we will at the ſame time ſhew the Direction and Law of the Forces, whereby thoſe Syſtems might ſubſiſt.

PROPOSITION LXXIX.

TO deſcribe the Semi-tichonic *Syſtem of the World.*

The maintainers of this Opinion, place the Earth, which is not moved progreſſively or out of its place, in the Center of the Sphere, in whoſe Superficies *ABCD*, [*Fig.*62.] they will have the Fixed Stars to be placed. But grant that the Earth is moved about its own Axis from Weſt to Eaſt by a Diurnal Motion. Beſides, in this Syſtem the Sun is moved about the Earth in the ſpace of a Year, at a Diſtance equal to that at which the Earth is about the Sun in the *Philolaic* Syſtem: And about the Sun, as a Center,

the five Planets, *viz.* Mercury, Venus, Mars, Jupiter and Saturn, are revolved towards the ſame way, and at the ſame Diſtances as they were in that. Moreover, the Moon revolves about the Earth, and about Jupiter and Saturn, their proper Satellites, in the ſame Periods as was ſaid before, at *Prop.* 15.

This Order, and theſe Motions of the great Bodies of the World pleaſe moſt of thoſe Philoſophers, that the other Syſtem explain'd at large before diſpleaſes, and which, from the reviver of it, *Copernicus*, is commonly call'd the *Copernican* Syſtem, as by *Longomontanus* and others. For in this the apparent Diurnal Revolution of all the Stars from Eaſt to Weſt, and the proper or ſecond Motions of the Planets are intirely the ſame as in the *Copernican* Syſtem, and the Earth continuing in the mean while in the Center of the Sphere of the Fixed Stars, is only moved about its own Axis. And tho' they place the Fixed Stars in the Superficies of a Sphere, whoſe Center is the Earth, that is to be looked upon rather as the Aſtronomical than the Phyſical one.

For the Fixed Stars ſcattered as it were thro' the whole Mundane Space, may be placed at different Diſtances, without any prejudice to this Syſtem; and their Motion *in conſequentia*, explained by aſcribing a very ſlow Motion to the Axis of the Earth, as we have explained it in *Prop.* 64.

PROPOSITION LXXX.

TO *explain the Direction and Law of the Forces whereby the* Semi-tychonic *Syſtem may be preſerved.*

'Tis evident firſt of all, that the ſame Forces and the ſame Direction are neceſſary here for the

the keeping the five Primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, in their Orbits about the Sun, and the Attendants of Jupiter and Saturn in their Orbits about Jupiter and Saturn, as in the *Copernican* Syſtem. The Law therefore of the Centripetal Force, whereby in this Syſtem, the Secondary Planets of Jupiter tend to Jupiter, the Satellites of Saturn to Saturn; and the Primaries Mercury, Venus, Mars, Jupiter, and Saturn, and their Satellites, towards the Sun, is known: For it is that Force which, in *Prop.* 47. we demonſtrated to be the ſame with our Gravity, which is reciprocally proportional to the ſquare of the diſtance. 'Tis farther evident, that this Syſtem being ſuppoſed, the Moon is kept in its Orbit by Gravity propagated towards the Earth by the ſame Law: For *Prop.* 46. where this was demonſtrated, holds as much, whether the Earth continue in the ſame place unmoved, or be carried by an Annual motion about the Sun. But ſince the difference between the *Copernican* and *Semi-tychonic* Syſtem is, that in the latter, the Sun (as it were, with its five attending Primary Planets and their Secondaries,) is moved about the Earth in an Orbit ſimilar and contrarily poſited to that, in which the Earth with its attendant is carried about the Sun in the former; beſides the Force of Gravity neceſſary in the *Copernican* Syſtem, that the Earth and its Attendant might be carried about the Sun, there is occaſion for another Force equal to it, but with a contrary Direction, that the Sun, together with the attending Planets, may be moved about the Earth at reſt in the *Semi-tychonic* Syſtem: For theſe two equal and contrary Forces impreſſed upon the Earth, keep it at reſt. (Nay, if one contrary impetus and not

to

to be repeated, do but put a ſtop to the progreſſive motion of the Earth and its attendant the Moon, along the Tangent of the *Orbis Magnus*, and afterwards that other Force now deſcribed, act upon the intire Solar Syſtem, the *Copernican* Syſtem will be turned into the *Semi-tychonic*. Therefore to preſerve the *Semi-tychonic* Syſtem, there is occaſion for both the above explained accelerative Gravity of all Bodies towards all others, which is, in a given diſtance, as the Body towards which it tends, and the Body being given towards which it tends, is reciprocally as the ſquare of the diſtance; and alſo of another certain Gravity, equal to that whereby the Earth tends towards the Sun, acting equally upon the Sun, Mercury, Venus, the Earth, Mars, Jupiter and Saturn, and their Attendants, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth. For I do not reckon among theſe Forces, that Force whereby the Earth is revolved upon its own Axe; becauſe only once to be impreſſed upon the Earth, as alſo upon the other great Bodies of the World revolving about their own Axes: Nor do I take in (for the ſame reaſon) that Force, whereby the Planets would deſcribe the Tangents of their Orbits: For we are now treating of thoſe Forces and their Law, neceſſary to preſerve the Syſtem, and not of the Forces whereby it may be produced.

PROPOSITION LXXXI.

TO *deſcribe the* Tychonic *Syſtem of the World.*

This differs from the Syſtem deſcribed in *Prop.* 79. only in this, that the Earth being deprived of all Motion, is abſolutely at reſt; and there-

therefore all the Stars, as well Fixed as Wandring, are not only carried by their ſecond Motions, but alſo by that common or firſt Motion from Eaſt to Weſt, in the ſpace of a Natural Day.

'Tis ſaid, that the great *Tycho* liked this Syſtem beſt; tho' others deny, that *Tycho* allowed expreſly a Diurnal Motion to the Stars. But it muſt be confeſſed, that he diſputes againſt the Diurnal Motion of the Earth as well as the Annual: And conſequently it may be ſafely concluded, that he aſcribed a Diurnal Motion to the Heavens and all the Stars.

PROPOSITION LXXXII.

TO *explain the Direction and Law of the Forces, whereby the* Tychonic *Syſtem may be preſerved.*

The ſame Forces that are required in the *Semi-Tychonic* Syſtem, are required alſo in the *Tychonic*, ſince the ſame motion of the Sun and Planets are ſuppoſed in both. And beſides theſe, ſome other Forces, whereby all the Stars, both Fixed and Wandring, are drawn from their Rectilineal motions and retained in their Orbits, which they deſcribe by their Diurnal motion about the Earth. But theſe Forces, ſince the Periods in all are the ſame, are (by *Corol.* 1. *Prop.* 26.) as the Semi-diameters of the Circles they deſcribe. And therefore, beſides the Forces explained in *Prop.* 80. there will be need of another Force tending towards the Earth, that increaſes in the receſs from the Earth as the diſtances increaſe; contrary to what happens in Gravity thro' the whole Solar Syſtem, and in all Natural Virtues propagated from a Center, or to a Center, in Right lines, every way thro' the Regions round about, which (by *Prop.* 48.) in

in its recefs from the Center, is leſſen'd in the duplicate ratio of the diſtance increaſed. And that this Force, whereby the Stars are retarded in their Orbits, deſcribed by their Diurnal Motions, may be the more eaſily eſtimated, let it be compared at the Moon with the Force of Gravity, whereby (by *Prop.* 46.) the Moon is retained in the Orbit, ſhe deſcribes in her Monthly Courſe round the Earth. And ſince the Circles deſcribed by the Moon in her Monthly and Diurnal Courſe, are equal, the Forces whereby the Moon is retarded in them, (by *Corol.* 3. *Prop.* 26.) are reciprocally as the ſquares of the Times; that is, the Force whereby the Moon, (when ſhe is carried in the Equator by a Diurnal Motion about the Sun) is kept from flying off in a Right line, which is a Tangent to the Orbit, is to the Force, whereby it is kept in the Orbit that it deſcribes about the Earth in a Periodic Month, as at 27 × 27 to 1; or as 729 to 1. But this Force tending towards the Earth, holds only in thoſe Stars, which are placed in the Equator: For the Force, whereby any Star declining on one ſide of the Equator, towards either of the Poles, is retracted from the rectilineal Motion, and kept in an Orbit parallel to the Equator, (by *Prop.* 12.) tends to the Center of that Circle, and not to the Earth, which is ſituated without the Plane of it; and it is, (by *Corol.* 1. *Prop.* 26.) as the Semidiameter of that Circle; that is, in a given diſtance from the Axe, equal or the ſame, and in a given diſtance from the Earth, as the Coſine of Declination.

To preſerve therefore, the *Tychonic* Syſtem, there is required 1*ſt*. That accelerative Gravity of all Bodies towards all others, which in a given diſtance, is as the Body towards which it tends; and

and the Body being given towards which it tends, is reciprocally as the Square of the Diſtance. *2dly.* Another accelerative Force equal to that whereby the Earth, by a ſuperior Force, tends towards the Sun, and acting equally upon the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and their Attendants, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun to the Earth. *3dly.* An accelerative Force, tending towards the neareſt Point of the Axis of the World, proportional to its diſtance from that Point, and reaching to the fartheſt Fixed Stars. *4thly.* Another Force like it tending to the neareſt Point of the Axis of the Ecliptic, and proportional to the diſtance from that Point. This is the Force, whereby the Fixed Stars are retain'd in the Orbits, which they deſcribe in their proper motion from Weſt to Eaſt, upon the Poles of the Ecliptic, and does not act at all in the Places that lie between the Center and the Fixed Stars, but begins to act at the Fixed Stars themſelves; and is (by *Corol.* 3. *Prop.* 26.) to the like Force in the Axis of the World, in the duplicate ratio of the Natural Day, to the Period of the Fixed Stars upon the Poles of the Ecliptic. And theſe four will ſuffice.

Proposition LXXXIII.

To *deſcribe the* Ptolemaic *Syſtem.*

In this Syſtem the Earth continues abſolutely at reſt in the Center: Next to the Earth, the Moon, then Mercury, afterwards Venus; above Venus the Sun, next Mars, then Jupiter, and Saturn ſurrounding Jupiter juſt as in the Scheme, [*Fig.* 63.] finiſhing their Revolutions under

der the Zodiac from West to East. Above Saturn the intire Sphere of the Fixed Stars, (whose Center is the Earth,) is also revolved very slowly from West to East, upon the Poles of the Ecliptic. But both the Fixed Stars and all the Planets, revolve upon the Poles of the Equator from East to West, in the space of a Natural Day. We don't reckon up the several Orbs, which the assertors of the *Ptolemaic* System, place above the Fixed Stars; since they are only of use in the *Theory*, and don't belong to the Aspectable World. And for the same reason, we omit likewise the Epicycles of each Planet: For in the Orbits here delineated, the Centers of the Epicycles themselves are carried about, in whose Circumferences the Bodies of the Planets themselves are moved.

In this Scheme of the Planets, we have not delineated the Orbits according to the proportion they have in the Heavens; as well because *Ptolemy* himself has not declared it in any, but the Sun and Moon; and because that tho' the Circles, that carry about the Centers of the Epicycles, are at some distance, yet the Heavens of the Planets are, in this Scheme contiguous, the inferior sometimes in the Apogæum, ascending from the Earth, to a distance equal to that, to which the next superior descends in its Perigæum: as because the Orbits of Mercury and Venus, (not to mention the Moon) taken from the Theories of *Ptolemy*, do almost vanish in respect of the Orbit of Saturn, and therefore are hardly to be described in this Scheme: For in this System, the distance of Saturn from the Earth, is an hundred and sixty times greater than the distance of Mercury from the Earth: But in the *Copernican* System, the distance of Mercury from the Sun, is only four and twenty

times

times leſs than the diſtance of Saturn from the ſame.

Tho' the Order of the Planets deſcribed above pleaſed the generality of Philoſophers and Aſtronomers, yet there were ſome, who placed the Sun next above the Moon, then Mercury, and then Venus; and this Opinion is uſually aſcribed to *Plato:* But *Porphyrius* placed Venus immediately above the Sun, and Mercury above Venus, the three others (Mars, Jupiter and Saturn) continuing as before: Perhaps induced by this reaſon, that it was better for the two Luminaries to be neareſt the Earth; whereas the *Ptolemeans* minding their motions more, placed the Sun in the middle of the Planets, being ſatisfied that the motions of the other Planets in their Epicycles depended upon the motion of the Sun.

PROPOSITION LXXXIV.

TO explain the Direction and Law of the Forces whereby the Ptolemaic *Syſtem may be preſerved.*

Firſt, for the keeping the Stars within the Orbits they deſcribe, by their Diurnal Motion about the Earth; the ſame Forces are neceſſary here, as were in the *Tychonic* Syſtem, for the producing the ſame Effect; namely, ſuch as may tend from any Point of the World to the neareſt Point of the Axis, and be in the ratio of the diſtances from it. Secondly, to contain the Fix'd Stars within the Orbits which they deſcribe, by their proper motion from Weſt to Eaſt, Forces like thoſe made uſe of in the *Tychonic* Syſtem, are neceſſary. Then to keep the Luminaries (the Sun and Moon) in the Orbits which they deſcribe by their ſecond Motions; ſince theſe Orbits are ſimilar to their Or-

bits

bits in the *Tychonic* Syſtem, the like Forces will be neceſſary in this Syſtem, to retain them in the ſaid Orbits, as in that; namely (by *Prop.* 80.) the accelerative Gravity of the three Bodies, Sun, Earth and Moon, towards one another; (that may not affect any other Mundane Bodies, tho' they lie between them,) which, in a given diſtance is as the Body towards which it tends, and towards a given Body, is reciprocally as the Square of the diſtance from it; and beſides, another ſort of Gravity equal to that, whereby the Earth would tend towards the Sun, acting equally upon the Sun, Earth, and Moon, by Lines parallel to a Right line, connecting the Sun and the Earth, and with a direction from the Sun to the Earth, but ſuch as affects no other Planet. For that Force, whereby the Sun is retained in its Annual Orbit, is not the ſame with that whereby the Moon is retained in its Menſtrual Orbit, propagated unto the Sun: For the Force, whereby the Moon is kept in its Orbit, is the ſame with Gravity, with us, (by *Prop.* 46.) But if the Sun were retained by the ſame Force, propagated ſo far as itſelf, the Cubes of the Diſtances of the Sun and Moon (by *Prop.* 27.) would have the ſame ratio as the Squares of their Periodic Times; and therefore ſuppoſing the diſtance of the Moon from the Earth to be 60 Semidiameters of the Earth, the Sun would not be above 340 Semidiameters of the Earth diſtant; and its Horizontal Parallax would be above 9 minutes, contrary to all Obſervations; and beſides, there would be no room for the Epicycles of Mercury and Venus, between the Heavens of the Moon and the Sun, where they are placed according to this Syſtem.

Some of this Sect of Philosophers are not willing to put any more than a mental distinction between the Motions of the Sun and Moon under the Zodiac, and the Diurnal Motions upon the Poles of the Æquator, so that the motion of each is really in the same simple Spiral Line, arising from the composition of the above-mentioned motions, into which these Celestial Motions are imagined to be divided by Astronomers, for the greater ease; and the most simple Motion in a Right line, may be compounded of two or more Motions. But because that Spiral is not a Line described in a Plane, a Body can't (by *Prop.* 11.) be retained in it by a single Force tending to an immovable Point. Either therefore, there is need of the several different Forces described above, or of a single Force equivalent to them, tending to a Point moved in the mean while forward and backward, and that in a very compounded Line.

Besides, to keep the other Planets (Mercury, Venus, Mars, Jupiter, and Saturn) within the Orbits they describe by their proper Motion, the Forces tending to the Earth are not sufficient, (as was demonstrated, *Prop.* 13;) nor are the Forces tending to the Sun, since the Orbits of Venus and Mercury do not surround the Sun in this System, and for that reason they are not concave toward him in all their Parts: Wherefore, from *Prop.* 11. the thing proposed is certain. And there is no Point in the System, (much less a Body worthy to be taken notice of,) that can be a Center, to which the Forces may tend, whereby the Planets are kept from going off in the Tangents; one of the above-mentioned reasons contradicting it.

But if this *Ptolemaic* Syſtem be underſtood literally, that is, if the motions deſcribed by *Ptolemy*, are not look'd upon as motions only ſeparated by the Mind, but as ſuch as are really in the Planets; then there will be need of a Force in the center of the Epicycle of every Planet, (Saturn, Jupiter, Mars, Venus and Mercury,) tending to the center of the deferent; that is, in an imaginary Point, (or where there is no Body at all,) to an imaginary Point: And beſides, there is need of a Force in the Planet itſelf, that may tend to the center of the Epicycle; that is again, a Force in a Body, that may ten to an imaginary Point. But even thoſe Forces whereby each Planet, and the center of its Epicycle are urged, are not propagated beyond its proper Point and Planet, nor do they affect any other Planets; and there is ſtill need of a peculiar Force for the keeping every center of an Epicycle, and every Planet in its proper Orbit; ſo that there is no Law which reſpects all the Planets: And the ordinary reſpective Gravity of the Satellites of Jupiter and Saturn, towards the centers of Jupiter and Saturn, continue in the mean time untouched.

After that the Phaſes of Venus and Mercury were obſerved by the help of Teleſcopes, the *Ptolemean* Syſtem of the World was ſoon thrown aſide by Philoſophers: becauſe 'tis evident from them, that the Orbits of Mercury and Venus, muſt encompaſs the Sun; as was ſhewn in *Schol. Prop. 6.* Such therefore as ſtill maintained, that the Earth was Immovable, made Mercury and Venus move in Epicycles round the Sun as a Center; thus far embracing the *Philolaic* Syſtem of the World, and bringing in again the old *Egyptian* one; that *Vitruvius* in his *Book of Architecture*

tecture, and Venerable *Bede* in his *of the Nature of Things*, approved of.

Ricciolus in his *Almagest*, adds Mars to the Attendants of the Sun; ſo that ſuch as have no Satellites of their own, are made Satellites to the Sun, and only Saturn and Jupiter that have Satellites, beſides the Luminaries, revolve about the Earth, as their immediate Center; and this Syſtem, which is almoſt the ſame with the *Tychonic*, he calls his own.

There is no need of explaining the Forces and their direction, that are requiſite to preſerve the *Ægyptian* Syſtem, or that other, which is little different from it, the *Ricciolic*: For that appears very eaſy, from what has been ſaid above. As they are compoſed of the Syſtems already deſcribed, ſo the Forces neceſſary to preſerve them, are parts of the reſpective Forces explained before.

SCHOLIUM.

This Section was ſubjoin'd, to the end, that not only Aſtronomers, (who are no ways afraid of a multiplicity of the Celeſtial Motions, and make it a Poſtulate, that any Star may be moved with any motion,) weighing the ſeveral different Syſtems, might embrace that which agrees beſt with the Phænomena: But that the Philoſophers may likewiſe chuſe ſuch a Syſtem as beſt agrees with Nature, namely, either the Old *Philolaic* or *Copernican* Syſtem, which agrees with the Phænomena, and may be preſerved by Gravity alone, propagated in the ſame tenor thro' the Univerſe; or by an accelerative force of all Bodies towards all others, that is, in a given diſtance, as the Body towards which it tends, and reciprocally as the ſquare of the diſtance, when the Body is given towards which it tends:

Or the *Semi-tychonic*, deſcribed in *Prop.* 79. that performs the ſame, as the *Copernican*, in regard of the Phænomena; but then beſides the ſame Gravity, as is requir'd in the Copernican Syſtem, (by *Prop.* 80.) it ſtands in need of another Force, whoſe Magnitude and Direction are perpetually changed; for this Force is equal to that, whereby the Earth would tend towards the Sun, and acts equally on all the Bodies of the Solar Syſtem, according to Right lines parallel to a Right line that connects the Sun and Earth, and with a direction from the Sun towards the Earth: Or (if it be reſolved upon, that the Earth ſhall be or is intirely at reſt) the *Tychonic Syſtem* deſcribed in *Prop.* 81. doing the ſame as to the Phænomena, in which (by *Prop.* 82.) beſides the two aforeſaid Forces, there is need of two more; namely, one equal in every point of the Axis of the World, that encreaſes as the Diſtances, and is at the Moon to the Force of Gravity in the duplicate ratio of the Periodic Month, to the Natural Day; and another Force like it, and ſimilarly propagated in every point of the Axis of the Ecliptic, that acts only upon the Fix'd Stars, and is at the Fix'd Stars to the former Force in the Axis of the World, in the duplicate ratio of a Natural Day, to the great Year, or period of the Fix'd Stars upon the Poles of the Ecliptic: Or the *Ptolemaic* Syſtem itſelf deſcrib'd in *Prop.* 83. But this does not agree with the Phænomena of the Heavenly Bodies, as has been ſhewn in *Prop.* 84. But if, ſetting aſide the Phænomena of the Phaſes of Mercury and Venus, we only attend to ſuch Phænomena as were known by *Ptolemy*, and his Syſtem be taken for true; (by *Prop.* 84.) beſides the two ſorts of Attractive Forces, equally diffuſed thro' the Axis of the World, and the Axis of the Ecliptic, there is

is need of an accelerative Gravity of the Sun, Earth and Moon towards one another, without affecting any of the other Planets, which if the diſtance be given, is as the Body towards which it tends, and towards the given Body, reciprocally as the ſquare of the diſtance from it; and beſides, there is occaſion for another Gravity, (equal to that whereby the Earth would tend toward the Sun,) acting equally upon the Sun, Earth and Moon, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth. Over and above, it ſtands in need of a particular Force in the center of the Epicycle of each of the Planets, Saturn, Jupiter, Mars, Venus, and Mercury, that tends to the center of its Deferent; and ſtill of another particular Force in the Planet itſelf, tending to the centre of its proper Epicycle, without any prejudice in the mean while to the ordinary Gravity of the Satellites of Jupiter and Saturn. But if the *Ægyptian* (or *Ricciolic*) Syſtem be moſt fancied, (which indeed may be made to ſolve the Phænomena,) then, (beſides the Forces reſiding in the Axes of the World and the Ecliptic,) there is need of a mutual Gravity of the Sun, Mercury, Venus, the Earth with the Moon, (and Mars,) as in the *Copernican* and *Tychonic*; but ſuch an one as does not reach ſo far as Jupiter and Saturn; and beſides, of a Force equal to that, whereby the Earth would tend towards the Sun, which acts equally upon the Sun, Mercury, Venus, the Earth together with the Moon (and Mars,) along Right lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth; and beſides of a particular Force in

the Center of the Epicycle of both Planets, Jupiter and Saturn, that tends to the Center of its Deferent, and of a particular Force in Jupiter and Saturn, tending to the Center of their proper Epicycle.

THE

THE

ELEMENTS

OF

Aſtronomy,

Phyſical and Geometrical.

The Second Book.

Of the Firſt Motion.

IN the preceding Book, we have laid down the Principles of a True and Phyſical Aſtronomy; that is, we have explained, in general, the motions of the Stars, as are really in nature: But ſome notice muſt be taken of the Appearance of them; that is, the Elements of an Aſtronomy fitted to them, are to be formed. For as it is out of all doubt to any Philoſopher that conſiders the Reaſons of Things, that the Earth is carried about among the Planets, ſo 'tis likewiſe certain to every one that enjoys his Sight, that the Earth appears at reſt, in the

middle of the Univerſe, and that the Heavens, and all contained by them, are moved as to Senſe, round about it. Now not only the reaſon of this Phænomenon is to be aſſigned, (which has already been done, in the foregoing Book,) but the Methods muſt be explained, that the Aſtronomers have uſed, to define the places of the Stars, ſeen from the Earth, who either thought, with the common People, that things were really ſo as they appear, or who thought that this Syſtem of Appearances ought to be retained, tho' they knew for certain, that things were not after this manner; and that 'tis according to this Syſtem that we muſt ſpeak with the common People; and not make our Reaſon and Philoſophy perpetually offer violence to our Sight and other Senſes.

To this end is this Second Book deſigned; namely, the explaining the Words us'd in Aſtronomy, and the giving the deſcription and uſe of Spheres, Globes, and other Inſtruments, made to ſet before our Eyes principally the Diurnal Motion, and in general to ſearch out in this Phyſical Syſtem, and to handle according to the apparent Syſtem, and Calculate all thoſe matters as are uſually ſaid by Aſtronomers to belong to the *Doctrine of the Sphere*: And tho' the Doctrine of the Sphere be uſually firſt treated of, by all Authors, we have not been afraid to invert the Order; ſince by this method better care is taken againſt young Perſons in learning Aſtronomy, their taking the Syſtem where the Earth is at reſt for true, to which they are firſt accuſtomed, and to which they are more inclined by their Senſes; and that the Doctrine of the Sphere, in *Kepler's* judgment, *ſtands in need of ſeveral things, that are borrowed of the Theoric part of Aſtronomy* (or Doctrine of the proper Motion of

of the Planets, a faint Draught of which you had in the preceding Book,) *by anticipation, whereas the Theoric Part may be deliver'd alone, without needing the aſſiſtance of the Diurnal or Firſt Motion, or of the Doctrine of the Sphere, which treats of it.*

SECTION I.

Of the Generation of the Circles of the Sphere, and of the Terms uſed in Aſtronomy, depending thereupon.

PROPOSITION I.

TO *explain the generation and nature of the Ecliptic and Zodiac, and the diviſion of them, alſo of the Secondaries of the Ecliptic, and their uſes, and the Terms uſed in Aſtronomy depending upon them.*

An Obſerver placed upon the Earth, by extending his Sight every way equally, if not hindered, bounds the World by a Spherical Superficies concentric with himſelf, (that is, with the Earth) and judges all remote Objects, whoſe diſtance is unknown, tho' really very unequally removed from one another, to be placed as it were in the ſame concave ſpherical Superficies; as for inſtance, he does the Fixed Stars, Sun, and Moon itſelf; as was ſhewn in *Prop.* 32. *B.* 1. This Sphere, concentric to the Earth and ſurrounding it, defined by the Sight, and beſpangled with the twinkling Fixed Stars, ſupplying the place of an abſolute Space, is conſider'd as at reſt; while we abſtract from the Diurnal Motion, whereby this intire Sphere, or rather the whole Heavens, are revolved from Eaſt to Weſt in the ſame Space of 24 Hours. But ſince by reaſon of the Annual Motion of the Earth about

about the Sun, the Sun ſeen from the Earth ſeems to move forward daily towards the more Eaſtern Stars (as was ſhewn in *Prop.* 2. *B.* 1.) the Sun will ſeem to move in this immovable Sphere or Mundane Space; and its Path marked among the Fixed Stars is call'd the *Ecliptic*, becauſe the Eclipſes of the Luminaries happen only in this Line; as was ſhewn above.

Having once ſettled the Ecliptic, 'tis evident that as all the Orbits of the Planets about the Sun, are not in one Plane, but each of the Planets ſeen from the Sun, makes an excurſion, ſometimes on this ſide, ſometimes on that ſide of the Way of the Earth, mark'd among the Fix'd Stars; ſo the ſame Planets ſeen from the Earth, make an excurſion ſometimes on this ſide, ſometimes on that ſide of the Ecliptic, and even in the ſame points of their Orbits ſeem to run off variouſly, according to the different ſituation of the Earth, (as was ſhewn in *Prop.* 5. and 8. *B.* 1.) But their greateſt excurſion is contained within the boundaries of ten Degrees; the Aſtronomers therefore have called that tract which the Sun, Moon, and Planets ſeem to adorn with their motions, and which is a Zone or Belt, twenty Degrees broad, (ten on each ſide of the Ecliptic,) by the name of the *Zodiac*, from the images of Animals, which the Fixed Stars, in that tract, ſeem to repreſent. The number of the images, pitched upon by them, is twelve; either becauſe this number ſeems the fitteſt, for its being diviſible into parts without a remainder, as two, three, four, ſix, and twelve; or becauſe, while the Sun ſeems to run thro' the Zodiac, in the ſpace of a Year, there happen near upon twelve Lunations, or the Moon runs thro' all her Phaſes twelve times. Theſe parts of the Zodiac are called *Signs*, and their

their marks are, Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagitarius ♐, Capricornus ♑, Aquarius ♒, Piſces ♓. Every one of theſe Signs is again divided into 30 equal parts, ſo that the whole Circle (like all others) is divided into 360. Tho' the Zodiac or any other Circle has no beginning nor ending, properly ſpeaking; yet Aſtronomers begin their reckoning from the firſt point of Aries, which the Sun is ſeen in when the Days and Nights are equal, and Spring begins to the Inhabitants of the Northern Hemiſphere, which alſo is the point of the common interſection of the Ecliptic and Æquator; as was ſhewn in *Prop.* 33. *B.* 1. The reckoning is made from ♈ to ♉, ♊, and ſo on, till a return is made to ♈, the beginning again towards the ſame way that the Sun ſeems to move about the Earth. And the motion or progreſs this way is ſaid to be *in conſequentia ſignorum*, or according to the order of the Signs; namely, that according to which the Sun, Moon, and almoſt the reſt of the Planets ſeem to move: But that Phænomenon, which is carried in the contrary order, or from ♉ to ♈, or from ♈ to ♓, is ſaid to be moved contrary to the order of the Signs, or *in antecedentia.*

Becauſe the Axis of the Earth after ſeveral revolutions about the Sun, goes a little off from a Site parallel to that it formerly had, deſcribing the Superficies of a Cone, the images of the Stars have removed from the Signs of the Zodiac, to which they originally gave names. And this happens not upon the account of the motion of the Stars or the Zodiac, but becauſe the Æquator of the Earth is moved together with the Axis of the Earth, ſo as that the interſections of the Celeſtial Æquator, with the

the Ecliptic, or Equinoctial points, (and consequently all the points of the Zodiac, as they are expressible by numbers) remove *in antecedentia*; and the images or the Fixed Stars seem to be transferred, in respect of them, *in consequentia*, as was shewn in *Prop.* 64. *B.* 1.

If innumerable Circles be supposed to be drawn thro' the Poles of the Ecliptic, cutting the Ecliptic at right Angles, which are called its *Secondaries*, any point in the Heavens may be referred to the Ecliptic, by the help of them; that is, any Phænomenon is understood to be in that point of the Zodiac or Ecliptic, in which such a Semicircle, passing thro' the Phænomenon, cuts the Ecliptic. And the Phænomena that are after this manner referred to the same point of the Zodiac, are said to be conjunct, or in *Conjunction*; but such as are referred to the opposite points, are in *Opposition*. If a Quarter of the Zodiac lie between the points to which the appearances are referred, they are said to be in a *Quartile* Aspect, but if a third part of the Zodiac, in a *Trine* Aspect; lastly, if a sixth part of the Zodiac lie between them, they are said to have a *Sextile* Aspect.

Besides, an Arc of the Ecliptic intercepted between the beginning of Aries and the said point of intersection, and reckoned according to the order of the Signs, is called, the *Longitude* of that Phænomenon or Point, as the Arc of a Secondary, intercepted between the Ecliptic and the said Phænomenon or point of the Heavens, is called its *Latitude*; on which account these Secondary Circles are called *Circles of Latitude*. The Latitude is either *Northern* or *Southern*; for the Heavens are divided according to Astronomers, by the Ecliptic, (a Celestial Circle, because originally consider'd in the Hea-

Heavens) into the Northern and Southern Hemiſphere.

PROPOSITION II.

TO give an account of the geneſis, nature, and uſes of the Celeſtial Equinoctial and its Secondaries and Parallels, and to explain the Terms uſed in Aſtronomy, depending thereon.

If the Plane of the Earth's Æquator or Circle, lying exactly betwixt the Poles, upon which the Earth is revolved by its Diurnal Motion, be produced every way, it will interſect the Spherical Surface of the Heavens, concentric to the Earth, in a correſponding Circle in the Heavens, and the Axis of the Earth produced will mark out the Poles in the ſame Sphere. And tho' the Earth, with its Axe and Æquator, be carried about the Sun in the Annual Motion, yet the Circle deſcribed by the Earth, is ſo ſmall, in regard of the Fixed Stars, that the Axis and Æquator of the Earth (every where parallel to themſelves) being produced, falls upon the ſame Fixed Stars, as to Senſe, and therefore all the other Fixed Stars, that retain the ſame ſituation in regard of one another, retain alſo the ſame ſituation in regard of the Æquator and Poles of the Heavens, excepting ſo far as the Axis and Æquator on the Earth do not continue parallel to one another; concerning which we ſhall ſpeak in the following Propoſition. And becauſe the Earth revolves upon its own Axis, in the ſpace of a natural Day, from Weſt to Eaſt, any Body ſeparate from the Earth will ſeem to deſcribe a Circle in the Concave Sphere, parallel to the Celeſtial Æquator in the ſame time from Eaſt to Weſt, the Eye being placed in the Earth, and judging its own Habitation to be unmoved; as was explained at large in *Prop.* 32. *B.* 1.

But

But becauſe the Earth is carried about the Sun in its Annual motion, the Earth's Æquator continuing always parallel to itſelf, applies itſelf only twice a Year to the Sun, ſo as that the Sun may be found in the Plane of it produced, and conſequently that the light of the Sun may reach to both the Poles of the Earth, and every point of the Earth turn'd about in the Diurnal motion, may be as long in the Light as in the Dark, (as was ſhewn in *Prop.* 33. *B.* 1.) that is, the Sun is ſeen only twice a Year to deſcribe the Celeſtial Equinoctial Circle in its Diurnal motion, in which caſe the Days will be equal to the Nights to all the Inhabitants of the Earth: For the Equinoctial interſects the Ecliptic in the Point in which the Sun appears among the Fixed Stars, when its Light reaches to both the Poles of the Earth, and cauſe the Equinox through all the Globe of the Earth. For in every other caſe, the Sun, in its Diurnal motion (like the other Celeſtial Bodies) ſeems to deſcribe Circles parallel to the Equinoctial, among which they are the moſt conſiderable, that the Sun ſeems to deſcribe in the Heavens, when the Poles of the Earth do moſt incline towards the Sun; that is, when the Sun is ſeen from the Earth in the beginning of the Signs of Cancer and Capricorn among the Fixed Stars; which therefore are called the *Tropics of Cancer* and *Capricorn*, becauſe the Sun returns immediately from thence towards the Equator. There are two other Circles alſo parallel to the Equinoctial Circle (namely thoſe which thePoles of the Ecliptic ſeem to deſcribe in their Diurnal motion; and by conſequence are as far diſtant from the Poles of the Equinoctial as the Tropics are from the Equinoctial,) called the *Polar Circles*, either becauſe

they

they are deſcribed by the Poles of the Ecliptic by the firſt or Diurnal motion, or becauſe they are near the Poles of the Equinoctial, which are called the *Poles of the World*; becauſe all the World beſides the Earth, ſeems to be moved about them in the ſpace of 24 Hours. The Equinoctial Circle, as uſual, is divided into 360 Degrees, and theſe Degrees are reckoned towards the ſame parts with the Degrees of the Ecliptic or Zodiac, namely, from Weſt to Eaſt, according to the order of the Signs, and from the ſame beginning; namely, the firſt Point of Aries. As each Point in the Heavens is referred to the Zodiac by the Secondaries of the Ecliptic, ſo by the Secondaries of the Equator, it is referred to the Equinoctial. And the Arc of the Secondary comprehended between the Point and the Equator, is called the *Declination* of that Point; *North* or *South*, according as it is towards this or that Pole. For a Point is ſaid to decline juſt ſo much as it is diſtant from the Primary Circle, being the middle and principal one of the Diurnal motion which moſt affects our Sight. This Diſtance or Declination, belongs to the Secondaries of the Equator, which are therefore called, the *Circles of Declination*: The chief of which are the two *Colures*, one of which paſſing thro' the interſections of the Ecliptic and Equator, or points of the Equinoxes, is called the *Equinoctial Colure*; and the other at right angles with the former, is called the *Solſtitial Colure*, becauſe it meets the Ecliptic in the remoteſt Points from the Equator, where the Solſtices are celebrated.

PRO-

PROPOSITION III.

TO *give an account of the nature and uſes of the Terreſtrial Equator, and its Secondaries and Parallels, and explain the Terms uſed in Aſtronomy and Geography depending thereon.*

The Equator is properly a Terreſtrial Circle, becauſe exiſting originally in the Earth, and transferred to the Heavens only upon the account of the Firſt Motion, (which is only an apparent one in the Heavens.) Its four principal Parallels, the two Tropics, and as many Polar Circles, may be underſtood to be on the Earth, either as ſo many Circles lying directly under the Circles in the Heavens of the ſame name, the generation of which we have ſhewn in the preceding Propoſition; or (which is more natural) as originally generated in the Earth itſelf. And then the Tropics will be Circles on the Earth, lying directly under the apparent Courſe of the Sun, when the Pole of the Earth, which is neareſt, inclines moſt to the Sun: But the Polars, ſuch as bound thoſe tracts of the Earth, as have at that time perpetual Day or Night, as was explain'd in *Prop.* 33. *B* 1.

Theſe four Circles parallel to the Equator, divide the Globe of the Earth into five *Zones*; of which, that is the *Torrid*, which is contained between both Tropics, becauſe lying directly under the courſe of the Sun, and receiving its direct and, by conſequence, moſt powerful Rays. The Inhabitants of this Zone are called *Amphiſcians*, becauſe the Shadow of a Perſon ſtanding upright, moves as well towards the right as the left hand of him that obſerves it, and becauſe the Noon Shadow at certain different times of the Year, is projected towards both the Poles. The tracts of Land included within the two

Polar Circles, make two other *Zones*; both of them *Frigid*, becauſe they receive only the oblique and conſequently the weakeſt Rays of the Sun, that are not therefore able to thaw or diſ- ſolve the cohæſion of the Ice, that froze during he Winter and long Night-time, in the follow- ng Summer, or as long a Day-time. The Inha- itants of theſe Zones, in regard of the Shadow, re called *Periſcians*, becauſe the ſhadow (the Sun ot ſetting) moves round about them. Between he Torrid, which is the middle, and the two rigid, which are the extreme Zones, lie two thers, call'd the *Temperate Zones*, becauſe par- aking of the affections of both the adjacent xtremes; the *Northern* of which is bounded y the Tropic of Cancer and the Arctic Polar Circle; and the *Southern*, by the Tropic of Ca- ricorn and the Antarctic Polar Circle. And ho' the Ancients ſaid that theſe only were emperate and habitable, yet experience now nforms us, that both the Frigid and Torrid ones are fit for the nouriſhment of Animals, nd Vegetables; ſeveral other Cauſes, partly hyſical, and partly Aſtronomical, making com- enſation for the too direct or oblique inci- ence of the Rays. The Inhabitants of the emperate Zones are called *Heteroſcians*, be- auſe their Noon ſhadow is thrown only to- ards one Pole, *viz.* the neareſt, and carried a- out only towards one quarter, as it is with us, ho inhabit the Northern Temperate Zone, and e our own ſhadow projected towards the right and.

Again, ſuch as live upon the Equator, have eir Days and Nights always equal, becauſe as ell the Equator as the Circle diſtinguiſhing e enlightened part of the Earth from the ob- ure are great Circles of the Sphere; and erefore, (by *Prop.* 11. *Book* 1. *Theodoſ.*)

they bisect one another; consequently, each Point in the Equator carried round about equably in its Diurnal motion, is as long in the light as in the dark. But to an Inhabitant living without the Equator towards either Pole, the Days are longer than the Nights, when the Pole of the Earth, that is nearest inclines towards the Sun, and are then the longest, when that Pole inclines most towards the Sun: And the farther an Inhabitant lives from the Equator, the longer are the longest Days to him, till at last at the Polar Circle in Summer, there is no Night at all; as has been shewn at large in *Prop.* 33. *B.* 1. This is the reason why *Ptolemy* and other Geographers, divided the Earth by Circles *Parallel* to the Equator, distant from one another, and their beginning (*viz.* the Equator,) so much as that the longest Day is a quarter of an Hour longer in one *Parallel* than in the other next *Parallel*. They take no notice of any division into lesser parts, because they would be scarce sensible; and they looked upon this space (tho' properly a little Zone) as indivisible in regard of Latitude, and therefore they call'd it a *Parallel*, Circle being understood. Making the Equator then to be the first Parallel; the second passes thorough those parts of the Earth, where the longest Day is $12\frac{1}{4}$ Hours; the third throrough them, where the Day is $12\frac{1}{2}$ Hours, and so on; and in the other Hemisphere after the same manner. Two such spaces make a *Climate*, which therefore differ from one another the length of half an Hour. Each Climate receiv'd a name from some considerable Place lying in it near its middle: and therefore there must be 24 of them reckon'd from the Equator to either Polar Circle, since the greatest or longest Day is 12 Hours longer. But the An-
cient

cients did not begin this reckoning from the Equator itself, but made the first Climate to pass thro' the Island *Meroe*, where the longest Day was found to be an whole Hour longer than 12 Hours, either because they left this Region near the Equator for a right Sphere, in regard of which, the other Regions on the sides of it, are called Κλίματα; or because perhaps they judged the interior parts to be uninhabited: Tho' *Ptolemy* says, there were several in his time, who maintain'd, that there were habitations under the Equinoctial itself, just as it were in a temperate Region; but the People of his Age, had not to that Day penetrated thither. *Ptolemy* has not given us an exact computation of the Parallels, so far as to the Polar Circle itself, they being more numerous thereabouts, and he not thinking it worth his while to be over nice in his enquiries in the more Northern Parts. As for the Climates on the other side of the Equator towards the Antarctic Pole, he has given each of them such a name, as signifies that this unknown Climate is as far distant from the Equator towards the South, as a certain known one is from the same Equator towards the North; as the Climate Ἀντι-δια-μερόης, Ἀντι-δια-Συήνης, &c.

As the Places of the Stars, or any Points in the Heavens, are computed as to their Longitude and Latitude, by the means of the Ecliptic (which is properly a Celestial Circle) and its Secondaries; so the Longitudes and Latitudes of Places on Earth, are computed in the Equator (which is properly a Circle on the Earth,) and its Secondaries passing thro' the Poles of the Earth. Any such Secondaries drawn thro' any Place upon the Earth, is called the *Meridian* of that Place, because when the Earth has in its

Diurnal revolution arrived to ſuch a ſituation, as that the Sun is in the Plane of that Circle produced, it is Noon Day; as ſhall be more largely explain'd hereafter. The *Latitude* of any Place is an Arc of the Secondary of the Equator, or of a Meridian, intercepted between the Equator and the Place: And it is either *North* or *South*, as the Place is diſtant from the Equator towards the North or South Pole. But the Longitude of a Place is computed from the Weſt towards the Eaſt in the Equator itſelf, becauſe the boundary of the Land towards the Weſt is better known than that towards the Eaſt. And becauſe the Equator has no beginning, nor any Cardinal Point, as there is in the Heavens, marked out by its interſection with the Ecliptic, the firſt Meridian, from whoſe interſection with the Equator the beginning of the reckoning is to be made, was left to the liberty of the Geographers to be ſettled. Now, they allow of no dry and habitable Land beſides the Contingent they themſelves inhabited, together with the adjacent Iſles; they feigned therefore a firſt Meridian paſſing thro' the moſt Weſtern Place of the Earth, that was then known, (as the *Azores* or *Fortunate Iſlands*; and from thence towards the Eaſt, they diſpoſed of all Places on the whole Surface of the Earth, in regard of Longitude; and called the *Longitude* of the Place, that Arc of the Equator which was intercepted between the *Firſt Meridian* and the *Meridian* of the Place. But when they found the Globe of the Earth was really Inhabited all about, and there was no ſuch thing as a moſt WeſternPlace, the above-mention'd way of reckoning the Longitude of Places was by degrees neglected, which would have been of great uſe in underſtanding the Monuments of the Ancients; (tho' it might

ſtil

ſtill be made uſe of;) and every one made the Meridian of his own City to be the chief, and took notice how the Meridians of other Places ſituated towards the Eaſt or Weſt ſtand in regard of this, or how many Hours there are between the more Eaſtern Meridians leaving the Sun, and the more Weſtern Meridians overtaking of it, or between the Sun's ſeeming to leave the more Eaſtern Meridian, and arrival at the more Weſtern.

Again, the Inhabitants of the Earth being compared together in regard of the Meridians and Parallels, ſome of them are called by Geographers the *Periœci*, and they inhabit in the ſame Parallel of the Earth, but in oppoſite Meridians. (The oppoſite Meridians are the oppoſite halves of the ſame Circle computed from the Poles, becauſe ſuch Places as are in ſuch oppoſite Semicircles are referred to oppoſite points of the Equator of the Earth.) On this account the *Periœci* enjoy the ſame Seaſons of the Year, by reaſon of the inclination towards the Sun of that Pole of the Earth which is neareſt to both, or its declination from the Sun; or ſpeaking according to the apparent reſt of the Earth, becauſe the Sun approaches after the ſame manner towards the Parallel of both Places, (becauſe the ſame) or recedes from it: But they have alternate changes of Noon and Midnight, according as the Meridian of the one or the other is turned to the Sun by the diurnal revolution of the Earth, or (which is all one,) as the Sun carried about us in its apparent diurnal revolution, approaches to the Meridian of the one or the other of them; unleſs they live in a Frigid Zone, where they enjoy the Day together. Again, other Inhabitants of the Earth are called *Antœci*, and they live in the ſame Meri-

dian, but in opposite Parallels, so that they have the same Noon and the same Midnight; because they have the same Longitude. But the Summer of the one is the other's Winter, according as the Earth in its annual motion turns sometimes its North Pole, and afterwards its South Pole, more towards the Sun; or as the Sun declines from the Equator to this or the other Pole. And lastly, others are call'd *Antipodes*, because they live in opposite Meridians and Parallels, and walk with their Feet diametrically opposite to ours. These have exactly the contrary things happening to them: The one has Summer and long Days or no Nights, at the same time as the other has Winter and short Days or perpetual Nights; 'tis Night here when 'tis Day there, and Night there when 'tis Day here. 'Tis evident an Inhabitant of the Equator, is an Antœcian to himself; and that the same Person is both Periœcian and Antipode to him; but that an Inhbitant of both the Poles is a Perœcian to himself, and that his Antœcian is the same as his Antipode.

POPOSITION IV.

TO give an account of the generation and nature of the Horizon and its Secondaries and Parallels, and to explain the Terms in Astronomy depending thereon.

Besides the lately mention'd Circles of the Equator and Ecliptic, in respect to which, Astronomers have determined the Places of the Stars viewed from the Earth, there is another called the *Horizon*, which is the great Circle, that every one standing in an open Plain, defines by turning of his Sight round about, and that divides the visible from the invisible part of the Heavens. This Circle is the most considerable

rable of all in a Sphere to appearance; becauſe immediately taken notice of and determined by any Obſerver, tho' the moſt unskilful. Its geneſis in the true Syſtem, has been deliver'd above in *Prop.* 32. *B.* 1. But becauſe the Spherical Superficies, to which we referred all Celeſtial Phænomena, is ſuppoſed to be concentric with the Earth and not the Eye, a Plane that paſſes thro' the Eye and touches the Earth, will not divide it into equal Segments; therefore the Horizon there deſcribed, generated by the Section of the ſaid Plane and Sphere, is properly called the *Senſible* Horizon; becauſe defined by Senſe: And the *Rational* Horizon of this Spectator, is that which is made by the Section of the ſaid ſpherical Superficies, by a Plane parallel to the Senſible Horizon, and paſſing thro' the center of the Earth. Theſe two parallel Planes produced, will mark out the ſame Circle in the Superficies of that very great Sphere, wherein the Fixed Stars are ſeen, becauſe the Earth compared to the Sphere of the Fixed Stars, is but like a Point. From whence 'tis evident, that the Horizon conſidered among the Fixed Stars is a great Circle in a Sphere, every way equidiſtant from the Point, exactly over the Head of the Perſon (whoſe Horizon it is,) called the *Zenith*, and from the oppoſite to it the *Nadir*: Which Points therefore, namely the Zenith and Nadir are the Poles of the Horizon, in which the Secondaries of the Horizon, drawn thro' all the points of the Heavens, croſs one another, which are therefore called *Verticals*, and ſometimes *Azimuths*; but the Parallels of the Horizon, whether ſuppoſed to be above or below, towards the Zenith or Nadir are called *Almicanters*. There are two of theſe Vertical Circles, that are moſt conſiderable, the one paſſing thro' the Zenith (the com-

mon Node of all the Verticals) and the Poles of the World, call'd the *Meridian*; namely the *Celeſtial*, becauſe being directly over the Terreſtrial Meridian: This Circle interſects the Horizon in the Cardinal Points of South and North, and marks them out. The other chief Vertical is at Right angles to this, and interſects the Horizon in the poins of Eaſt and Weſt; and becauſe the former, tho' a Vertical, is called likewiſe by another name, *viz.* a Meridian, this latter has the name of the *Primary Vertical* left intirely to itſelf. By the help of the Horizon and its Secondaries, any point of the Heavens whatever is diſpoſed of according to its Altitude above or Depreſſion below the Horizon, and Azimuths. That is, the *Altitude* or *Depreſſion* of any Point is an Arc of a Vertical Circle intercepted between the ſame Point and the Horizon: And the *Azimuth* is an Arc of the Horizon intercepted between the North or South Cardinal Point and that Point wherein the Vertical drawn thro' the Phænomenon, meets the Horizon; which is *Eaſtern* or *Weſtern*, according as you reckon from the Meridian towards the Eaſtern or Weſtern part of the Heavens. Sometimes the Azimuth is reckoned from the Eaſtern or Weſtern Cardinal Point towards the North or South. But the *Eaſtern* or *Weſtern Amplitude* is always reckoned from theſe Points, which is an Arc of the Horizon, reckoned from the Eaſt or Weſt to the Riſing or Setting point, and therefore both may be *Northern* or *Southern*; tho' this way of naming them ſeems to have flown firſt from intire Conſtellations; and the queſtion was, how large a ſpace of the Horizon any Conſtellation took up in Riſing or Setting.

That portion of the Convex Surface of the Earth, that the Spectator ſtands upon, being taken as a Plane parallel to the Horizon; the ſeveral Quarters of the Winds are conſider'd in it, and the Cardinal ones are the North, South, Eaſt and Weſt; lying under the Meridian and Prime Vertical, deſcribed above, and are the Sections of the Horizontal Plane with the Planes of the ſaid Circles. The Motion of the Heavens will point out to us theſe four Quarters; namely the Eaſt, where the Sun riſes in the Equinoxes; the Weſt, where the Sun ſets that Day: The North, where the Pole of the World is ſeen by us the Inhabitants of the Northern Hemiſphere of the Earth, and the Stars call'd the *Septem Triones*, always appear: And laſtly, the South, whence the Sun ſhines upon us at Noon. For the Terms in Aſtronomy are accommodated to ſuch tracts of the Earth as are ſituated in the Northern Temperate Zone, becauſe Aſtronomy was firſt cultivated in thoſe parts. There are as many lying exactly betwen the four Cardinals, that have names (in the *English* and other Tongues a-kin to it) made up of the Cardinals that are on their ſides, ſo as that the name of the chief Cardinal Point is ſet firſt. Betwixt theſe eight are placed eight more, having Names made up of the Names of the eight preceding, ſo as that each of them is made up of the two laterals next it, and the principal Cardinal ſet firſt: Hence it comes to paſs, that the name of the chief Cardinals is preſently doubled, and the names of the other Cardinals in the beginning and end of the Word thus compounded. Between the ſixteen Points named, there are ſixteen others having compound Names, each from one of the eight firſt, to which the name of the Cardinal Point towards which

it declines, is connected by a Propoſition; and thus we have two and thirty Wind's Ways, or Points, which are the common interſections of as many Verticals with the Plane of the Horizon, and ſerve to diſtinguiſh the Winds accurately enough. But we uſe the Degrees of the Horizon, in reckoning the Azimuths of the Celeſtial Phænomena, beginning from one of the four Cardinal Points, as was ſaid above. Beſides theſe Points, (while they are conſidered in the Plane of the Horizon) are looked upon as Right lines; but if the ſurface of the Earth be looked upon as Spherical (as it really is,) none of them produced upon the ſurface of the Earth are Right lines, and only the Cardinals are Circles; namely, one pointing out the North and South, being a great Circle, and the ſame with the Meridian of the Place, from whence it takes its beginning; the other ſhewing the Eaſt and Weſt, being a leſſer Circle, paſſing thro' the Place, and parallel to the Equator; excepting when the Place is in the Equator itſelf, in which caſe it is a great Circle; becauſe the Equator itſelf. The other Points of the Compaſs being produced upon the ſurface of the Earth are Spirals *ſui generis*, cutting all the Meridians at given Angles, and are called *Rhumbs* or *Loxodromics*.

PROPOSITION V.

TO *explain the generation and nature of the Celeſtial Meridian and other Hour Circles.*

In *Prop.* 3, we explain'd what ſort of a Circle the Meridian of each Place conſidered upon the Earth was; namely, a Circle compaſſing the Earth and paſſing thro' the ſaid Place and both the Poles; and conſequently a great Circle, becauſe paſſing thro' the oppoſite Points. And the Celeſtial Meridian of the ſame Place is that

in the Heavens, which lies directly over and and anſwers it. And becauſe the Obſerver continuing in the ſame place, looks upon the Earth, and therefore the place where he ſtands, and the point over his Head as unmoved, he likewiſe conceives of the Celeſtial Meridian paſſing through the Poles and the Vertex of the Place, which are unmoved, as a Celeſtial Circle unmoved: And ſeeing the Heavens and all the Stars ſeem to move round in the Diurnal motion, he conceives of the Meridian of the Place where he is, as what has no ſhare of that motion, and as it were, without the movable Heavens, and of the Heavens, as if they were revolved within it. And it is always Noon at that place, when the Sun, by the Diurnal revolution of the Heavens, ſeems to have arrived at the above-mentioned immovable Celeſtial Meridian above the Horizon; and Midnight, when the Sun has reached the other part of it below the Horizon. For ſince this Celeſtial Meridian paſſes thro' the Poles of the Horizon and Equator, and (by *Prop.* 15. *B.* 1. *Sphæric. of Theod.*) interſects the Horizon and the Equator together with its parallels at right angles; 'tis evident that the Meridian divides the Segments of all the Circles parallel to the Equator made by the Horizon, into equal parts; and therefore ſince the Sun deſcribes in its Diurnal motion, one of theſe Parallels, (the Arc above the Horizon in the Day-time, and the Arc or Portion of it below the Horizon in the Night,) 'tis evident that Mid-day or Noon is then made, when the Sun comes to the Meridian above the Horizon, and Mid-night when it arrives at its oppoſite part, lying concealed below the Horizon: from whence it has its name. And for the ſame reaſons the middle point of the continuance of any Star above or below

below the Horizon happens, when that Star comes to the Meridian: Where, at the ſame time, it has its greateſt elevation, call'd, its Meridian Altitude.

Again, ſince the ſpace of time between the two neareſt Noons is ſuppos'd to be divided into 24 equal parts, (called *Hours*,) and by reaſon of the equable Revolution of the Earth about its own Axis, the Sun ſeems to deſcribe equably the Equator, or ſome parallel to it upon the Poles of the Celeſtial Equator; beſides the Meridian, there are eleven other *Hour Circles* to be conceived of, paſſing thro' the Poles of the Equator, and together with the Meridian, dividing the Equator into 24 equal parts; and theſe are look'd upon (in regard of the ſame place) as immovable, and like the Meridian, placed without the Sphere, the Heavens in the mean while revolving equably under them. From whence it is evident, that an Arc of the Equator intercepted between any two next of theſe Circles is 15 Degrees, that is, 24th part of an intire Circle. But when the Terreſtrial Meridian is changed, the Obſerver changes all his Hour Circles with it. They are called Hour Circles, becauſe the Sun, ſeen to have arrived at any of them, by its apparent Diurnal Motion, cauſes the Hour of the Day to be that before or after Noon, which this Circle is in order, more Eaſtern, or more Weſtern than the Meridian. An infinite number more of theſe ſort of Circles may be imagined, according as we ſuppoſe the Hour to be divided into 60 Minutes, each of which is again divided into 60 Seconds, and ſo on. Theſe Hour Circles are the ſame in poſition, with the Circles of Declination; (of which in *Prop.* 2.) becauſe they are Secondaries to the Equator: But they differ

in

in this, that the Circles of Declinations revolve, together with the Stars and Points of the Heavens, whoſe Declination they meaſure; but theſe Hour Circles (as was ſaid before) are look'd upon as immovable. Hour Circles alſo in the Heavens, anſwer to Meridians on the Earth; and indeed the Meridians on the Earth, are the real Hour Circles, and are only apparent ones in the Heavens. For as 'tis Noon in a given Place, when the Earth, revolving by its Diurnal Rotation, arrives at ſuch a ſituation, as has the Plane of the Meridian produced falling upon the Sun: ſo it is ſuch or ſuch an Hour, before or after Noon, as that Terreſtrial Meridian, in order from the Meridian of the Place, is, in whoſe Plane produced the Sun is then found.

PROPOSITION VI.

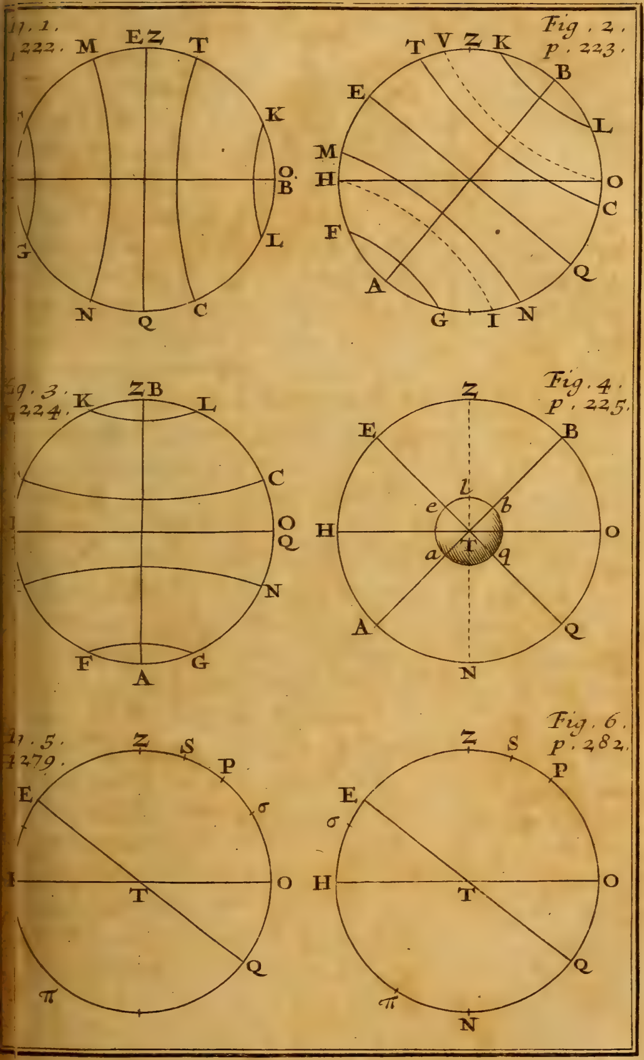
TO explain the various appellations of the Sphere of the World, and other Terms uſed in Aſtronomy, depending upon the various inclination of the Horizon to the Equator.

Since according to the diverſity of Places upon the Earth, the Horizon, and conſequently the Face of the Heavens, (that is, the Sphere of the World,) is different; Aſtronomers make this diverſity threefold, according to the three different kinds of Poſition of the Horizon to the Equator. For either the Horizon of the Place is right to the Equator, cutting it at right angles, or it is oblique, or elſe does not cut it at all, but coincides with it. Such as have the firſt Poſition of the Sphere, are ſaid to inhabit a *Right Sphere*; ſuch as have the ſecond, *an Oblique*; and ſuch as have the third, a *Parallel Sphere*, becauſe each Star ſeems to deſcribe a Circle, parallel to the Horizon in its Diurnal Motion.

In

In a right Sphere, (namely, where the Equator E ♎ is erect or perpendicular to the Horizon HO, as in the 1ſt Figure,) the Equator (by *Prop.* 15. *B.* 1. *Theodoſius*) will paſs thro' the Poles of the Horizon, the Zenith, and Nadir; the Place therefore itſelf, on the Surface of the Earth, will be in the Terreſtrial Equator; for ſuch only have the Zenith and Nadir, in the Celeſtial Equator; all ſuch therefore, inhabit a Right Sphere, and only ſuch as are ſpread along the length of the Terreſtrial Equator. Their Horizon paſſes thro' the Poles of the World A and B (by the abovecited Propoſition of *Theodoſius*) and every Point in the Heavens will ſeem to aſcend right or perpendicularly above the Horizon; becauſe it deſcribes a Circle parallel to the Equator, (which is right or perpendicular to the Horizon,) in its apparent Diurnal Motion. And the Points, that riſe together, come to the Meridian alſo together, and ſet together; becauſe the Poles, about which the Diurnal Motion is made, by which the Stars ſeem to riſe, come to the Meridian, and ſet, are as well in the Horizon as in the Meridian of this Sphere. And from this ſituation of the Sphere, the way of reducing the points of the Heaven to the Equator, derives its original. That is, the *Right Aſcenſion* of any point in the Heavens is an Arc of the Equator, reckoned towards the Eaſt, intercepted between the beginning of Aries, and the point of the Equator that riſes, together with the ſaid point in the Heavens, in a Right Sphere: After the ſame manner we are to underſtand what the *Right Deſcenſion* is: and inſtead of the Horizon of a Right Sphere, the Meridian of any Place may be aſſumed. Now, becauſe only the Inhabitants of the Terreſtrial Equator have a Right Sphere,

Plate 1, Book 2.



Sphere, what the affections of this Sphere are, is evident from what has been ſaid above; (eſpecially in *Prop.* 33. *B.* 1.) namely, that the Nights are always equal to the Days, and every point of the Heavens is as long above as below the Horizon.

In an Oblique Sphere, where the Horizon *HO* (*Fig.* 2.) cuts the Equator *E* ♋ at oblique Angles; neither of them paſſes thro' the Poles of the other, and therefore one Pole of the Equator or World is above the Horizon, the other below it; the former therefore is always viſible, but the latter always inviſible: Nor will the Equator paſs thro' the Vertex *Z*, but the Vertex will lie between the Equator and the viſible Pole. There are two kinds of this Sphere; for either the North Pole *B* is elevated above the Horizon *HO*, and the South lies unſeen; or on the contrary, the South Pole is elevated, and the North depreſſed. All ſuch have the Sphere of the former kind, as live between the Terreſtrial Equator and the Arctic Pole, and ſuch as live between the Equator and Antarctic have the latter. The chief Phænomena of both of them, as to Summer and Winter, may be ſeen, already explained, in *Prop.* 33. *B.* 1. according to the true Syſtem of the World. And in the apparent Syſtem, ſince each point in the Heavens, by its equable diurnal Motion deſcribes either the Equator or a Parallel to it; and only the Equator, but not one of the Parallels is divided into two equal parts, by an oblique Horizon; the Sun, and all the Stars, that decline towards the viſible Pole, continue longer above the Horizon than below; and on the contrary, ſuch as decline towards the inviſible Pole, continue hid longer below the Horizon, than they appear above the Horizon.

And

And this holds to a certain limit towards both Poles. For if the declination be increafed till the diftance of the Star from the vifible Pole, is lefs than the elevation of the Pole above the Horizon, the Star does not fet at all: But if its Diftance from the invifible Pole be lefs, it will not afcend above the Horizon; that is, it will not rife at all. And the Parallels *OV*, *HI*, bounding the always vifible and the invifible Stars in a given Place, and touching the Horizon, are called, by fome of the Ancients, (as *Euclid* in the *Phænomena*, and *Manilius* in his *Aftronomy*) the *Polars* of that Place, which are therefore greater and more remote from the Celeftial Pole, according as the Place, whofe Polars they are, is farther off from the Terreftrial Equator; that is, as the Sphere is more oblique. All the Stars that lie between thefe Polars have the viciffitudes of Rifing and Setting, Afcenfion and Defcenfion. The *Oblique Afcenfion* of a Star or any Celeftial Point, is an Arc of the Equator, reckoned towards the Eaft, intercepted between the beginning of Aries, and that point of the Equator that rifes together with the Star, in a given oblique Sphere: and this is various according to the variety of obliquity of the Sphere. And the *Afcenfional Difference* is the difference betwixt the right and oblique Afcenfion. And the fame things are to be underftood concerning the *Oblique Defcenfion*.

In a Parallel Sphere, where the Horizon *HO*, [*Fig.* 3.] coincides with the Equator *EQ*, and makes one of the Parallels of the Diurnal motion, their Poles alfo (namely, the Zenith and Pole of the World) will coincide; and therefore this pofition of the Sphere agrees only with the Inhabitants of the two Poles of the Earth; and all the Celeftial Bodies or Points, that

that deſcribe Circles parallel to the Equator by the diurnal motion, will likewiſe deſcribe Circles parallel to the Horizon. Upon this account, there will be no riſing or ſetting at all by the diurnal motion, and therefore no Aſcenſion or Deſcenſion in this Sphere, nor any Meridian, ſince it paſſes thro' the Pole and Vertex of the Place; and ſince theſe two coincide in this Sphere, they can't determine the Circle, each Vertical having an equal right to claim the name of a Meridian. The Phænomena of the Sun ariſing from the Annual motion of the Earth, viſible in a parallel Sphere, are explained in *Prop.* 33. *B.* 1; namely, ſuch as agree to the Inhabitants of either Pole. In the apparent Syſtem 'tis evident, that the Sun, during its declining from the Equator towards that Pole, which is in the Zenith, is always ſeen, and makes the Day half a Year long; as on the contrary 'tis half a Year Night, while the Sun declines towards the other part of the Equator, and conſequently of the Horizon; and that the beginning of the Day and Night falls upon the Sun's ingreſs into the Equinoctial Points.

PROPOSITION VII.

THE Latitude of the Place is an Arc ſimilar to the Arc of the Elevation of the Pole above the Horizon.

Let *aebq* [*Fig.* 4.] be the Earth, its Center *T*, Poles *a* and *b*, Equator *eq*. Let any Place on the Surface be taken, as *l*, whoſe Latitude is *le*, an Arc of the Meridian intercepted between it and the Equator. Let the Celeſtial Sphere be concentric, and its Poles, Equator and Meridian, *B*, *A*, *E* ♎, and *BEA* ♎, anſwer to the Poles, Equator and Meridian of the Earth. Let *Z* be the Zenith, and *N* the Nadir of the

Q Place

Place *l*; namely in the intersections of the Right line *T l* produced, with the Superficies of the Celestial Sphere. Therefore the great Circle *H O*, in the Sphere, described upon the Poles *Z* and *N*, is the Horizon of the Place; and the Arc of the Meridian *O B*, is the height of the visible Pole *B* above the Horizon; and this, I say, is similar to the Arc *l e*, namely, to the Latitude of the Place. For, since *A* and *B* are the Poles of the Circle *E Q*, the Angle *B T E* is right; (by *Prop.* 10. *B.* 1. of *Theod. Sphær.*) and for the same reason, the Angle *Z T O* is right. If therefore from the equal Angles *ETB*, *ZTO* you take the common Angle *Z T B*, the remaining Angles *E T Z*, *B T O*, will be equal. And therefore (by *Prop.* 33. *Elem.* 6.) the Arcs *e l*, *B O*, they stand upon at the Center, are similar.

PROPOSITION VIII.

TO explain the Causes of the Crepusculum or Twilight, and to define its Limits.

The Twilight is that dubious light, which we have before the Sun rises and after the Sun sets. If there were no Atmosphere about the the Earth, nor any brightness from the *Aura Ætherea* that is near the Sun, so soon as by the diurnal motion of the Earth any place upon its surface comes within the shadow of the Earth by the diurnal motion of the Earth, or so soon as the Sun descends below the Horizon of that Place, there would be nothing but mere Darkness; because the Spectator being forsaken by the Rays of the Sun, is left destitute of Light. But not only the Sun, but the Ethereal *Aura* that is spread about the Sun very nearly to it (and its inflamed Atmosphere as it were) do also shine and give some light: Now, this spending

ing more time than the Sun does in riſing or ſetting, before Sun riſe the Aurora ſhines out in a manifeſtly Circular figure, riſing into the brightneſs of the ſame Figure with that of the Segment of the Circle, that is already riſen of the Atmoſphere of the Sun, and intirely different from that, which proceeds from the illumination of the Atmoſphere of the Earth, made by the Sun: Which is to be underſtood in like manner concerning the Twilight after Sun-ſet. Becauſe the matter that thus ſhines, by reaſon of the nearneſs of the Sun, ſhines ſometimes more, ſometimes leſs, the boundaries of the Twilight, which ariſes from thence are not ſo certain, eſpecially when it acts in conjunction with the other and more powerful cauſe of this dubious Light. For after that the Inhabitant of the Earth ſtanding upon its ſurface ſomething beyond the bounds of Light, is revolved into the ſhadow of the Earth, the Atmoſphere that ſurrounds the Earth and is expanded to a conſiderable diſtance above the Earth, is ſtill inlighten'd, and does alſo inlighten the Place where the Obſerver ſtands with its reflected Rays. This cauſe is often upon the change, according as more or fewer Particles fit for reflecting or other ways conveying the rays of the Sun to us, are found ſuſpended in the Air; and according as theſe Particles get up to a greater or leſs height, which depends upon the gravity of the Air (it being the Fluid in which they ſwim,) that is ſhewn by the Barometer. For if they hang low and very near the ſurface of the Earth, even theſe alſo ſoon ceaſe to receive any of the Rays of light, being revolved preſently afterwards, together with the Place over which they hang, into the Earth's ſhadow. If they are ei-

ther very rare as they float in the expanded space, or unfit to reflect the Light, they will return to us either such a Light from the Sun as is nothing to speak of, or at least so weak and thin as not capable of affecting our Sight, as we really experience in all the expanded space that lies without the Earth's shadow; for tho' it lies perpetually open to the Sun's Rays, yet it sends back to us so weak and faint a Light, that it scarce deserves that name.

Tho' the duration of the Twilight depending upon both these causes is various, yet 'tis certain, that the beginning of the Morning Twilight happens generally about the time that the Sun is not above 18 Degrees below the Horizon; and that the Evening Twilight ends about the same time, or when the Sun is gone to the like degree of depression below the Horizon. *Tycho* would have this depression of the Sun that bounds the Twilight to be 16 Degrees. Others extend it to 19; that is, till the least Fix'd Stars become visible. *Cassini* from his own Observation, extends it only to 17 Degrees. *Ricciolus* finds by his Observations, that 'tis not the same in the Morning as it is in the Evening, and that it is different in different Seasons of the Year.

SCHOLIUM.

There is a Light that seems very near a-kin to the Twilight, which was first observed by the Quick-sighted Mr. *Cassini*, in the Year 1683, a little before the Vernal Equinox in the Evening, and extended along the Ecliptic from the Sun towards the East. The Observations made afterwards by Mr. *Cassini* and *Fatio*, evince that this Light is diffused from both sides of the Sun almost along the Ecliptic, but ordinarily deflects a little from it towards the North rather

tha

than the South. Its form is pointed on both ſides; its two points being ſometimes more, ſometimes leſs diſtant from the Sun, at firſt about two Signs or a little more; but three Years after, its diſtance from the Sun encreaſed to three Signs or 100 Degrees: Its breadth is above 30 Deg. near the Horizon. But it can't be ſeen where it is broadeſt; namely, juſt at the Sun it itſelf along the Circle of its breadth. Its ſides are ſtreight, bating its being gibbous a little ſometimes about the middle, between the Sun and either Point, and they are inclin'd to one another, ſometimes at a greater and ſometimes a leſs Angle. This Angle in its mean bigneſs is about 21 Degrees. From the whole, it is evident, that this Luminous Phænomenon moves together with the Sun, juſt as that does, thro' the Ecliptic; which is alſo true of both the Points, allowing for their greater or leſs diſtance from the Sun during the increaſe or decreaſe of the whole. The brightneſs of it is much like that of the Milky Way, or the Tail of a Comet, and is pellucid like the latter; 'tis greater in the middle, leſs towards the extremes, and ſenſibly decreaſing, till it vaniſhes in the ſurrounding Blue of the Heavens: From hence it appears to different Obſervers, according to the ſharpneſs of their Sight, at the ſame Time and Place, of a different magnitude, and according as the Heavens are clear or cloudy, and the true Twilight and brighter Stars are preſent or abſent, it is more or leſs extended and variouſly terminated, and appears fainter always in the Morning than in the Evening. This light, by reaſon of the Twilight laſting all Night long, can't be ſeen in the midſt of Summer in the Regions about either Pole; but in the Morning and Evening of the the ſame Day

it may be ſeen about the middle of Winter (i the Moon be away.) In places near the Equa tor it may be ſeen at any time of the Year And in any place it is brighter, the more e rect it is, in regard of the Horizon; becauſe i is then more out of the Twilight and Vapour that hang about the Horizon: And therefore i is beſt ſeen in theſe Northern Regions in th Morning after the Vernal Equinox, in the be ginning of *October*; and in the Evening, at th end of *February*; the Ecliptic along which it i ſpread, being then more erect in regard of th Horizon,(in the beginning and end of the Twi light, which is then ſhorteſt) the Equinoctia Points being at that time in the Horizon.

TheBody (or rather collection of Corpuſcles whoſe appearances theſe are, ſeems to ſurroun the Body of the Sun in the form of a Lens; a Mr. *Fatio* conjectures. The Plane thro' th edge of the Lens is in the Plane of the Ecliptic or at leaſt not far from it: The Edge itſelf i between the Orbits of Venus and the Earth, bu nearer to the latter. The Particles which fil up this lentiform ſpace (like the particles of the Sun's Atmoſphere, of which we ſpoke but juſt now) ſeem to produce the abovemention'd ap pearance, by reflecting the light of the Sun.

For a Ring whoſe oppoſite Faces are plane and parallel, would appear in the form of an Ellipſe, more obtuſe towards the Points than this Phænomenon. Mr. *Caſſini* takes theſe par ticles reflecting the Sun's light, contained in this ſpace, for an infinite number of Planets, ſince being feparated, they would, like the Planets, exert their motions about the Sun; by which method, this morning and evening light owes its original to innumerable Planets, juſt as the Milky Way does to innumerable Fix'd Stars.

Mr.

Mr. *Fatio* suspects that this light is coeval with the World: *Cassini* on the contrary, that it was produced a little before the first Observation he had made of it, and that it was not in being two Years before, since he had often viewed a Comet very intensely with his Eyes, in the same place as this Phænomenon ought to possess. That it has appear'd formerly, and afterwards disappeared, he thinks is very probable from some ancient Histories that seem to give a description of it; but especially from the Observation of Mr. *J. Childrey*, who no doubt saw it, as is evident from his description, *pag.* 183, 184. in the Advertisement at the end of a Book, published *Anno* 1661. intitled *Britannia Baconica*; because it appears even now in the same place, and near the same Constellations, as it did at that time of the Year. His Words are these: *There is something more, that we would recommend to the Observation of the Mathematicians; namely, that in the Month of* February, *and a little before and after it (as I have observed for several Years) about six a Clock in the Evening, when the Twilight has entirely left the Horizon, a Path of Light tending from the Twilight towards the Pleiades, and touching them as it were, presented itself very plainly to my Sight. This Path is to be seen whenever the weather is clear; but best of all in a Night when the Moon does not shine.* And a little after: *I am apt to believe that this Phænomenon has been formerly, and will hereafter appear always at that above-mentioned time of the Year. But the Cause and Nature of it, I can't so much as guess at; and therefore leave it to the enquiry of Posterity.*

Proposition IX.

To explain what is meant by the Poetic Rising and Setting of the Stars, and the kinds of it; namely, the Cosmical, Achronical and Heliacal

In the foregoing Propositions we have treated of the true Rising and Setting of the Stars; namely of their ascent above the Horizon of a certain Place, and descent below it, or rather of a depression of a given Horizon below a Star, and elevation of the same above it; and that without any consideration of the Sun. But now the Rising and Setting of the Stars are to be compared with the motion of the Sun along the Ecliptic, and consequently with the Daylight and Seasons of the Year. For the ancient Husbandmen, and from their traditions the Writers of Husbandry, as also Physicians, Poets and Historians, have made use of these marks to express the Seasons of the Year by, and consequently none of them can be understood without an explication of these Terms.

There are usually reckoned three kinds of the Poetic rising and setting of a Star, The *Cosmical*, *Achronical* and *Heliacal*. A Star is said to rise *Cosmically*, when it rises together with the Sun, but to set Cosmically, when it sets when the Sun rises; so that the Cosmical rising and setting is all one with the Morning rising or setting, as if the beginning of the Artificial Day, or the Rising of the Sun, were the same with that of the World. A Star is said to rise or set *Achronically*, that rises or sets when the Sun sets; and consequently the Achronical rising or setting is all one with with the Evening ones. *Kepler* maintains that these words are to be taken in another sense; namely, so as that to rise and set *Cosmically*, signifies the same as to ascend

above

above or deſcend below the Horizon; but that to riſe or ſet *Achronically*, is the ſame as to riſe or ſet in the Sun's oppoſite, or in the other Achron or extreme of the Night: In which ſenſe *Ptolemy*, and to this day moſt Aſtronomers ſay, a Planet is Achronical, when it is oppoſite to the Sun and bright all the Night: So that to riſe Achronically is, as uſual, the ſame as to riſe when the Sun is ſetting; but to ſet Achronically, is to ſet when the Sun is riſing; which is commonly called the Coſmical ſetting.

A Star riſes *Heliacally*, when it before lay hid under the Rays of the Sun, ſo as that it did riſe and ſet together with the Sun, but now gets ſo much out of the Rays of the Sun, as when the Sun is below the Horizon the Star becomes viſible, being to ſet ſoon after under the Horizon, or to diſappear by the ſupervention of the Daylight. That Star is ſaid to ſet *Heliacally*, which was lately ſeen above the Horizon when the Sun was not much below the Horizon, but now has hid itſelf ſo much among the Rays of the Sun, that the next Day being riſen above the Horizon, or juſt ſetting below it, it can no more be ſeen. If there were no Twilight, this Heliacal riſing and ſetting of the ſame Star (or rather Apparition and Occultation,) would not be above one Day diſtant from one another at moſt. For in that caſe a Star would be ſeen either before Sun-riſe, if its Oblique Aſcenſion in a given Horizon were never ſo little leſs than that of the Sun, or it would be viſible after Sun-ſet, if but never ſo little greater. Nay, if there were no Atmoſphere at all, the ſmalleſt Stars would be viſible even in the Daytime when the Sun ſhines. For the reaſon why they don't appear is this: The particles of the Atmoſphere enlightened ſtrongly by the Sun, affect the Eye of

of the Spectator with ſo vivid a light, that the Retina (or whatever it be that is the Senſor of Sight) is ſcarce moved by the very weak image of the Star, and conſequently can't take notice of it or ſee. But if there were no Atmoſphere ſpread about the Earth, nor any ſurrounding Terreſtrial Bodies that reflect Light; if the direct Rays of the Sun were turn'd off from the Eye, the Eye thus free from the Rays of a Body either very bright in itſelf, or ſtrongly enlightened, would ſee clearly (as it does now in the Night) even the ſmalleſt Stars, unleſs they are hid under the Atmoſphere of the Sun: For this together with the Body of the Sun, make up one lucid Body. Since proportionable to the leſſer brightneſs of a Star, there is need of a greater depreſſion of the Sun below the Horizon, to make it viſible; 'tis evident that the Heliacal ſetting of the Sun is ſooner, and the riſing later. That the leaſt Stars may be ſeen, the Twi-light muſt be quite over, or the Sun muſt be full 18 Deg. below the Horizon: For the ſeeing of a Star of the ſixth Magnitude, the Sun muſt be 17°; and ſoon, 'till you come to the Stars of the firſt Magnitude, which are to be ſeen in that part of the Horizon which is towards the Sun, when the Sun is 12°. The Planets ſhine with a brighter and fuller light, and therefore have no need of ſo great a depreſſion of the Sun below the Horizon, ſo that 11 Degrees are enough for the ſeeing of Mars and Saturn; about 10 for Jupiter and Mercury; and 5 deg. are commonly look'd upon as neceſſary for Venus, tho' it has been frequently ſeen while the Sun ſhone, and not at an Heliacal Riſing or Setting. But theſe things in the Planets depend upon their various diſtance from the Earth, and the more or leſs fullneſs of the Orb of the Inferior.

All

All the Fix'd Stars under the Zodiac, and the superior Planets, Saturn, Jupiter and Mars, the Sun in its Annual Motion towards the East getting before them, rise Heliacally in the morning, a little before Sun-rise; that is, a few days after their Cosmical rising: They set Heliacally in the evening; namely, a little before their Achronical setting. But the Moon, which always gets before the Sun, rises Heliacally in the Evening; when its very old and hastens to a Conjunction with the Sun. The inferior Planets Venus and Mars, sometimes getting before the Sun, and at other times being left behind by him, (as has been shewn at large in *B.* 1.) sometimes rise Heliacally in the morning, namely, when they are Retrograde; sometimes in the evening, when they are Direct; which is to be understood in like manner of the Heliacal setting of them. But the Fix'd Stars, plac'd a great way from the Zodiac towards the elevated Pole, may in the same Day both rise and set Heliacally together, and undergo other changes in regard of this kind of Rising or Setting; as will appear to any Person that shall consider it.

The *Grecians* and *Romans* anciently used a Year not exactly fitted to the motion of the Sun; and by that means sometimes they were before the Sun, and sometimes behind him. But the Seasons of the Year not returning with their erroneous Calendar, but with the Sun and the Solstices; that their Country, Domestic, or Military Affairs might be dispatched in their proper Seasons, the Ancients proposed the rising and setting of the Stars instead of a Calendar. For the Fix'd Stars moving from the Equinoctial Points but very slowly, and by a motion not then taken notice of, they made

no

no queſtion, but that the Sun returning to the ſame place in reſpect of the ſame Fix'd Star, did alike come again to the ſame Station in reſpect of the Equinoxes or Solſtices; that is, when the ſame Star riſes or ſets Coſmically, Achronically, or Heliacally, (which they chiefly regarded,) that the ſame ſeaſon of the Year was reſtored to the Globe, and conſequently that the ſame labours or affairs were to be returned to; from hence they called the ſtated Seaſons of the Year *Sydera*, or the Stars. But it is found that the Fixed Stars, after a long ſeries of Years, remove out of their places; and therefore, if the Day of the Riſing or Setting of the Fix'd Star given among the Ancients is to be reduced to our Calendar, an account is to be taken of the motion of the Fix'd Stars in the intermediate time. And now-a-days Calendars may eaſily be had, and the *Roman* Calendar (which we uſe,) comes nearer to the Motion of the Sun, upon which the Seaſons of the Years depend and are ordered, than the Riſing and Setting of the Fix'd Stars; the obſerving their Riſing and Setting came to be neglected by degrees, and is not uſed by any but the Poets, who are wont to deſcribe and paint out as it were the ſeveral Seaſons by the circumſtances of the various Riſing and Setting of ſo many Stars, than which nothing is more beautiful and affecting; tho' it be generally done by them very erroneouſly, becauſe they deſcribe at this time the Day of our Calendar by that Riſing of the ſame Star by which it would have been juſtly deſcribed in the time of *Cæſar*; whereas the difference between theſe two times is nearly 14 Days.

SECTION II.

Of the Division of Time, and ſeveral other things depending thereon.

PROPOSITION X.

TO *explain the Diviſion of Time into Hours, Days and Weeks, and the Terms uſed in Aſtronomy depending thereon.*

We have thus far been explaining the generation of the Circles of the Sphere in the apparent Syſtem of the World, and the more uſual Terms in Aſtronomy depending thereon: But ſeeing there are no terms in Aſtronomy that occur oftener than thoſe, whereby Time or any ſpace of it is expreſſed; 'tis neceſſary to treat a little concerning them,and to ſhew, by the way, the Civil diſpoſition of Time, ſo far as it is uſed in an Aſtronomical Calculation.

The moſt conſiderable parts of Time are, a *Day*, an *Hour*, *Month* and *Year*. The firſt of them is a *Day*, (becauſe a ſpace of Time moſt obvious and known by us;) and it is either Natural or Artificial. A *Natural* Day the duration of an intire apparent revolution of the Sun about the Earth. The *Artificial* is that part of the Natural, during which the Sun is above the Horizon; and it is oppoſed to the *Night*, which is taken for the Sun's continuance below the ſame: But the Natural Day takes in both. The Natural Day is either the Aſtronomical or Civil; and they differ from one another only as to their beginning, according to the cuſtom of the People, and ſettlement of the Aſtronomers. The Aſtronomical Day is the ſpace of time which flows between

between the Sun's leaving a given Celeſtial Meridian, and next return to the ſame; that is, the ſpace wherein a revolution of the whole Celeſtial Equator is performed, and of that part of it beſides which anſwers to a portion of the Ecliptic, which the Sun deſcribes in the mean while by its Annual motion towards the Eaſt. Becauſe this portion of the Equator to be added to the intire Equator is not every where equal, (tho' its mean Quantity is nearly that of one Degree,) as well on the account of the Obliquity of the Ecliptic, as that the apparent Annaual motion of the Sun about the Earth is not equable, the Natural or Aſtronomical Days are not preciſely equal: But of this inequality of the Natural Days, I ſhall treat in its proper place afterwards; taking no notice at preſent of this nicety. Moſt Aſtronomers begin the Day at noon: But *Copernicus* following *Hipparchus* begins it at Night, and this beginning is retained in the *Prutenic* Tables. Differing Nations make different beginnings of the Day. The *Babylonians* begin their Day with the Riſing of the Sun; The *Jews* and *Athenians* with the Setting of the Sun, as the *Italians* do ſtill. The *Egyptians* at Midnight; which is the cuſtom of the *Britains*, *French*, *Germans*, and ſeveral other *European* nations. And anciently the ſame was done by the *Jews*. For we naturally refer the Night, which we ſpend in ſleep and ſilence, partly to the preceding, and partly to the followlowing Day; that is, we begin the Νυχθήμερον, at Midnight. The *Umbri* anciently made the Noon to be the beginning of their Day, and this the *Arabians* obſerve at this time.

Tho' the Artificial Day was hardly divided anciently any other way than into the Morning and the Evening, and the Night into

Watches; yet afterwards the Natural Day was divided more accurately into 24 parts, called *Hours*. Hours are either equal or unequal. An *Equal* Hour is the four and twentieth part of a Natural Day. This sort of Hour was always used by Astronomers, and is now in use among almost all Nations. The *Italians* reckon the Hours from One to 24, from the beginning of the Night; (that being the beginning of the Natural Day,) and their Clocks are made after that manner. But We, the *French*, *&c.* reckon not 24, but twice 12 Hours; perhaps that the number of the Strokes struck by the Clock might not be tedious. So that we divide the Day into the Hours before Noon and the Hours after Noon. An Hour is commonly divided only into four Quarters (which are called Points by some,) a more minute division being in most businesses unnecessary: But the Astronomers (and now all the Politer sort of People) subdivide it into 60 Minutes, and each Minute into 60 Seconds. An *unequal Hour* is the twelfth part of an Artificial Day, as also the twelfth part of the Night. 'Tis call'd Temporary, because of a various length in different Times or Seasons of the Year: Thus the Summer Diurnal hour is longer than the Winter, and the Nocturnal shorter; and the Diurnal hour in Summer is longer than the Nocturnal, and in Winter shorter: At the time of either Equinox, the Diurnal hours are equal to the Nocturnal, and then these unequal hours become the same with the equal ones described above; upon which account these equal hours are called by Authors *Equinoctial* Hours. The *Jews*, *Grecians*, and *Romans* made use of these Temporary hours. Such as begin the Civil Day, at the Morning or Evening, do it because the Sun

is

is then in the Horizon, the most sensible of all the Circles of the Sphere. They have this advantage, that by the number of their equal Hours they know how much of the natural Day, that is, how much time is gone since Sun rise; but they have this disadvantage, that the times of the Sun-set, Noon and Midnight, in different Seasons of the Year, are marked with different numbers of the Hours, and they can't be known but by Calculation: They likewise have this advantage, they immediately know how much time remains till Sun-set, and so know how to accommodate their Labours or Travail to it; and this disadvantage again, that they can't reckon the Hour of Sun rise, Noon, or Midnight, without computing. Such as begin the Civil Day from Noon or Midnight, do it, because the Sun is then at his greatest Elevation or Depression; but the name of the Hour of Sun rise or Sun set, they get only by Computing. Lastly, such as shou'd use the unequal Hours, just now described, could always tell Sun rise or Sun set by the twelfth hour; Noon and Midnight always by their sixth; but could know the Quantity of the Hour only by Calculation.

A *Week* is the ancientest collection of Days as ever was; as is evident from the Sacred Writings. The *Jews* used this collection of old, and since the reception of Christianity, other Nations too: Every Day of it has a name given to it from some of the Planets, after the following manner: The Hours of the natural Day are four and twenty, the Planets but seven, and in this order in the common System, Saturn, Jupiter, Mars, Sol, Venus, Mercury, the Moon; they began at the first Day of the *Jewish* Week, ascribing to the Sun (the Author of the Day) the

the first Hour of it, to Venus the second, to Mercury the third, to the Moon the fourth; then beginning from Saturn, they assign'd the fifth Hour to him, and so on; by this means the first Hour of the following Day happens to the Moon, and she therefore bestows a name upon that Day of the Week from her own; which method is continued in the following Days to the end of the Week.

POPOSITION XI.

TO *explain the division of Time into Months, Years, and the various sorts and collections of them.*

Tho' a Month be properly that space of time wherein the Moon goes thro' the Zodiac; yet since there are about twelve Months that pass while the Sun runs once thro' the Ecliptic, that space of time also wherein the Sun runs thro' one Sign of the Zodiac, is call'd a *Solar Month*; which is about 30½ Days. Again, the *Lunar* Month is either *Periodic*, in which the Moon having departed from any Point of the Zodiac returns to it again, and is something less than 27 Days; or *Synodic*, which is something more than 29½ Days, wherein the Moon runs thro' all its several Phases, and on that account is of principal use in the marking of Time. Both the Solar and Synodic Lunar Month, is either *Astronomical* (of which we have spoken already) or *Civil*, according to the different custom and institution of each Nation. For some, as the *Egyptians*, liked the Solar Months, and made each of them to consist of 30 Days; and to compleat the Year, at the end of twelve such Months they annexed five Days, made up of those twelve times ten Hours, that each Solar Month exceeds 30 Days by.

But moſt of the Ancients fancied the Synodic Lunar Month moſt, ſuch as the *Jews* anciently, the *Grecians* and *Romans*, to *Julius Cæſar*'s time; and the *Mahometans* at this time. But that theſe ſort of Lunar Months, that do not conſiſt of whole days, might be fitted to Civil uſe (where pieces of days can't be taken notice of, and conſidered,) they made their Civil Months alternately of 30 and 29 days, (calling the former *Full*, and this *Cave* Month;) ſo that two ſuch Months are equal to two Lunar Months of 29$\frac{1}{2}$ days a piece; and the New Moon would not ſenſibly remove from the firſt day of the Civil Month in the courſe of ſome Years.

A Year is ſometimes taken for the time of the revolution of a Planet through the Zodiac; in which ſenſe a Month is ſometimes called a Year: and ſometimes for the time of an intire apparent revolution of the Fix'd Stars thro' the Zodiac, which they call the *Annus magnus*, or *Great Year*. But a *Year* is properly that time which the Sun takes to run thro' the Zodiac in. And it is of two ſorts, the Aſtronomic and the Civil: The Aſtronomic is alſo twofold, according to the two different bounds of the Sun's revolution; namely, the Sydereal and Tropical. The *Syderial* Year is the ſpace of time, that the Sun having departed from a Fix'd Star returns to the ſame in; and it is 365 Days, 6 Hours, and 10 Minutes nearly. The *Tropical* Year is that wherein the Sun departing from one of the Cardinal points, the Equinoctial or Solſtitial, returns to it again; and is ſomething leſs than the Sydereal, becauſe the Cardinal points of the Ecliptic themſelves go backwards, and as it were meeting the Sun, make the Sun to return to the ſame point of the Ecliptic ſomething ſooner than to the ſame Fix'd Star, where

that

that point of the Ecliptic was, when the Sun was before in the said Point. This Tropical Year is 365 Days, 5 Hours, 49 $\frac{1}{5}$ Minutes very near; and wants about 21 Minutes of the Sydereal.

The *Civil* Year is the space of time, that the motion of the Sun or Moon, or both, points at, received by the settled Custom of any Nation. Hence 'tis evident, that there are three forms of the Civil Year; namely, either purely *Lunar*, or purely *Solar*, or *Luni-Solar*, which is made up of both. The *Lunar* Year consists of twelve Lunations or Synodic Months that are finished in 354 Days, at the end of which the Year begins again. And since this Year wants near 11 Days of the Tropical Year, that brings the Seasons over again; it comes to pass, that the beginning of this Year falling upon the Spring now, in 8 Years time would fall upon Winter, and after as many more upon Autumn, then upon Summer, and lastly, at the end of 33 Years return to the Spring again. And this is called the *Wandring Lunar Year*; because its beginning wanders thro' all the Seasons of the Year, and that in the memory of one Man. 'Tis called also *Annus Solutus*, because free and disengaged from the motion of the Sun, which is not at all considered in the ordering of it. This sort of Year is in use among the *Turks*, and seems to be found out at first in those Countries, where the differences of Summer and Winter are not very evident and sensible; and where, for want of Astronomy, the return of the Sun to the Cardinal points of the Zodiac, that defines the Solar Year, is not easily known: On which account taking the Signs of the Seasons from the Moon, they took the beginnings of their Years from the beginnings of her Phases

or Appearances; now, there is no number of Lunations, that comes ſo near to the Solar Year (which they had ſome ſmall knowledge of) as the Twelve. I am apt alſo to think, the reaſon of the beginning the Civil Day from the Sun-ſet is to be taken from the Moon: Becauſe it ſeems neceſſary for them to begin the Civil Day with their Year and Month, that is, with the New Moon, which at firſt they knew no otherwiſe than by the help of their Eyes; which was, when they ſaſt the New Moon in the Evening immediately after Sun ſet. And upon this account, the firſt, ſecond, third, *&c.* Day of the Month, was called the firſt, ſecond, third Moon, and the day of the Full Moon was called conſequently, the fourteenth Moon.

Such of the Ancients as were willing to retain the Lunar Year, thought it notwithſtanding proper to adapt it ſo to the Sun, that its beginning might be in a manner fix'd, an- that it might keep the Cardinal points of the Year ſhewn by the Seaſons, from receding much from their Months; and to this purpoſe they us'd Intercalary or Embolimean Months. And firſt, in the ſpace of three Years, they made up an intire Month to be intercalated every third Year, out of the eleven odd Days in every Solar Year. But becauſe this artifice was not exact enough, in every eight Years they intercalated three Months, every one of which conſiſted of 30 days, (namely at the end of the third, fifth and eighth Year, tho' ſome others otherwiſe,) and at laſt eight Months in 19 Years. Hence 'tis evident that ſome of theſe Years are ſimple, namely of 354, or 355 days, the num ber that there is in the Wandring Lunar Year; ſome embolimen or intercalary, of 384 or 384 days, when there are thirteen Lunations

tions in the Year. These Luni-solar or fix'd Lunar Years, are used by the *Jews* and the Clergy of the Roman Communion.

There are three sorts of Solar Years, or such as are fitted to the motion of the Sun alone and the vicissitudes of the Seasons depending thereon; the *Egyptian*, *Julian*, and *Gregorian*. The *Egyptian Year* consists of 365 Days, which they divided into twelve Months of 30 days apiece, and five other days (called ἐπαγόμεναι) added at the end of them, as was said above. In the constitution of this Year there is no regard had to the motion of the Moon: But because this Year wants 5 Hours 49 Minutes, or almost 6 Hours of the Solar Tropical Year; in four Years time it gets near a whole day before the Solar Tropical one, and in 1460 Years its beginning wanders thro' all the Seasons of the Year; on which account it may in a manner be called a *Wandring* Year, altho' in the Age of one Man, or 60 Years, it does not anticipate above 15 Days.

Julius Cæsar finding the *Egyptian* Year to get before the Tropical, because the six Hours, whereby it wants of the Tropical, are intirely neglected in its constitution; did therefore add those 6 Hours to each *Julian* Year; so that a *Julian* Year consists of 365 Days, six Hours. And because this piece or quarter of a Day, can't be considered or taken notice of in Civil use, he added the Day made out of them in four Years to every fourth Year, between the 23d and 24th of *February*, (because the *Romans* while they used the Lunar Year before *Cæsar*'s time, intercalated the Embolimean month;) therefore from that time they writ *bis sexto Kalendas Martii*, or the sixth of the Kalends of *March*

twice; from which custom the Year had the name of the *Bissextile*.

And this *Julian* Year consisting of 365 Days, and adding in four Years one Day, so that after three simple Years there may follow a fourth of 366 Days, is the best fitted for Astronomical Computations; because it is a mean between the Natural or Tropical Year of 365 Days, 5 Hours and 49 Minutes, and the Sydereal Year of 365 Days, 6 Hours and 10 Minutes, and does, as it were, give an ocular Demonstration of the natural regress of the Equinoxes. This form of the Year held common among all Polite Nations from *Augustus* (who restor'd it when it was almost lost) unto the Year 1582, when the *Julian Calendar* was reformed by *Gregory* the XIIIth. But it is still in use among the *Britains*, *Irish*, and others. Yet it must be confess'd, that the quantity of the *Julian* Year is too big, on which account the beginning of this Year also by little and little creeps forwards in regard of the Seasons, or (which is all one) the Times of the Equinoxes and Solstices creep backwards in regard of the Days of this Year. And since this regress is about $10\frac{4}{7}$ Minutes every Year, in about 133 Years it will be a matter of a Day, and consequently from the Year of Christ 325, wherein the *Council of Nice* was held, to the Year 1582, wherein the Pope reform'd the Calendar, namely 1260 Years, this regress was a matter of 10 Days. Hence it came to pass, that whereas in the time of the *Nicene Council* the Vernal Equinox happened near about the 21st day of *March*, in the Year 1582 it was found to have crept up to the 11th of *March*. That he might therefore bring the Equinox to its former place, ten days were suppressed in the Month of *October*, in the Year 1582, and the 5th day

day was call'd the fifteenth; by this means what would otherwise have been the 11th day of the *March* following, and the time of the Equinox, becomes the 21st of *March*; that is, the Equinox happens the same day, *viz.* 21st of *March*, as it did in the time of the *Council of Nice*, which settled the time of keeping *Easter*. Still if no other reformation should be made, but the taking away this ten days, the Cardinal Points would again remove after the former manner, in some Years time. The Pope therefore to avoid this, order'd that once in 133 Years (in which time the excess of a *Julian* Year above the Tropical rises to a whole Day) one day should be taken away from the Calendar; that is, that three days should be taken out of four hundred Years; and this he appointed to be done, by making every hundredth Year of the Christian Æra common, which according to the *Julian* Account is always *Bissextile*; but every four hundredth, which was to continue *Bissextile*, as in the *Julian*. The New form of the Year settled by the Authority of Pope *Gregory* XIIIth, was call'd from him the *Gregorian*, and is observed in *Italy*, *France*, *Spain*, *Germany*, wheresoever the authority of the Pope reaches; and in the close of the last Age, was admitted, as to the Civil Months, by several of the Reformed in *Germany*: As for the finding the *Paschal Moon*, they kept themselves still to *Kepler's Rudolphine Tables*, till something more certain could be fix'd upon and settled.

As the form of the Year among different Nations is various, so likewise is its beginning. The *Jews* begin their Ecclesiastical Year with the New Moon of that Month, whose Full Moon happens next after the Vernal Equinox: The Church of *Rome* begin their Year with

the Sunday that falls upon the ſaid Full Moon, or that happens next after it; or with the *Feaſt of the Reſurrection* of our Lord, which was the cuſtom in *France* before the Year 1564. The *Jews* begin their Civil Year with the New Moon, that has its Full Moon happening next after the Autumnal Equinox. The *Grecians* began their Year with the New Moon that happens next after the Summer Solſtice: And the *Romans* anciently began theirs with the New Moon next after the Winter Solſtice. And this ſeems to be the reaſon why *Julius Cæſar* did not fix the beginning of his Year (tho' a Solar) in the Winter Solſtice it ſelf, but tarried for the following New Moon, where he might place the beginning of his Year or the Calends of *January* according to the cuſtom then receiv'd. Tho' perhaps the ancient opinion was not yet intirely loſt, that the Solſtices were celebrated in the eighth parts of the Signs of Cancer and Capricorn, and not in the very beginning of the Signs: by which no more was meant among the Learned, than that the Sun was really in the eighth parts of the Signs, when he was commonly thought to be in the beginning; which is all one as to ſay that the Equinoxes and Solſtices were anticipated for 6 or 7 Days. And if the Sun had been in the firſt point of Capricorn on the 24th of *December*, he would be exactly enough in the eighth Degree of it on the 1ſt of *January*. By this Method he brought it about that the Celebration of the Olympic Games ſhould fall upon the firſt Day of the Month *Quintillis*, which he call'd by his own Name, *July*: And perhaps theſe Games themſelves were by ancient Rite inſtituted on the account of the Summer Solſtice.

Tho'

Tho' the firſt of January is now look'd upon as the beginning both of the *Julian* and *Gregorian* Year almoſt thro' all *Europe*; yet others aſſign other beginnings to it both formerly and now-a-days. We have ſpoken of the moveable Feaſt of *Eaſter* already. The *Venetians*, *Florentines*, the Inhabitants of *Piſa* in *Italy*, and of *Treves* in *Germany*, make the Vernal Equinox to be the beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feaſt of the Annunciation of the Bleſſed Virgin, that is, of the Incarnation of Chriſt, to be the beginning of the Year: Which the *Church of England* ſtill retains as the beginning of the Year. And the Civil Year begins here alſo, tho' the Vulgar begin it at the firſt of *January*, with the neighbouring Nations.

The more celebrated collections of Years or Times, meaſured by a repetition of Years, are an *Olympiad*, of 4 Years; a *Luſtrum*, of ſometimes 4, ſometimes 5 Years; a *Jubilee*, of 49 or 50 Years; a *Seculum*, or an Age, of 100 Years; an *Annus Magnus*, or Great Year, ſtanding for an intire apparent Revolution of the fix'd Stars, till after having departed from a fix'd Point, they return to the ſame again, which is 25000 or 26000 Years, or thereabouts.

PROPOSITION XII.

TO enumerate the chief and moſt noted Epochas, and to explain their relation to each other.

As the Aſtronomer in computing the motion of the Celeſtial Bodies, aſſume certain fix'd Points to begin at; ſo they do certain moments of Time, from which as Roots, their Computations may begin and proceed. Theſe Roots, which they alſo call *Epochas* and *Æras*, commonly have a name from ſome famous Action or

or Event, that happen'd near the beginning o[f] each. The most considerable, and which we ar[e] best acquainted withal, is the *Epocha of Christ'[s] Nativity*, or the first day of the *January* that w[e] suppose to follow next to the Nativity of ou[r] Lord: And 45 years before is the *Epocha of Julius Cæsar*, or the beginning of the *Julian Year* at which time *Cæsar* rejecting the old *Roman* Year, that had been used from *Numa*, as the[y] believed, ordered the *Julian* Year to be observed thro' the *Roman* Empire. 'Tis true, thi[s] Year was not observed always from the beginning, but after various and confused intercalations extraordinarily; in about the 10th Yea[r] after its beginning, that is, the 5th of the Vulgar Æra, or Birth of our Saviour, it was restored by *Augustus* in the same order as if it had been continued without any interruption from the beginning.

Tho' the Epocha of the Nativity of Christ above described be established by common use, and now almost universally received, yet *England* and *Ireland*, in affairs both of Church and State, use an Epocha a full year after it; and suppose Christ to be born, not on the 25th of *March*, near the ending of the forty fifth *Julian* Year, but of the following Year; namely the forty sixth *Julian* Year; and they suppose Christ to be conceived on the 25th of *March* of the forty sixth *Julian* Year; and number the Year not named from the Nativity, but from the *Incarnation* of Christ, that is, the Conception; and by consequence the same in number, for the greatest part of the Year, with the remaining round Number, that shews the Year from the Nativity. But in the Months almost between the first of *January* and the 25th of *March*, they reckon a different year from the rest, as they

they ought according to their Æra. That this difference between the Æras that take their name from Chriſt, may be better underſtood, we are to take notice, that *Dionyſius* called *Exiguus*, was the Author of this Æra above five hundred Years after Chriſt, from which time they began to reckon from the Nativity or Incarnation of Chriſt, whereas before they uſed to number their years by the Conſuls, and from the Building of *Rome*, in the *Roman* Empire; and out of it by the Years of each King's Reign. This Æra, ſometimes call'd from its Author, the *Dionyſian*, is by no means to be looked upon as exact and agreeable to Hiſtory. For 'tis not yet ſettled, whether it was the firſt, ſecond, third, fourth, or fifth Year, before the Vulgar Æra that Chriſt was born, as may be ſeen in *Kepler*'s Book *of the Year wherein Chriſt was born*. But this ſuppoſed Æra is not too far diſtant from the time, which is ſufficient for an Æra. And according to *Dionyſius*, the Author of this Æra, Chriſt was conceived the 25th of *March* of the firſt Year current of this Æra, and born the following Winter, at the end of the 46th Year of *Julius Cæſar*. This computation of the Year was at firſt univerſally received, but is now left and retain'd only in *England*, together with the beginning of the Eccleſiaſtical and Civil Year, (the Feaſt of the Annunciation,) which obtained in thoſe days: For in the Chriſtian World beſide this Epocha was tacitly departed from; for now the common opinion is, that Chriſt was born the Winter before the *Dionyſian* Annunciation, namely at the end of the 45th of *Julius Cæſar*, and therefore they reckon the Years from the Nativity; and ſo make Chriſt one Year older than *Dionyſius* the Author of the Æra did, and the *Engliſh* with him.

Since moſt Aſtronomical Tables are now fitted to the more common Chriſtian Æra, the other more celebrated ones are to be compared and reduced to it. And the firſt of them is the *Epocha of the Creation*, about which there are ſeveral controverſies. Some ſay the World was created 3950 Years before the common Æra of Chriſt, and theſe ſays *Gaſſendus* come neareſt the truth; which is likewiſe approved of by the *French* Aſtronomers. Others 3983 years; and with theſe *Petavius* agrees. *Kepler* 3993 years, in the middle of Summer, from Aſtrology. *Hevelius* in the 3963d Year currrent before Chriſt, the 24th of *October* of the *Julian Year* continued backwards, at 6 a Clock in the Evening, in the Meridian of *Eden*; becauſe he found in his Tables of the Sun, that at that time the Sun was in the beginning of ♎, and its Apogæum in the beginning of ♈. Others make it much more ancient, and place it 5199 Years before Chriſt.

The Epocha of the *Olympiads*, of all Profane ones, is the moſt Ancient and Celebrated: It is placed in the Summer of 777 years before Chriſt, the firſt of *July*, the *Julian* Year being continued backwards. Not much after is the *Epocha of the Building of Rome*, which is generally fix'd in 752 years before Chriſt.

The *Æra of Nabonaſſar*, always famous among the Aſtronomers, began at the 26th day of *February* of a *Julian* Year produced backward, and in the Year 747 before Chriſt. And becauſe this was the firſt of the *Ægyptian* Years, *Ptolemy*, and after him the other Aſtronomers, and *Copernicus* himſelf, compute from thence the motion of the Stars by *Ægyptian* Years. For the *Ægyptian* Years conſiſting of 365 Days, tho' unfit for common Uſe, becauſe its Cardinal Points wander thro' all the Seaſons, yet is very fit

fit for Aſtronomical Computations, becauſe it is diſturbed with no intercalation.

The *Epocha of the Death of* Alexander *the Great* follows next in order, on the 12th day of *November*, in the year 324 before Chriſt. And this alſo being the firſt of the *Ægyptian* Wandering Years (for ſo the Founders of Æras will have it, tho' the Event happen'd before or after the beginning of the Year) *Theo*, *Albategnius*, and others that uſed this Epocha, computed the *Ægyptian* Years from hence. Between the Æras of *Nabonaſſar* and *the Death of* Alexander *the Great*, there are 424 years pecifely.

I paſs over the *Æra of the Abyſſinians*, call'd alſo the Æra of *Diocleſian* and *the Martyrs*: As alſo the *Æra of the Arabians*, that is, the *Hegira*, or *Flight of Mahomet*: And the *Epocha* of the *Perſians*, or as it is call'd *Jeſdegird*; and others, (being ſuch as are of ſmall uſe to us) which may be ſeen at large in Writers upon that ſubject; and in fine, the Epocha of the *Julian* Period of 7980 years, invented by *Scaliger*, beginning 4714 years before the Chriſtian Æra, and conſequently before the World was created.

SECTION III.

Of Spheres and other Inſtruments invented to repreſent the firſt Motion of the Heavens, and of their Uſe.

PROPOSITION XIII.

TO deſcribe the common Inſtruments, by which the Firſt or Diurnal motion is imitated, and demonſtrated to the Sight, namely the Celeſtial Globe and Sphere.

For

For the better conception of the aforesai[d] Circles in the Heavens, Artists present us wit[h] the Image of the Starry Heaven upon the Ce[-]lestial Globe, which is a Sphere made of Me[-]tal, Paper, or any other like substance, easy t[o] be moved upon two Poles. Upon the Conve[x] surface you have marked the Places of the chie[f] Fix'd Stars, so disposed in regard of one ano[-]ther and the Poles of the Sphere, as the Fixe[d] Stars themselves, which they represent, are dis[-]posed in respect to one another and to the Poles about which the Heavens seem to revolve i[n] the diurnal motion. The Images also of the Asterisms are drawn upon it, that these Star[s] seem'd to represent to the Ancients, drawn to i[t] either by the force of Fancy or Religion.

There are drawn also upon this Globe, the Equinoctial Circle exactly between the Poles, and the two Tropics parallel to it, on each side of it at $23\frac{1}{2}$ Degrees distance, and as many Polar Circles at as great a distance from both Poles. The Ecliptic also is drawn reaching to both the Tropics, and intersecting the Equi-noctial upon the Surface of the Globe, in the points so related to the Places of the Stars, as the points of the Celestial Equinoctial (where the Sun appears in the time of the Equinoxes) are related to the Stars themselves, in the Age wherein the Globes are made: As well the E-quinoctial as the Ecliptic is divided into De-grees, beginning at the point that represents the Vernal Equinox; the Equinoctial into 360, without any break, but the Ecliptic into 12 Signs, distinguished by their proper Marks, every one of which is subdivided into 30 Degrees: And the Secondaries of the Ecliptic drawn thro' the beginning of each Sign, and meeting in the Poles, divide the whole Globe it self into as

many

many Signs. Only two Secondaries of the Equator, namely, the Colures, are drawn upon the Celeſtial Globe, one of which, namely, the Solſtitial Colure, is the Secondary alſo of the Ecliptic. Such a Globe as this repreſenting the Starry Firmament, is made movable upon the aforeſaid Poles, ſo inſerted into the Meridian, a Brazen immovable Circle placed without the Globe, as to fit that ſide of this Meridian, which is divided into Degrees; which ſide therefore is to be taken for the true Meridian, the thickneſs of the Circle only ſerving for ſtrength.

Beſides, the Globe incompaſſed with the Meridian (as was ſaid) is let into a Wooden contrivance repreſenting the Horizon, ſo as that half of it is above the upper Surface of this Wooden contrivance, which is therefore to be looked upon as the Horizon. And that the Globe may come near enough in every part to the Horizon, there are notches in the Horizon for the inſertion of the Meridian, that is, prominent without the Globe. Upon the breadth of this artificial Horizon, which is large enough, beſides the inmoſt Circle divided into its proper Degrees, reckon'd from convenient beginnings, there are the *Julian* and *Gregorian* Calendars, with the Degrees of the Signs of the Ecliptic, that the Sun is in nearly upon thoſe Days in that Age: There is alſo a Circle of the Winds or Points of the Compaſs.

The Horizon is ſupported by four or more little Pillars faſten'd to a Foot or Baſe, upon which there is placed a Mariner's Compaſs and Needle, to ſet the Globe pretty nearly agreeable to the Quarters of the World. You have alſo a Quardant of Altitude that may be faſten'd to the vertical Point of a Meridian by the help of

of a Screw, its Limb divided into Degrees, is to be taken for a true Quadrant: and 90 Degrees are reckon'd from its extremity that touches the Horizon upwards towards the Zenith. In the Globes made now a-days, there is alſo a Circle called the *Hour* Circle, faſten'd to the Meridian, having the Pole for its Center. This little Circle is divided into twice twelve hours, that it might be fitted to our way of reckoning the hours, beginning at the Meridian. And ſince the Pole carries the Index along with it, the Index will point out the hours according to the revolution of the Globe; if it be at twelve a Clock of the day, at the ſame time as the place where the Sun is at that time is under the Meridian.

What is commonly call'd a *Sphere* differs from a Globe in this, that its ſurface is not united and perfect, as 'tis in a Globe, but the more principal Circles made of Armillæ or Hoops (from whence it has the name of an *Armillary Sphere*) ſhadow out a Spherical ſurface. In ſuch a Sphere, beſides the Horizon and Meridian, (between which being immovable, it is revolved like a Globe,) there are four great Circles; namely the Equator and Ecliptic, or rather Zodiac, (for like a Belt or Swath, it is 20 deg. broad;) and the two Colures and four leſſer Circles, *viz.* the two Tropics, and as many Polar Circles: And thus the number of the Circles of a Sphere came to be reckoned Ten. This Sphere is ſo excavated, that by its tranſparence as it were, it is a repreſentation of the World, and preſents us with the little Globe of the Earth in its Center at reſt, and ſuſpended by the Axe, which could not be done ſo well in a Globe, whoſe Superfices is intire and united. From what has been ſaid, 'tis evident, that in

an

an Armillary Sphere, the above deſcribed pla-
ces of the Fixed Stars can't be marked at all,
excepting ſome few, that appear in the Zodiac,
tho' they likewiſe are commonly taken no no-
tice of; and therefore the Problems relating
to the Fix'd Stars can't be ſolv'd by ſuch a
Sphere. Tho' ſince the ſaid Circles of the
Sphere are delineated upon the ſurface of the
Globe, all the Problems that are done by the
Armillary Sphere, may be performed very ea-
ſily by the Globe: And therefore Spheres, tho'
of great name formerly, when the ſtudy of A-
ſtronomy began to revive, are very ſeldom uſed
now, but to help the imagination of the young.

To theſe Armillary Spheres there are ſome-
times annex'd the Planets made to be immova-
ble in different Circles, in the order as they are
reckoned by the Aſtronomers of the *Ptolemaic*,
Tychonic, or *Copernican* Sect. And therefore in the
Ptolemaic and *Tychonic* Sphere, the Planets are
carried in different Circles about the Earth at
reſt, in the Plane of the Zodiac, within the
hoops that repreſent the Starry Heaven. But
ſince there is required a Spherical ſurface (ſha-
dowed out by the hoops) concentric with the
Earth, to repreſent the Firſt or Diurnal motion,
and this is movable in the *Copernican Sphere*;
therefore the Makers do place in it the Planets
without the hoops compaſſing the Earth, which
is movable about the imaginary Sun. Sometimes
Spheres of this kind and Globes are made to go
by a Weight or Spring, ſo as to imitate the riſing,
ſetting, and all the Diurnal motion of the Fix'd
Stars. Sometimes alſo the Planets perform their
revolutions in their proper time: But things of
this nature don't belong to this place.

PROPOSITION XIV.

TO *describe the Terrestrial Globe, as it is com-monly made.*

The Terrestrial Globe does not differ from th above described Celestial Globe but in this, th instead of the Stars and Constellations compo sed of them, Lands and Seas are delineated up on it. And the Tracts, Bays, Promontories, an other things belonging to the Earth, are painte upon the convex spherical Surface, in regard o one another and the Poles upon which th Globe is turned in the Meridian, just as th Places themselves are upon the Earth, whic they represent, in regard of one another and th Poles upon which the Earth really revolves b the diurnal motion. These places of the Eart thus imitated will really have that situation, each of them be transferr'd upon this little Eart agreeable to their Longitude and Latitude b Observation. The Equator, and its Secondari or Meridians passing thro' every ten Degree are also painted upon the Terrestrial Globe; on of which, namely that which passes thro' th *Azores*, or *Fortunate Islands*, is the most conside rable, and called the first Meridian; it bein that Meridian from which the Longitude of place is reckoned, as was said above; and by consequence from whence the numbering o the degrees of the Equator towards the East i continued. The Parallels also to the Equator namely the Polar Circles and Tropics, and seve ral others at ten deg. distance from one another together with the Rhumbs (in the spaces repre senting the large and vast Seas) shewing the Courses of Ships guided by the Mariner's Com pass and Needle, are all drawn upon it. And that the Terrestrial Globe, besides its own uses,

might

might supply the place of a Celestial Sphere (when such one is wanting,) the Makers have fitted an Ecliptic to it. But because this Circle is purely a Celestial one, and has no proper place upon the Earth, therefore they have placed its intersection with the Terrestrial Equator, (which intersection they might have placed any where) at the point where the first Meridian intersects the Equator.

PROPOSITION XV.

TO resolve several Problems relating to the Situation of places of the Earth to one another, and to the Diurnal motion of the Stars, by the Globes; that is, to shew the chief Use of both Globes.

Tho' the solution of the following Problems by the Globes is not accurate and exact, yet it serves very much to form and instruct the minds of Juniors; and we shall dispatch them in one Proposition.

And here the Celestial Globe is to be looked upon as the great Machine of the World itself; the Sun and all the Stars, and whatever we see in the concave Firmament, are to be conceived as upon the convex surface, view'd by an Eye plac'd without that surface: But there is no need of such a fiction in the Terrestrial Globe. And because the Globes are the Instruments we make use of in this Proposition, they are to be look'd upon as perfect; as for the manner of making them so, we are not now to enquire, but to apply them as they are already made to the solution of the following Problems and others like them.

1. *To find the Longitude and Latitude of any given Place upon the Terrestrial Globe.*

Bring the given Place to the Meridian, (always understand that side of it as is divided into

Degrees, which, as we ſaid before, is the tru Meridian) and the degree of the Equator, then found under the Meridian, ſhews the Longitude of the Place. For the beginning of reckoning thoſe degrees is taken from the firſt Meridian as was ſaid before. Beſides, reckon from the Equator in the Meridian to the given Place, the degrees of Latitude: For the Meridian is divided into four times 90 degrees, two of which 90 begin to be numbered from the Equator towards both Poles; the reckoning of the two other 90 degrees begins from both Poles, and ends in the Equinoctial under the Horizon The Latitude is North or South, according as the Place given is to the North or South of the Equator.

2. *The Longitude and Latitude being given, to find the Place itſelf upon the Terreſtrial Globe.*

Look in the Equator for the degree of the given Longitude; 'tis evident that the Place ſought is in the Meridian drawn thro' this point taken in the Equator. Apply this point therefore to the divided Meridian, and reckon the degrees of the Latitude given toward the North Pole, if it be northern, or towards the South Pole, if it be ſouthern, and the point at the end of the reckoning will be the Place ſought.

3. *To rectify both the Globes to a given Latitude together with the Quadrant of Altitude; and to diſpoſe the Globes agreeable to the four Quarters of the World*

If the Latitude of the Place given be North let the North Pole be raiſed above the Horizon if South, the South Pole, till the Arc of the Meridian intercepted between the Pole and the Horizon, be of as many degrees as the Latitude of the place given is. For by *Prop.* 7. the

Eleva

Elevation of the Pole is an Arc ſimilar to the Latitude of the place; that is, containing as many degrees. From whence 'tis evident to the ſight what Stars always appear; namely, ſuch as upon the turning of the Globe quite round, don't deſcend below the Horizon: As alſo what on the contrary, never appear. Then compute from the Equator in the Meridian upwards, the degrees of the Latitude given, and the point where the reckoning ends, ſhall be the Vertex of the given Place: (For the height of the Pole is equal to the diſtance of the Equator from the Vertex; ſince the ſame Arc, namely the diſtance of the Pole from the Vertex is the complement of both to 90 deg.:) Fix the Quadrant of Altitude by means of the Screw, to the Vertex thus found, ſo as that its divided edge, (which ſerves inſtead of a Quadrant,) may proceed as it were from thence. Laſtly, by the help of the Mariners Compaſs fix'd to the foot of the Globe, let the Right line that is the common Section of the planes of the Horizon and Meridian be placed in the Meridian line, ſo as that the elevated Pole of the Globe may look towards the elevated Pole of the World. For we ſuppoſe the Magnetic Needle to be directed along the Meridian line; at leaſt, that its declination (that hinders it) is known. By this means, the Meridian of the Globe will coincide with the Meridian of the Place; and its other points with the correſpondent points of the Place. But ſuch an Horizontal plane muſt be pitched upon for the Globe to ſtand upon, as will make the Horizon of the Globe to be parallel to the Horizon of the Place, as it ought to be.

4. *To find the Degree in the Ecliptic that the Su is in at any given time, by the help of the Calendar and Circle of Signs drawn upon the ſurface of the Horizon, and by that to aſſign it Place in the Ecliptic itſelf.*

Look into the Calendar for the Month and Day propoſed, and oppoſite to it you will find the Sign and degree thereof,that the Sun is therin. Then in the Ecliptic drawn upon the Globe look for that Sign of the degree of it, and that will be the Place of the Sun ſought. If you would be more exact, you muſt get the Place of the Sun, the help of an Ephemerides,and find that out, in the Ecliptic drawn upon the Globe After the ſame manner may the Places of the Planets found by ſuch Ephemerides, be marked upon the ſurface of the Globe ; by ſeeking firſt the Longitude of the Planet in the Ecliptic, and by protraction its Latitude from the Ecliptic either way, by the help of a pair of Compaſſes. To find the degree in the Ecliptic the Sun is in pretty nearly at a given time, you need only to know what Day of each Month the Sun enters a Sign of the Ecliptic, and compute one degree for every Day from thence. For this purpoſe there are ſome memorial Verſes compoſed, which may be ſeen in the Writers of the Doctrine of the Sphere.

5. *To find the right Aſcenſion and Declination of the Sun, or Star, or any given Point ; and conſequently to find the Time of the Culmination of a given Star or Point.*

The right Aſcenſion of the Sun or any point upon the Celeſtial Globe, is found after the ſame manner as the Longitude and Latitude of a place on the Terreſtrial Globe ; (by *Probl.* 1.) For the right Aſcenſion and Declination in the Celeſtial, is all one with the Longitude and Lati-

Latitude in the Terrestrial Globe. Besides, having brought the Place of the Sun to the Meridian, bring the Index to twelve a Clock, (we reckon it twelve a Clock when the Sun is in the Meridian,) and turn the Globe round till the Star or given Point comes to culminate, then the Index shews the hour, when that will happen. But because the Hour Circle is too small, the part of an hour can't be easily distinguish'd in it: To be more exact, you must do thus. Because the intire Equator (together with a certain portion of it, corresponding to the proper motion of the Sun during that time) passes the Meridian in the space of a natural day; 'tis evident that a 24th part of the Equator, or 15 deg. pass the Meridian in one hour, and one deg. in four Seconds of time, and so on. If therefore from the right Ascension of a Star, you take away the right Ascension of the Sun, you will have that portion of the Equator which passes the Meridian, in the time between one culmination of the Sun and the next following culmination of the Star: If instead of that, you take the time necessary for the passage of the said Arc of the Equator by the Meridian, (allowing an hour for fifteen degrees, and four minutes for one degree, as was said before; which Operation is called the *Converting an Arc of the Equator into Time*;) you will have the time of the Culmination of that Star from Noon. But if the right Ascension of the Star be lesser than the right Ascension of the Sun, so that the latter can't be Subtracted from the former, you must add to it a whole Circle or 360 degrees.

6. The Place of the Sun or any Star being given, to find its Oblique Ascension or Descension, as also its Eastern and Western Amplitude.

Bring the given Place of the Sun or Star to

the Eastern part of the Horizon, (having re ctified the Globe for the Latitude of the Place by *Prob.* 3.) and mark the degree of the Equato that ascends with it: For the Arc of the Equa tor, intercepted between the beginning of Arie and the Point thus marked (expressed by the number annexed to the point) is the Oblique Ascension of the Sun or Star it self. And the Arc intercepted in the Horizon between the East (that is, the intersection of the Equator and Horizon) and the Place of the Sun or Star is the Eastern Amplitude of that heavenly Body. If you bring the same Place of the Sun or Star to the Western part of the Horizon, the degree of the Equator then descending, will be the Oblique Descension; and the Arc of the Horizon, intercepted between the Western point of the Horizon, and the setting Celestial Body will be its Western or setting Amplitude.

7. *The right and oblique Ascension of the Sun, or a Star being given, to find half the time of its continuance above or below the Horizon; and if it be the Sun, whose Ascensions are given, to determine the Length of the Day and Night, and the hour of the Rising and Setting of the Sun; but if it be a Star, to find the hour of its Rising or Setting, by the time of its Culmination being given: And on the contrary, the Place of the Sun and Length of the Day being given, to determine the Places on the Earth to whom they agree.*

If you fix the Index to twelve a Clock when the Sun's place is under the Meridian, and turn the Globe backwards till the Sun's place is at the Eastern part of the Horizon, the Index will tell you the Hour of Sun-rise; and if the Place of the Sun be brought to the Western part of the Horizon, the Index will point out the Hour of its setting; the same Index will likewise shew the

the Culminating, Riſing or Setting of the given Star, when the Star marked on the Globe comes to the Meridian, Eaſtern or Weſtern part of the Horizon: For in all theſe inſtances Nature itſelf is imitated. And the Hour of the riſing or ſetting of the Sun or Star being known, the continuance of them above or below the Horizon is ſelf-evident.

To find the ſame more exactly by computation without the Globe, the Aſcenſions being given, do thus: Let the Aſcenſional Difference be converted into time, and the addition of it to ſix hours, if the Celeſtial Body decline from the Equator towards the Pole that is elevated, or the ſubtraction of it from ſix hours, if it decline towards the Pole that is depreſſed will give half the continuance of that Celeſtial Body above the Horizon: And this added to the time of its Culminating will give the Hour of its Setting, ſubtracted from it will give the Hour of its Riſing. And conſequently its whole continuance above the Horizon, (which is the Length of the Day, if the Celeſtial Body be the Sun) will be given, which ſubtracted from 24 hours, gives its continuance below the Horizon.

If the Place be required where the Day is of a given Length, when the Sun is in a given point in the Ecliptic; bring the Place of the Sun to the Meridian, and the Index to twelve a Clock; turn the Globe till the Index ſhow the Hour of the Riſing or Setting of the Sun agreeable to the given Length of the Day; then elevating or depreſſing the Pole, let the Globe be brought to ſuch a ſituation as the Sun's place may be in the Horizon when the Index is at the ſaid hour; for in this caſe, the Globe is in that ſituation, wherein the Sun being in a given point of the Ecliptic, makes the Day of the given

given Length. Therefore all the Places of the Earth, whose Latitude is equal to the height of the Pole of the Globe in this position, will answer the question. And if the given Place of the Sun be in the Tropic towards the visible Pole; (that is, if the Place be required whose longest day is a given length,) this Problem will serve to distinguish the Parallels and Climates.

8. *The Latitude and Sun's place being given, and besides any one of these three, namely the Hour of the day, the Altitude of the Celestial Body, and the Azimuth of the same be given, to find the other two.*

Rectify the Globe for the Latitude of the Place given, by *Prob.* 3; bring the Sun's place to the Meridian, and the Index to twelve a Clock; then if the Hour be given, turn the Globe about, till the Index point at that Hour, or move it accurately, till as many fifteen degrees of the Equator pass the Meridian forwards or backwards, as there are Hours since or before Noon; and if the given Hours have minutes, for each four minutes, make one degree pass by the Meridian. Fixing the Globe in this position, let the Altitude and Azimuth of the Sun or Star, be measured by the Quadrant of Altitude first fixed to the Zenith: Namely the Altitude in the Quadrant, reckoning upwards from the Horizon; but the Azimuth in the Horizon from South or North point to the intersection of the Quadrant with the Horizon. If the Altitude of the Sun or Star be given, turn the Globe about till the Star arrive at the given Altitude, (which may easily be try'd by the Quadrant of Altitude,) and the foot of the Quadrant will shew the Azimuth, and the Index the Hour. When the Azimuth is given, bring

bring the foot of the Quadrant to the Azimuth given, and turn the Globe till the given Star come to the edge of the Quadrant, and immediately the Altitude is known, being reckoned upon the Quadrant, and the Hour by the Index: Unleſs it be better to find it by the number of degrees of the Equator, that have paſs'd the Meridian, ſince the Sun was upon it. By Hours we here underſtand Equinoctial ones reckon'd from Noon to Midnight, tho they may be eaſily reduced to *Italic* or *Babylonic* Hours, by finding (by preced. *Prob.*) the Hour of the riſing or the ſetting of the Sun, from whence they (being likewiſe Equinoctials) are reckoned: 'Tis an eaſy matter alſo to reduce any given Equinoctial hour, reckoned from any beginning, to an unequal or *Planetary* hour, by the length of the day. And if the places of the Planets (found for that time by the help of an Ephemerides) be mark'd upon the ſurface of the Globe, the Conſtellations and Planets may be eaſily found and diſtinguiſhed in the Heavens by the Globe, if rectified to the hour of the Night, and in a clear ſerene night be compared with the face of the Heavens, and turn'd about gradually as the night paſſes, to keep the ſame ſituation with the Heavens.

9. *To find the time when a Fix'd Star riſes or ſets Coſmically or Achronically.*

Bring the given Star to the Eaſtern part of the Horizon, and mark the point of the Ecliptic that riſes with it; the time when the Sun is in that point (which you may eaſily find upon the Horizon of the Globe by the Circle of Signs and the Calendar annex'd) is the time of the Coſmical riſing of that Star: But if the time of the Coſmical ſetting were required, mark in the Globe the point of the Ecliptic riſing when the given

Star

Star comes to the Weſt part of the Horizon, and the time wherein the Sun enters that point, is the Time ſought. To find the Achronical Riſing, mark that point of the Ecliptic which ſets when the Star riſes, and the Time wherein the Sun paſſes that point, is the Time ſought. And if that point of the Ecliptic be taken notice of, that comes to the Weſtern part of the Horizon together with the Star, that day the Sun is in that Degree, the Star ſets Achronically. And this is ſufficient, if by the Coſmical and Achronical Riſing and Setting be underſtood Morning and Evening riſing and ſetting, as it commonly does. But if the Coſmical riſing and ſetting (as *Kepler* would have it) ſignify the ſame as its aſcent above the Horizon of a given place, and deſcent below it, then this happens daily (if at all) to any Fix'd Star, the time of which is found by *Prob.* 7. And the Achronical Riſing of a Star (which according to the ſame Author, ſignifies a Star's riſing when the Sun ſets) will be the ſame, and found after the ſame way as the former: But the Achronical ſetting comes to the ſame thing with the Coſmical Riſing in the common ſenſe.

10. *To find the time, when a given Fix'd Star riſes or ſets Heliacally.*

Since all the Fix'd Stars riſe Heliacally in a Morning, and ſet in an Evening, to find the time when a given Fix'd Star (for inſtance, of the firſt Magnitude) riſes Heliacally, that point of the Ecliptic is to be found, which is 12° below the Eaſtern part of the Horizon when the Star riſes: For when the Sun is in that point of the Ecliptic, he is depreſſed below the Horizon, juſt as much as is required, that ſuch a Star may be viſible when it is juſt come above the Horizon. To find this by the Globe, bring the

the Star to the Eaſtern part of the Horizon, and find with the Quadrant of Altitude that point of the Ecliptic which is elevated 12 degrees above the Weſtern part of the Horizon: The point oppoſite to this is the point ſought, *viz.* one depreſs'd 12 deg. below the Eaſtern part of the Horizon: That day therefore wherein the Sun poſſeſſes that point of the Ecliptic (which may be eaſily found in the Calendar annex'd to the Circle of Signs in the Horizon of the Globe) the ſaid Star of the firſt Magnitude riſes Heliacally. If the Star be of the ſecond magnitude, a point of the Ecliptic muſt be found, that is depreſſed thirteen degrees below the Horizon, when the Star is juſt at it: If of the third, 14 degrees, and ſo on; as is evident from *Prop.* 9. If the time of the Heliacal ſetting of a given Star of the firſt Magnitude be required, bring the Star to the Weſtern part of the Horizon, find as before, that point of the Ecliptic which at that time is elevated 12 degrees above the Eaſtern part of the Horizon, and the point oppoſite to it (namely that which is depreſſed as many degrees below the Weſtern part of the Horizon) is the Sun's place in the Ecliptic, when the ſaid Star ſets Heliacally. After the like manner you may find the Place of the Sun in the Ecliptic for any given time paſt, when a given Fix'd Star did at that time riſe or ſet Heliacally; and conſequently the times anciently deſcribed and marked out by the Writers of Husbandry and Medicine, may be compared with ours: For inſtead of a given Star, as it is painted upon the Globes, you are to make uſe of that place of it, in which it was in that Age; leſſening the Longitude of the Star one degree and a third part for every Age ſince the given, but keeping the ſame Latitude.

11. *To*

11. *To determine the beginning of the Morning and end of the Evening Twilight for any time given.*

Rectify the Globe for the Latitude of the Place, (which is ſuppoſed in this and all the preceding Propoſitions to be given,) bring the Place of the Sun at the time given to the Meridian, and the Index to twelve a Clock; then turn the Globe about towards the Eaſt, till the point of the Ecliptic oppoſite to the Sun's place, be elevated 18 degrees above the Weſtern part of the Horizon, in which caſe the Sun itſelf is depreſſed as many degrees below the Eaſtern part of the Horizon; and therefore (by *Prop.* 8.) the Morning Twilight is beginning: and the time of it ſhewn by the Index. But 'tis better to turn the Arc of the Equator, that paſſes the Meridian, (while the Globe is moved about into this ſituation from that which it had when the Sun's place was at the Meridian,) into time, to be ſubtracted from 12 hours, for finding the Time of the beginning of the Morning Twilight. After the ſame manner the end of the Evening Twilight is found, by turning the Globe about till the point oppoſite to the Sun's place be elevated 18 degrees above the Eaſtern part of the Horizon, *&c.*

12. *To find the Diſtance of any two Places on the Earth, that are given, as alſo the Angle of Poſition; or that which a great Circle paſſing thro' the given Places, makes with the Meridian of either of them.*

The neareſt diſtance of any two Places, which is always to be computed in a great Circle, is found by taking their diſtance as they are drawn upon the Globe; (by a pair of Compaſſes form'd for that purpoſe,) and applying that interval to the Equinoctial or firſt Meridian, to know the num-

number of degrees between the two Places; and reckoning for every Degree, 60 *English* or 15 *German* Miles. For the magnitude of the Earth is ſuch, that a degree of a great Circle deſcribed upon its ſurface, is nearly 60 *Engliſh* or 15 *German* Miles.

To find the Angle of Poſition, the Globe muſt be turn'd about till the firſt of the two Places becomes Vertical,(which it will be, when it arrives at the Meridian of theGlobe,being re-ctified for the Latitude of that Place;) then move the Quadrant of Altitude faſten'd to the Vertex, and conſequently to the firſt Place, till it touch the ſecond Place; and the Arc of the Horizon intercepted between the Meridian and Quadrant of Altitude, will be the meaſure of the Angle of Poſition between the Meridian of the firſt Place and the Great circle (namely the ſaid Quadrant) paſſing through the two Places, becauſe the Zenith is the Pole of the Horizon. The Arc of the Quadrant intercepted between the two given Places converted into miles, (without the help of the Compaſſes uſed above) will give the diſtance of the Places. But if, repeating the former Praxis,the Angle be found, that the ſame Circle paſſing thro' the two Places makes with any number of intermediate Meridians; (paſſing thro' for inſtance, the ſeveral degrees of the Equator;) the *Rhumbs* will be determined, along which you are to ſail to the intermediate Places ſituated in the ſaid Meridians, if you would ſail nearly in anArc of a great Circle (that is, the ſhorteſt way) thro' given Places.

13. *Given the Hour of the day according to the reckoning of one given Place, to find the Hour at that time in another given Place; and to find the Place where a given Hour is reckon'd.*

Bring

Bring the Place to the Meridian, and the Index to the given Hour according to the reckoning of that Place, then turn the Globe about till the other given Place comes to the Meridian, and the Index will point at the Hour ſought. For the ſecond caſe, bring the Meridian and the Index to the given Hour, as before; then turn the Globe till the Index point at the other given Hour; and all the Places then under the Meridian will reckon or have the given Hour of the day.

If you are for calculating it, you will find it more accurately. If the two given Places are under the ſame Meridian, or have the ſame Longitude, they will alſo reckon the ſame Hour from Noon or Midnight; But if their Longitudes differ, the difference of their Longitudes found by the Globe and turn'd into Time, gives the diſtance of the Hours in thoſe Places reckoned at the ſame moment, ſo as that the Place which is towards the Eaſt reckons the greater of thoſe two Hours. In the latter caſe, where the difference of time is given, let it be converted into an Arc of the Equator: And this Arc will be the difference of the Longitude, which being added to the Longitude of the given Place, if it reckons the former Hour, or ſubducted from it, if the latter, will give the Longitude ſought; and all the Places of the Earth under the Meridian having that Longitude, will anſwer the Queſtion. What is here ſaid of our Hours reckoned from Noon or Midnight, may eaſily be applied to *Italic* or *Babylonic*, or even *Planetary* Hours, by reduceing them; as was ſhewn in *Prob.* 8.

14. *To find the Place where the Sun is vertical at a given time; and all the Places where the Sun at that time appears in their Horizon riſing or ſetting.*

Having rectified the Terreſtrial Globe for the Latitude of your Place and the Cardinal points of the World, turn the Globe about till that Place become vertical; and let the Globe in his poſition be expoſed to the Beams of the Sun, (by removing the Horizon, that hinders he Rays of the Sun,) and at one view it will ap-pear in what places of the Earth it is Day, and where it is Night; where the Sun is riſing and where it is ſetting. For ſince the Globe and Earth it ſelf are ſimilar in Figure and ſimilarly oſited, they will be ſimilarly enlightened by he Sun, whoſe immenſe diſtance may be look-d upon as infinite. That Place, which is the Pole of the Circle, marked out by the confines f Light and Shade, is the Place where the Sun vertical, and may be determin'd mechanically, y ſeeking the place, where the Shadow of a tyle erected perpendicularly upon the Surface f the Globe vaniſhes, or falls upon the Foot f the Style it ſelf. If the ſame things be re-quired when the Sun does not ſhine, find (by receed. Prob.) the Meridian of the Places here it is then Noon, and in this Meridian ke that place of the Earth, whoſe Latitude to-ards the ſame Pole is the ſame with the Sun's eclination at that time; and that is the place here the Sun is vertical, and if it be made rtical, the Horizon will ſhow the confines of light and Shade, or all the places where the in appears in the Horizon; and then 'tis eaſy distinguiſh the Places where he is riſing, from e places where he is ſetting.

15. *To make an Horizontal Dial for a given Place by the help of a Globe.*

Rectify the Globe (as before) for the Latitude of the Place and the Cardinal points of the World: If besides the Meridian, there were eleven other Hour Circles (that is, 24 Hour Semicircles) immovable, standing without the Globe, and the whole Globe, besides the Arc about which it revolves, were pellucid; 'tis evident that the Sun placed in the Meridian would cast the shadow of the Axe upon the other half of the Meridian, and placed in the One a Clock Hour Circle after Noon, would cast the shadow of the opake Axis upon the One a Clock at Night Hour Circle, or the other half of the same Meridian beyond the Poles, and so on in the other Hour Circles. If besides all this, you imagine an Horizontal Plane shewing the shadow of the Axe cast upon it by Reflexion, when the Sun is in the Meridian, the shadow of the Axis projected upon the horizontal Plane, will be in that part of the common section of the Meridian and Horizon, which is beyond the Center; that is, in a Right line connecting the Center with the intersection of the Meridian and Horizon: When the Sun is come to the One a Clock Circle, the shadow of the opake Axis projected upon the horizontal Plane, will be that part of the common Section of the One a Clock Hour Circle with the Horizontal Plane, that lies beyond the Center, or which connects the Center with the common intersection of the One a Clock at Night Hour Circle; and which does therefore contain with the Meridian or Twelve a Clock line, an Angle, that is measured by the Arc of the Horizon, which is intercepted between the Meridian and One a Clock Circle

after

fter Noon or Midnight. And after the ſame manner the Sun, when it is at the Two a Clock Circle after Noon, caſt the ſhadow of the Axe upon the Horizontal Plane in a Right line connecting the Center and interſection of the Two a Clock at Night Hour Circle and Horizon; which therefore makes with the Meridian line an Angle, that is meaſured by an Arc of the Horizon intercepted between the Meridian and the two a clock Circle: and after the ſame manner in the other Hour lines upon the Horizontal plane, every one of them makes an Angle with the Meridian, which is meaſured by an Arc of the Horizon intercepted between the Meridian and Hour Circle it belongs to. That a Sun-Dial therefore may be drawn upon an Horizontal Plane, upon any Point as a Center, let a Circle be deſcribed repreſenting the Horizon of the Globe, (for the Center of it repreſents the Center of the Earth, or in the preſent caſe, the Center of the World:) Thro' the Center draw a Meridian line; that is, one that being produced would reach to the Cardinal Points of North and South, and upon the Center erect an opake Right line repreſenting the Axe of the World, tending to the Pole; that is, ſo as in the plane of the Meridian to make an Angle with the Meridian line equal to the Angle of the Elevation of the Pole above the Horizon of the place. Then let the one a Clock line after Noon be drawn from the Center of the Circle on the Eaſtern ſide of the Meridian line making an Angle with it, meaſured by the Arc of the Horizon intercepted between the Meridian and One a Clock Hour Circle. But becauſe the Hour Circles are not to be found in Globes, let ſome certain Secondary (for inſtance, one of the Colures) be pitched upon, and

let the Globe be ſo placed, that this aſſume Secondary may be inſtead of the firſt Hou Circle; that is, that 15 degrees may be inte cepted between the fix'd Meridian and this a ſumed one; then let the degrees intercepte between this aſſumed one and the fix'd Meri dian, be reckoned in the Horizon. Turn th Globe then, till the aſſumed Secondary of th Equator come to be in the place of the Two Clock Circle in the Afternoon; that is, till 3 degrees of the Equator be between it and th fix'd Meridian, and the degrees of the Horizo intercepted between the Meridian and the Cir cle aſſumed thus poſited, being numbered; le lines be drawn to the Eaſtern ſide of it from th Center of the Sundial, making Angles with th Meridian line meaſured by the ſame number of degrees, and they will be the Hour lines o One and Two a Clock. And proceeding, afte the ſame manner muſt the Lines of the follow ing Hours be drawn, till you come to tha Hour that the Sun ſets at in the Longeſt day Where, 'tis evident, that the Hour line of ſix Clock is perpendicular to the Meridian line ſince in any Latitude whatever, there are 9 degrees intercepted between the Meridian an the ſix a clock Hour line. After the ſame manne are the Angles to be determined, that the line of the half Hours or any other parts of Hour contain with the Meridian line upon the Hori zontal Plane.. Having done drawing the Hou lines upon one ſide of the Meridian, for inſtance for the Hours after Noon, the Morning Hou lines are to be drawn ſo, as that the lines repre ſenting the equally diſtant Hours from Noon may make equal Angles with the Meridian but lying on the other ſide. After the like man ner (by the help of the 13th. *Prob.*) may a Sun

di

ial be drawn upon any Plane: For any Plane the Horizontal Plane of ſome place.

SECTION IV.

About the determining by Obſervation, the ſituation and reſpect, that the Circles of the Sphere have to one another.

PROPOSITION XVI.

TO determine by Obſervation, the Plane of the Meridian Circle in any Place.

The Place being given, the Horizon is alſo given: for that is determined by the Sight. And this being the only one that comes under the notice of our Senſes, the others muſt be determined by the reſpect they have to this; and firſt the Meridian. Let two planes of Vertical Circles be found by Obſervation, that a Celeſtial Body, which does not change its Declination, is upon, when it is at the ſame Altitude before and after its greateſt Elevation above the Horizon; the Vertical Plane biſecting the Angle contained by theſe two Planes, will be the Plane of the Meridian, and produced unto the Heavens by the ſight will mark out the Meridian ſought. Since (by the ſuppoſition) the Celeſtial Body does not change its declination, it will deſcribe by its Diurnal motion a Circle parallel to the Equator: The higheſt point thereof of this Circle (as alſo of the Equator itſelf) is in the Meridian Circle, and the points that are at equal Heights on either ſide, will be equi-diſtant from the Meridian. Wherefore converſly, that Circle from which the Verticals, in which the Celeſtial Body was at the ſame Alti-

Altitude, are equidiſtant on both ſides, is thi Meridian; that is, the Circle, whoſe Plane bi ſects the Angle contained by the abovementi oned Verticals is the Meridian itſelf. *Q. E. I.*

As for the Practice, this is a good way enougl of ſolving the Problem: Upon a Plane paral lel to the Horizon, that is, one to which Plummet is perpendicular (ſince the Zenith anc Nadir, upon which the Plummet falls when pro duced, are the Poles of the Horizon,) erect Style, on whoſe lower extremity, as a Center, let a Circle be deſcribed upon the Horizonta Plane, and let a point be marked in the Cir cumference of it, upon which the ſhadow o the top of the Style falls before Noon; and a gain that point of it upon which the ſhadow o the ſame falls in the Afternoon: A Right line connecting the center of the Circle with that point which biſects the Arc lying between the two points thus marked, is the common Se ction of the plane of the Meridian with the Horizontal Plane, which is alſo called the *Meridian line*; becauſe it ſhews the North and South Cardinal points by its direction. A Plane therefore erected upon this line, perpendicular to the Horizontal plane, is the Plane of the Meridian ſought. When the Ce leſtial Body made uſe of in the ſolution of this Problem is the Sun, 'tis evident that it muſt be done, when the Sun does not ſenſibly change his declination; (for this is a neceſſary quali fication in the Celeſtial Body pitched upon for this purpoſe;) that is, in one of the Solſtices. Be cauſe theſe Obſervations are to be made before and after Noon, the Summer Solſtice is to be preferred, ſeeing that in the Winter Solſtice, by reaſon of the Sun's nearneſs to the Horizon, the

uncer-

uncertain Refractions will render the operation doubtful; and besides, then the Sun ascends and descends too obliquely, altering its Vertical much more than its Altitude; which is incommodious in this operation, where the Vertical is to be determined by the Altitude. Therefore when the Sun is in the Summer Solstice, a Circle is to be drawn upon the Horizontal plane, so as that the extremity of the shadow of the Style or Gnomon may fall upon it, when the Sun alters its Altitude pretty much, and his Azimuth but little; and therefore several concentric Circles are to be drawn, that the most commodious may be chosen.

To determine the Meridian line drawn for the fixing your more accurate and perfect Instruments, this operation should be repeated, and others made use of, to be seen amongst several others, or easily thought upon by any one: And this is to be understood of any other Observations, that are fundamental, and upon which a long train of others are built. The longest Styles or Gnomons are the fittest for the making this Observation: And instead of an opake extremity or top, in a large Building, a hole letting the Rays of the Sun pass thro' it into the Ædifice, would be more proper; as *Cassini* did in the Church of St. *Petronius* at *Bononia*.

PROPOSITION XVII.

TO *determine by Observation the Height of the Pole, and Situation of the Equinoctial, in regard of the Horizon.*

Where one of the Poles has a considerable Elevation, [*Fig.* 5.] let the greatest and least Altitudes *O S* and *O σ* above the Horizon *H O* of any Fix'd Star that never sets, (both of which happens in the Meridian,) be observed by an

Instrument fix'd in the plane of the Meridian, (determin'd by the preced. *Prop.*) And PS or $P\sigma$ half the difference $S\sigma$ of these two Altitudes, is the distance of the Star from the Pole: This distance $P\sigma$ therefore, or PS added to the lesser Altitude $O\sigma$, or subtracted from the greatest OS, gives OP the Height of the Pole sought. If the Star, when it is at its greatest Height is situated upon the other side of the Zenith Z, between Z and H; then instead of the greatest Altitude of the Star, you are to make use of its complement to a Semicircle, in the foregoing Calculation.

Great care and diligence is to be used in making this Observation: For upon this are all the other observations of the Sun and Fixed Stars, and consequently all Astronomy is founded. Such a Fix'd Star therefore is to be chosen as is least liable to Refraction in its least Altitude; that is, the nearest to the Pole, which does not approach the Horizon too much: For the Refraction of Stars are liable to make their places observed near the Horizon uncertain; as shall be shewn hereafter.

If neither of the Poles be elevated enough above the Horizon, then the thing is to be done by the Meridian Altitude of a Star, whose Declination is known; as shall be shewn in the *Corollary* to the following *Proposition*. And in most places, where but few Observations are design'd to be made, this method is usually taken. But since the Observation of the Declination of the Stars, depends upon the knowledge of the Height of the Pole, 'tis more proper to lay down such a manner of observing the latter as does not depend upon the former; which also must necessarily be used by all cautious Observers, that design to determine the places of the

Stars,

Stars, and without trusting the Observations of others at all, since they may repeat them.

The Height of the Pole being found, the Altitude of the Equator, or *HE* the Arc of the Meridian intercepted between the Horizon *HO* and the Equator *E*♋ is determined; For it is the Complement of the former to a quarter of a Circle: For because *HZO* is a Semicircle, and *PE* a Quadrant, *PO* and *HE* together will be equal to a Quadrant. The situation therefore of the Equator in regard of the given Horizon *HO*, is determined: For it intersects it in the points of the true East and West, which are marked out by a Right line perpendicular to the Meridian (found by the preced. *Prop.*) and the Plane of it is inclined to the Plane of the Horizon by an Angle, whose measure is the Arc *HE* lately found.

SCHOLIUM.

By the help of these two Propositions the magnitude of the Earth may be known. If a journey made along the Meridian (drawn by *Prop.* 16.) be exactly measured, and by an Observation of the Altitude of the Pole made at the beginning and end of the journey, the ratio of the Arc of the Terrestrial Meridian intercepted, to the intire Meridian be determined; the Circumference of the intire Terrestrial Meridian will not be unknown: And consequently, the Earth's Diameter, and from thence (by Geometry,) its Surface and Solidity will be known.

Besides, because in practise, a journey taken under any Meridian, or the length of the Way, is every where very nearly proportional to the change of the Elevation of the Pole by Observation; 'tis evident that the common Section of the surface of the Earth, and of the plane of any Meridian is a Circle, and consequently that the

figure

figure of the Earth is Spherical. We abstract now from the little elevation at the Equator (concerning which in *Prop.* 31. *B.* 1.) which in the before-mentioned operation will be insensible.

PROPOSITION XVIII.

TO observe the Declination of any Celestial Body.

Let the Meridian Altitude of the Star proposed be observed. If the Star be in the quadrant *ZH* of the Meridian [*fig.* 6] that the Equator *E♋* intersects the Meridian in, as at *σ*; the difference of the Altitude of the Equator *HE* and of the Altitude of the Star *Hσ* (namely *Eσ*) is the Declination of the Star sought. But if the Star be in the quadrant *ZO* of the Meridian, in which the Pole *P* is, as in *S*; then the difference of the Altitude of the Pole *P*, and of the Altitude of the Star *OS*, (namely, *PS*,) or the distance of the Star from the Pole, is the complement of the Declination sought. *Q. E. I.*

We add nothing at all here about the actual making the Observations and the construction of the Instruments, in which the chief difficulty of these things consists. For we suppose the Maker very well versed in *Practical Geometry, Mechanics* and *Optics*. And it is evident that it will be most commodious for any one that is about to determine the Places of the Fix'd Stars, to have a Quadrant or rather a Semicircle intirely immovable in the Plane of the Meridian, and divided into Degrees, and their lesser parts according to art, and an Index moving upon its Center, furnished with Telescopic Sights, for the ready measuring the Meridian Altitude of any Star, when it comes to the Plane of the Meri-

Meridian: For this Obſervation of the Declination of the Stars is of great uſe in correcting their Places.

COROLLARY.

As the Declination of an Heavenly Body is found by the Elevation of the Pole or Equator, and the Meridian Altitude of the Celeſtial Body being given; ſo, on the contrary, the Elevation of the Pole or of the Equator may be had, by the Meridian Altitude and Declination of the Star being given. For in the quadrant *ZEH*, the ſum or difference of the Lines *Hσ*, *Eσ* gives the Elevation of the Equator *HE*. But in the Quadrant *ZPO*, the ſum or difference of the Lines *OS* and *PS* gives the Altitude of the Pole.

PROPOSITION XIX.

TO obſerve the Inclination of the Ecliptic to the Equator or the Obliquity of the Ecliptic, and to determine its Situation and conſequently that of the Colures, Tropics and Polars, in reſpect of the Equator.

Let the Sun's Declination be daily obſerved about both Solſtices: (by the preced.) This, where it is greateſt, is the Obliquity of the Ecliptic ſought. For ſince the Ecliptic is the way of the Sun, as big as the greateſt Declination of the Sun is, ſo big is that of the Ecliptic it ſelf; conſequently the one being obſerved, the other is alſo obſerved. *Q. E. F.*

But ſince from the preceding Praxis, the Declination of a Celeſtial Body can't be obſerved, excepting when that Star is in the Meridian; if the Solſtice ſhould happen to be celebrated when the Sun is not in the Meridian of the Obſerver, (which will generally happen) there will be ſome error; but ſo ſmall as to be not worth regarding. For we gave inſtructions that

that the Declination ſhould be taken any Noon; and conſequently the greateſt error will be when the Solſtice is celebrated at Midnight; that is, at that point of time which is moſt remote from Noon. The error therefore, when it is greateſt, will be equal to what the Sun's Declination 12 hours before or after the Solſtice wants of its greateſt Declination: But this does not ariſe to above four Seconds; tho' the Obliquity it ſelf is not determined to a minute, or even two, by Artiſts themſelves. Of the Solſtices that is to be choſen, in which the Sun may be beſt obſerved without any danger of error, and in this regions near the North Pole, the Summer Solſtice; becauſe the Sun in the Winter Solſtice, even at Noon it ſelf, is not out of all danger of Refraction; which is a matter of very great moment in this kind of Obſervation. There are ſome places, where to find the Obliquity of the Ecliptic, it may be adviſable to ſubtract the Meridian Altitude of the Sun, when in one of the Solſtices, from its Meridian Altitude in the other, if the Sun in both caſes be on the ſame ſide of the Zenith; but if on different ſides, from the Complement to a Semicircle of the latter Meridian Altitude, to have the diſtance of the Tropics remaining, half of which is the Obliquity ſought.

To get the Obliquity of the Zodiac with great exactneſs, Obſervations continued for ſeveral Years, ſays the ſkilful and induſtrious *Hevelius*, and ſuch as are compleat on all accounts, taken by your larger Inſtruments, and ſuch as may be depended upon, are neceſſary. And upon this account it is that now-a-days, when Inſtruments are both great and accurately divided and furniſhed with Teleſcopic Sights, there is a ſenſible difference obſervable between

the

the Obliquities of the Zodiac ſet down by different Artiſts. For *Hevelius* makes it $23^{deg.}\ 30'.\ 20''$. The ſame does *Ricciolus* in his *Reformed Aſtronomy*. *Mouton*, who has writ almoſt an entire Book upon this Argument, chooſing, after ſeveral repeated Obſervations, the middle between the extremes, fixes upon $23^{gr.}\ 30'$, as *Ricciolus* in his *Almageſt* does from ſome ſelect Obſervations: This is what Mr. *Street* alſo, in his *Aſtronomia Carolina*, ſets down, and moſt Aſtronomers now make uſe of. Notwithſtanding Mr. *De la Hire*, in his *Aſtronom. Tables, Part firſt, of the Motions of the Sun and Moon*, makes it an intire minute leſs; namely, only $23°.\ 29'$, and that from Obſervations near the Zenith, and out of all danger from Refraction.

And were the judgment of the Ancients about this matter conſulted, ſo great a difference would be found between the Obliquity obſerved by them and that taken from the Heavens now, that ſome Authors have thought it to be changable, beſides the change made in it twice a Year; which we don't here diſpute about, becauſe almoſt inſenſible: Since from the time of *Ariſtarchus* the *Samian*, who flouriſhed near 300 Years before Chriſt, it was reckon'd to be about 24°: But in *Ptolemy*'s time, about 140 Years after Chriſt, not above $23°.\ 51'$. And again, in *Albategnius*'s time, near 900 Years after Chriſt, only $23^{gr.}\ 35'$. And *Tycho* by his Obſervations makes it to be but $23°.\ 31'.\ 30''$. Yet the moſt induſtrious obſervers of the preſent Age, will have it, notwithſtanding, to be conſtant; namely, *Gaſſendus*, *Ricciolus*, *Horrox* and *Hevelius*, who does not think, in his *Prodromus*, that we ought to depend ſo much upon their Obſervations, ſince they did not make uſe of the beſt Inſtruments, nor proceeded in the ſhorteſt method.

Nay,

Nay, ſome of the Ancients, and *Ptolemy* himſelf, in his *Almageſt*, think ſo too; *Albategnius* for the ſame reaſon with *Hevelius*, prefers his own Obſervations to thoſe of the Ancients. And *Gaſſendus* in the *Life of Peireskius*, ſhews, by ſome Obſervations of his own making, that the Obliquity of the Zodiac is the ſame now-a-days, as it was in the time of *Alexander the Great*; becauſe he found by an Obſervation made at *Marſeilles* by himſelf, that the ratio of the Gnomon to its ſhadow at Noon in the Summer Solſtice, was the ſame with that obſerv'd in the ſame City at the ſame time of the Year by *Pytheas Maſſilienſis*.

Beſides, the points in the Heavens, where the Ecliptic interſects the Equator are they, in which the Sun appears when the days are equal to the nights to an Obſerver, to whom either of the Poles is elevated above the Horizon. This therefore is the ſituation of the Ecliptic, that it may cut the Equator in the ſaid points, and be inclined to the Equator in the Angle above determined, half of it tending to the North Pole, in which the Sun is, when the Days are longer than the Nights to the Inhabitants of the Northern Hemiſphere of the Earth. The ſituation of the Ecliptic being given, the ſituation of the Tropics is immediately given; becauſe they are parallel to the Equator, and touch the Ecliptic, and are as far from the Equator, as the Obliquity of the Ecliptic is great, and the Polar Circles are as far diſtant from the neareſt Poles. The ſituation of the Colures is deduced from hence, for they paſs thro' the Equinoctial and Solſtitial points found before, and interſect one another at right angles in the Poles already determined by Obſervation.

PRO-

PROPOSITION XX.

THE Obliquity of the Ecliptic being given, to find by calculation, the Right Ascension and Declination of a given point in it, as also the Angle that the Ecliptic makes with the Meridian at the said point; and conversely, to define the point of the Ecliptic, to which one of these agrees, and consequently to determine the place of the Sun in the Ecliptic, by an Observation made in Prop. 18.

Let *E* ♋ [*Fig.* 7.] represent the Equator, whose Pole is *P*; *E L* the Ecliptic; *E* the point of one of the Equinoxes; *L* a point given in the Ecliptic, thro' which let a Circle of Declination, as *P L* ♋, be supposed to be drawn. In the Spherical Triangle *E L* ♋, there is given the Side *E L*, namely the distance of the point given from *L*, and the Angle ♋ *E L*, the Obliquity of the Ecliptic; the Angle ♋ also is given, namely a right one, because *P* is the Pole of the Circle *E* ♋: The side ♋ *L* therefore will be found (by Spherical Trigonometry,) which is the Declination of the point *L* sought, and is North or South, according as the adjacent Pole is: In like manner the Side *E* ♋ will be found, which (according to the situation of the point *L* in this or that Quadrant of the Ecliptic, and the nature of the point *E*) will be either the Right Ascension sought, or its complement to a quarter, or half, or three quarters, or an intire Circle. For the Angle ♋ *L E* is given, that the Ecliptic makes in the said point with the Circle of Declination or Meridian. The converse of this is done after the like manner: Yet we are to take notice, that there are two points of the Ecliptic, whose Declination towards the given Pole is the same; and therefore some sort of a determination is necessary.

SECTION

SECTION V.

Of the Fix'd Stars, and the determining their Places by obſervation, and ſome other things that belong hereto.

PROPOSITION XXI.

TO explain the ſeveral Orders of the Fix'd Stars, into which they are divided on account of their different apparent Magnitude; and to give an account of this difference.

In ſome of the preceding Propoſitions we have laid down the method, whereby the poſition of the chief Circles of the Sphere are determined in reſpect to the ſenſible Horizon; namely, of the Meridian, Equator, Zodiac, both Colures, Tropics and Polar Circles: We ſhould now ſhew the way, how the Places of the Fix'd Stars in reſpect of theſe Circles are to be found. But before that, we muſt ſay ſomething about the Fix'd Stars themſelves, their differences and order.

And firſt, there appears a vaſt difference among the Fixed Stars as to their apparent Magnitude and efficacy of their Rays, to any one that ſhall but look up to the Heavens in a clear night when the Moon does not ſhine. On this account Aſtronomers have made ſix Claſſes, or Orders of them, the greateſt of which they call *Stars of the firſt Magnitude*; the leaſt, *Stars of the ſixth*, and the intermediate in the ſame order. Some make the difference to lie in the real magnitude of the Stars, aſſerting ſome of them to be really bigger than others, while they are all equally diſtant; becauſe placed according to them

hem in the ſame Spherical ſurface: Others following the Ancients, make their different diſtance o be the cauſe of their different Magnitude.

And this opinion is very much favoured by he number of the Fix'd Stars of the firſt and econd Magnitude. For if every Fix'd Star id the office of a Sun, to a portion of the Mundane ſpace nearly equal to this that our un commands, there will be as many Fix'd Stars f the firſt Magnitude, as there can be Syſtems f this ſort touching and ſurrounding ours; hat is, as many equal Spheres as can touch n equal one in the middle of them. Now, is certain from Geometry, that thirteen pheres can touch and ſurround one in the iddle equal to them, (for *Kepler* is wrong in ſſerting, in *B.* 1 of the *Epit.* that there may be twelve ſuch, according to the number of the ngles of an *Icoſaedrum,*) and juſt ſo many uncontroverted Stars of the firſt Magnitude are aken notice of by Obſervation. For Aſtronomers are not as yet agreed upon their number: *Hevelius* reckons the *bright Star in the Eagle's Shoulder* of the ſecond magnitude, whereas *Tycho* made it of the firſt; and on the contrary *Hevelius* makes the *little Dog* and *the right Shoulder of Orion* of the firſt magnitude, and *Tycho* of the ſecond. And there are others that *Hevelius* himſelf doubts of.

Again, if it be asked how many Spheres equal to the former can touch the firſt Order of Spheres, ſurrounding the Sphere placed in the beginning, (or rather a Sphere comprehending thoſe former thirteen together with a fourteenth in the Center;) the number of theſe will be found to be 52, or 4×13. For the centers of theſe Spheres of the ſecond Order are in a Spherical ſurface, which is quadruple of that in which

the centers of the Spheres of the firſt Orde touching the inmoſt are, ſince the Diameter c the one is double that of the other; and nearl as many Fix'd Stars of the ſecond magnitud have been taken notice of; *Hevelius* makin them to be about 50; *Kepler* following *Tych* 58; and *Ptolemy* not above 45; now we aſſum the middle between them: For there is greater difference among Aſtronomers abou Stars of the ſecond magnitude than abou Stars of the firſt. Nor will there be an great difference from Obſervation in deter mining the number of the Fix'd Stars of th other magnitudes according to this method For the farther conſideration of this matter, i only remains that we ſhew, that there is th ſame Order obſerved among the Fix'd Stars o the firſt rank, as there is between the centra Bodies of the Spheres touching and ſurroundin the inmoſt Sphere (near whoſe Center the Ey is placed;) and the ſame between the Fix'd Star of the ſecond magnitude, as there is betwee the central Bodies of the Spheres ſecond in or der from theſe; and ſo on. And here indee the matter does not go on ſo well, (which mad *Kepler* of another opinion,) as is evident eve from hence, that upon the firſt caſt of our Eye upon the Heavens, ſome tracts of the Firma ment appear fill'd with innumerable Fix'd Stars whereas others are found to be almoſt empty and void of any.

But there is no great error committed i the order of the Stars of the firſt and ſecon magnitude, as will appear to any one tha makes a compariſon: For there are ſix Stars o the firſt magnitude in the Zodiac, three to th North, and four to the South, nearly, as it ough to be according to this Theory.

PRO

PROPOSITION XXII.

TO *enumerate the Constellations, into which the Fix'd Stars are distributed by Astronomers.*

The Ancients either following the Shape and igure of the disposition the Stars appeared in, led by Religion, have distributed all the Fix- Stars visible in our Temperate Zone, into 48 nages: Twelve of which are situated along ngth of the Zodiac, and give names to its odecatemories: And there are in the Northern half, *Aries, Taurus, Gemini, Cancer, Leo* and *Virgo*; the Southern, *Libra, Scorpio, Sagittarius, Ca- ricornus, Aquarius* and *Pisces.* The other Images placed in the Hemispheres separated from e another by the Zodiac: in the Northern , namely *the little Bear, the great Bear, Draco, Cheus, Bootes, the Northern Crown, Hercules, La, Cygnus, Cassiopeia, Perseus, Andromeda, the Tiangle, Auriga, Pegasus, Equuleus, Dolphin, Sa- gita, Aquila, Serpentarius, Serpens.* To these were afterwards added the Constellations of *Antinous* made out of the unformed ones near *Aquila*, be- tween it and *Capricorn* and *Sagittary*, and of *Coma Berenices* out of the unformed ones near th *Lyon's Tail. Ptolemy* makes *Antinous* belong to *Aquila*, and *Equuleus* to *Pegasus.* On the Sothern part of the Zodiac there are 15 Con- steations known to the Ancients, namely *Cetus, Erlanus, Lepus, Orion, the Great Dog, the Little Do, the Ship Argo, Hydra, the Cup, Corvus, the Ceaur, Lupus, Ara, the Southern Crown, and Pisces Aurinus.*

To these are lately added 12 Constellations, th are invisible to us, being near the South Pole, naely the *Phœnix, Grus, Pavo, Indus, Apus, South Tringle, Musca, Chamæleon, Piscis volans, Toucan,* or *anser Americanus, Hydrus, Xiphias* or *Dorado.*

There are ſome other Stars, which are not with in the compaſs of the Images deſcribed, or re ducible to them, theſe therefore they call *Un formed*; and out of theſe, ſome of the more con ſiderable Obſervers of the Fix'd Stars, make now and then ſome new Conſtellations, and give them proper names. Thus the Accurate Mr. *Ed mund Halley*, who firſt of all obſerved with ex actneſs the Stars near the South Pole, while he was at St. *Helena*, has taken this liberty, in form ing a new Conſtellation into the Image of a Oak, out of the unformed Stars that lie betwee the Ship *Argo* and the *Centaur*, and conſecrate it under the name of *Robur Carolinum*, or *th Royal Oak*, tranſlated into the Heavens in per petual memory of the Preſervation of Kin *Charles* II. inthe hollow thereof. Thus *Bartſchi* in his Globe of four foot diameter, has two othe new Conſtellations, *the Camelopard*, and *the Mo noceros*, that *Hevelius* retains.

But *Hevelius* himſelf has gone farther in thi matter, and has made up more new Conſtella tions out of the unformed ones, than any of th Moderns beſides, as is evident from his *Firma ment*; where he has plac'd *Leo minor* betwee Leo and Urſa major, and *Lynx* between Urſa ma jor and Auriga above Gemini; the *Canes Vena tici* after Urſa major and under its Tail; ſo tha what to *Tycho* was an unform'd Star betwee the Tails of Helice and Leo (afterwards call'd C *Caroli* by the *Engliſh*,) by *Hevelius* is ſaid to b *in Annulo Armillæ Charæ*, or *Collar of Chara*; th *Lacerta* or *Stillio* between Andromeda and Cyg nus; the *Sextans Uraniæ* between Leo and Hy dra; *Scutum Sobieſciannum* between Aquila an Serpentarius; *Vulpecula cum Anſere*, betwee Aquila and Lyra under Cygnus; *Triangulm mi nus* between Triangulum Boreale and the Hea

of Aries. He has altered ſome of the ancient Conſtellations a little, and added others to them; as he has armed Antinous with a *Bow* and *Arrow*, who was before unarmed, and placed *Cerberus* above the bending of *Hercules*'s left Arm, and *Mons Mænalus* under the feet of Bootes. The Conſtellations are ſet in the places deſcribed, as a continuation of the ancient Fables of the adjacent Conſtellations, leſt he ſhould break too much in upon or diſorder the Rules and Judgments of the Aſtrologers, or to perpetuate the memory of ſome noble Action.

Others again are for giving the Stars new Names, ſuch as are *Chriſtian* ones; a ſpecimen of this we have in the Venerable *Bede* in the Signs of the Zodiac; and *Julius Schillerus* has continu'd in his *Cælum Stellatum*, or Starry Heaven, publiſhed in the Year 1627. This deſign has been very juſtly diſapproved of by *Copernicus*, *Tycho*, and *Hevelius* in his *Firmament*, and other Aſtronomers, who think the ancient names ought to be retained, for fear of confuſion, and by degrees loſing the Ancient Aſtronomy. For by this method, which has been conveyed down to us from the original of Aſtronomy, we obtain what was of the greateſt importance in this matter; namely (as *Copernicus* ſays) *that ſo vaſt a multitude of Stars might be diſtinguiſhed into parts, and that theſe may have ſome names to expreſs them by.*

The *Via Lactea*, or *Milky Way*, comes under this head of Conſtellations; for the preſent Age has found it to be a collection of innumerable little Fix'd Stars, as ſome of the Ancients believed, as *Manilius* informs us: 'Tis a broad Circle white like Milk, compaſſing the whole Heavens; ſometimes in a double, but generally a ſingle path, paſſing thro' Caſſiopeia, Perſeus, Auriga, the

U 3 Feet

Feet of Gemini, Orion's Club; the fore parts of Monoceros, the Tail of Canis Major, Arg Navis, Robur Carolinum, Crux and feet of th Centaur; after which, it is divided into tw parts, over against Ara; its Eastern part passe thro' Ara, the extremity of the Tail of Scorpio the Eastern foot of Serpentarius, the Bow of Sagittarius, Scutum Sobiescianum, the feet of Antinous and Cygnus, where its greater part joins the other: But the Western part pas ses thro' the former part of the Tail of Scorpio the Right of Serpentarius and Cygnus; and ends its course in Cassiopeia, where *Manilius* begins the description of it. There are othe lesser divisions of the *Galaxy*, the which may b seen in *Hevelius*'s *Firmament*. There are als two *Nubecula* somewhat resembling the Milk Way, situated near the South Pole, invisible i *Europe*, and called by Sailors, the *Magellan Clouds*, are exactly like the whiteness of the *Ga laxy*, as Mr. *Halley* assures us; and being view'd by a Telescope, they present us here and ther with little *Nebulæ*, and small Stars. The greate of these two is situated between Hydrus and Dorado, the less between Anser Americanus o Toucan and Hydrus.

PROPOSITION XXIII.

THE *Theory of the Sun and a Pendulum Clock being given, to determine the Right Ascension of the Fix'd Stars by observation.*

Having explained the difference of the Fix'd Stars, as well on the account of their apparent magnitude, as of the figure of the Constella-tions, into which they are commonly divided; what remains is to shew the method of deter-mining the Place of any of them, in regard of the Circles of the Sphere, whose situation has been

ɟeen determined before, and first in regard of he Equator; for which purpose, we have shewn ɩow to observe the Declination of any one of hem in Prop. 18; it remains therefore that we ind the Right Ascension: But, because the beginning of the Equator, from whence the Right Ascension of the Stars is reckoned, is its interſection with the Ecliptic; in which, though the un be found once a Year, *viz.* in the beginning of our Spring, yet it presently departs rom thence, and leaves that point obscure or narked out by no appearance that may come nder the cognizance of our Senses; and lastly, uch an one as that the Ascensions of the Stars an't be immediately deduced from thence by Observation; some Phænomenon therefore is ɔ be pitched upon, whose Right Ascension is iven every moment, that the Right Ascensions f the others may be observed from this. And f all others, the fittest for this purpose is the un, because its motion is the most simple, and s Right Ascension is quickly found, by *Prop.* 20, s place in the Ecliptic being given. If its Theory therefore be given, that is, if its place ı the Ecliptic at a given time be given, and a Clock also whose Index moves round equably, ıe Right Ascensions of the Fix'd Stars may be ıus observed.

First, let the Clock be order'd so, as that the ıdex may go over the 24 Hours, during the me that any Fix'd Star, departing from the Meridian, returns to it again, which is a little ſs than a natural Day. Let the Index of a Clock thus order'd, be set to twelve a Clock, hen the Sun is in the Meridian, and let the our the Index points at, be taken notice of, that ıe fix'd Star whose Right Ascension is sought, omes to the Meridian at. Let these hours and

parts be turn'd into degrees of the Equator and their parts, as was ſhewn before: And you will have the difference between the Right Aſcenſions of the Fix'd Star and the Sun; this added to the Right Aſcenſion of the Sun, gives the Right Aſcenſion of the Fixed Star ſought.

If in the Clock fitted to the diurnal Motion of a Fix'd Star, inſtead of the 24 Hours, there are placed 360 Degrees, and their Sexageſſimal parts, and if while the Sun is in the Meridian, the Index be placed to the number of Degrees and Minutes, the Right Aſcenſion of the Sun then conſiſts of; it will then point at the Right Aſcenſion of the Fix'd Star, without any farther reduction, whenever it arrives at the Meridian.

PROPOSITION XXIV.

THE Declination and Right Aſcenſion of one Star, and the Declination of another together with their Diſtance being given, to find the Right Aſcenſion of the latter; and vice verſa, *to find the Diſtance of two Stars, whoſe Right Aſcenſions and Declinations are given.*

Let *E*♋ repreſent the Equator, [*fig.*8.] whoſe Pole is *P*; and the Stars *A* and *B*, whoſe Declinations *CA*, *DB* being given, their Complements alſo are given, or the diſtances from the Pole, namely *PA*, *PB*, and by ſuppoſition, the Diſtance *AB* of the Stars is given. Wherefore in the Triangle *APB*, the Sides being given, the Angle *APB* will be found, and conſequently its meaſure the Arc *CD*; namely the difference of the Right Aſcenſions of the Stars *A* and *B*, which therefore added to, or ſubtracted from the Right Aſcenſion given of one of them; (according as the Circle of Declination paſſing thro' this, is to the Eaſt or the Weſt of the

Plate 2. Book 2.

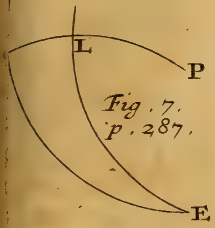


Fig. 7.
p. 287.

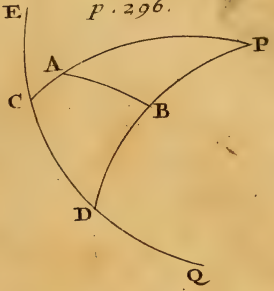
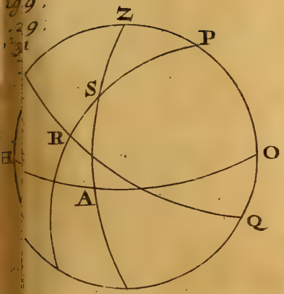


Fig. 8.
p. 296.



ig. 9.
. 29.

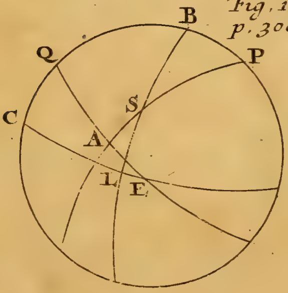
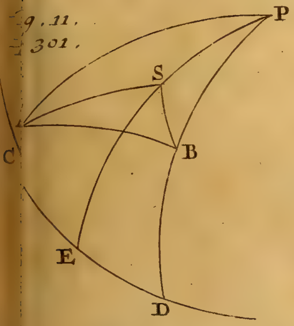


Fig. 10.
p. 300.



g. 11.
301.

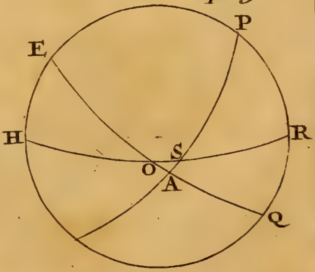


Fig. 12.
p. 320.

the Circle of Declination paſſing thro' that,) gives the Right Aſcenſion ſought of the other.

And on the contrary, the Right Aſcenſions of the Stars *A* and *B* being given, the difference of them *C D* is alſo given; that is, the Angle *C P B*, whoſe meaſure that Arc is: And in the Triangle *A P B*, the Sides *P A*, *P B*, the Complements of the given Declinations, together with the Angle *A P B* contained are given; the other Angles will conſequently be found, and the Side *A B* the diſtance of the Stars ſought.

After the like manner you may find the Diſtance computed in a great Circle of two places in the Surface of the Earth, whoſe Longitudes and Latitudes are given, and the Angles which that great Circle (which anſwers the common Celeſtial Azimuth) makes with both Meridians.

PROPOSITION XXV.

TO determine by Obſervation the Right Aſcenſion of a given Star, by a Phænomenon viſible by day and night.

Let the Declination of the Phænomenon viſible by day and night, be obſerv'd (by *Prop.* 18. and the diſtance as well from the Sun as from a Star; the latter in the night, the former in the day. From the Declination of the Phænomenon, the Diſtance from the Sun, and the Sun's Place, the Right Aſcenſion may be deduced, by the preceding; from that and the Declination of the ſame being obſerved, together with the Declination of the Star and Diſtance from the Phænomenon being obſerved, the Rignt Aſcenſion of the Star ſought may after the ſame manner be found. *Q. E. F.*

SCHO-

SCHOLIUM.

The Ancients, who had not an accurate meaſure of Time,(ſuch as *Prop.*23.requires, their Hourglaſſes, the inſtruments they made uſe of, being too fallacious,) ſought after the places of the Fix'd Stars by the Moon, taking the diſtance of the Moon from the Sun, computed according to the Equator or Ecliptic a days, and the diſtance of a Star from the Moon a nights, from whence they concluded upon the diſtance between the Star and the Sun, and conſequently (by the Sun's Theory) the diſtance of the Star from the beginning of the Ecliptic.

The Moderns, inſtead of the Moon uſe Venus, becauſe its Diameter being leſs will make the error leſs in obſerving the diſtance; and becauſe the Moon's place comes often to be obſerved by the places of the Fix'd Stars firſt corrected, that its Theory may be corrected, which is not as yet accurate, eſpecially about the Quadratures, which was the times that the Ancients made theſe Obſervations in; and alſo becauſe the Motion of Venus is neither ſo ſwift, nor ſo uncertain. Therefore it will be beſt to obſerve Venus in its greateſt Elongation from the Sun, when its Motion is nearly equal to that of the Sun; and when by reaſon of their Elevation the Sun and Venus are leaſt obnoxious to Refraction. In making theſe Obſervations ſpecial care muſt be taken to repeat them, as well when Venus is to the Eaſt of the Sun as when it is to the Weſt, ſo that by theſe different Obſervations conſpiring to determine the ſame place of the Fix'd Star, the true place may be at laſt certainly had. *Ricciolus* is for uſing *Sirius* inſtead of Venus, which, as he ſays, ſhines ſo bright as about the Equinoxes is viſible above eight minutes before Sun-ſet in *March*, and

after Sun-riſe in *September*. But both *Regulus* and *Arcturus* have been obſerved by the naked Eye in the Meridian, while the Sun ſhone, by the Aſtronomers of the *French* Royal Academy of Sciences, even when the Sun was elevated above the uncertain Limits of Refraction: Which may be done every day in any of the brighter Fix'd Stars by the help of the Teleſcope.

PROPOSITION XXVI.

THE Height of the Pole, Hour from Noon, and Place of the Sun being given, if beſides, the Altitude and Azimuth of a Star be known by Obſervation, to find the Right Aſcenſion and Declination of that Star.

Let *PZHO* repreſent the Meridian; [*fig.* 9.] *HAO* the Horizon, whoſe Pole is the Zenith *Z*; *ER*♎ the Equator, whoſe Pole is *P*; *S* the Star, thro' which we are to imagine the Vertical Circle *ZSA*, and Circle of Declination *PSR* to be drawn. In the Triangle *SZP*, there are given the Side *ZP*, the Complement of the Elevation of the Pole *OP*, the Side *ZS*, the Complement of the Altitude of the Star *SA*; and the Angle *SZP*, that the Arc *AO*, given by the Azimuth, meaſures; therefore the Side *PS*, the Complement of the Arc *RS*, of the Declination of the Star ſought is found; as alſo the Angle *ZPS*, and conſequently its Meaſure, namely the Arc *ER*. But the Hour of the Day being given, the Arc of the Equator intercepted between the Meridian and Circle of Declination, drawn thro' the Sun, is alſo given, conſequently the ſum and difference of the given Arcs are given. But this ſum, if the Sun and Star be on different ſides of the Meridian; or difference, if on the ſame, is the difference of the Right Aſcenſions of the Sun and Star.

And

And the Sun's Place being given, its Right Aſcenſion (by *Prop.* 20.) is given; conſequently the Right Aſcenſion of the Star ſought is alſo become known.

PROPOSITION XXVII.

TO *find the Longitude and Latitude of a given Star.*

In Figure 10, let *E ♋* repreſent the Equator, whoſe Pole is *P*; *EC* the Ecliptic, whoſe Pole is *B*; the Point *E* the common interſection of theſe Circles, the beginning of ♈ or ♎; *S* the given Star, thro' which the Secondaries *B S L*, *PS A*, of the Ecliptic and Equator are to be imagined to paſs. And the great Circle *PB♋C* connecting the Poles is the Colure of the Solſtices. Let *A S* the Declination of the Star be found (by *Prop.* 18,) then you will have its Complement *PS*; and (by *Prop.* 23 or 25) *E A* the Right Aſcenſion of the Star (at leaſt from the given Aſcenſion, the Right Aſcenſion *E A* will be given,) from whence you will have its Compliment *♋A*, and the Angle *♋PA*, whoſe meaſure it is. Therefore in the Triangle *B PS*, there are given the Side *P S* and the Side *P B*, namely the diſtance of the Poles of the Equator and Ecliptic equal to the Obliquity of the latter, together with *BP S*, the Angle contained by them; the Side *BS* therefore is known, and conſequently its Complement *S L*, the Latitude of the Star ſought. The Angle *P B S* alſo will be found, or *C B L* the next to it, and its meaſure the Arc *C L*, and therefore its Complement *L E*. But this Arc either is the Longitude itſelf, which is ſought, or its Complement to one, two, three or four Quadrants.

In like manner, by the ſituation of a Star in reſpect of the Ecliptic being given, its ſituation,

tion, in regard of the Equator will be determined.

PROPOSITION XXVIII.

THE Longitude and Latitude of two Stars, and the Diſtance of the third from both being given, to find the Place of that third.

Let *CED* (in *fig.* 11.) repreſent the Ecliptic, whoſe Pole is *P*; *A* and *B* the Stars, whoſe Longitude and Latitude are given, thro' which let the Circles of Latitude *PAC*, *PBD* be imagined to be drawn: The Arc of the Ecliptic *CD* is the meaſure of the Angle *CPD*, contained under them, the difference of the given Longitudes, and conſequently is given. In the Triangle *APB*, the Sides *PA*, *PB*, the Complements of the Latitudes *CA*, *DB*, and the Angle *APB*, being given, the Side *AB*, and Angle *PBA* will be found. Again, in the Triangles *ASB*, all the Sides being given, namely *SA*, *SB*, the diſtances of the Star *S* from the two Stars *A* and *B* being given, and *AB* being firſt found, the Angle *ABS* will be found, and conſequently *SBP*, the difference of this and *PBA* found before: And laſtly, in the Triangle *SBP*, two Sides *BP*, *BS*, with the Angle *SBP* contained between them, being given, *PS* will be found, and conſequently its Complement *ES*, namely the Latitude of the Star *S*; and the Angle *BPS*, and its ſure *DE*, namely the difference of the Longitudes of the Stars *B* and *S*, to be added to the Longitude of the Star *B*, that the Longitude of the Star *S* may be had, if *PSE* be more to the Eaſt than *PBD*; but to be ſubtracted, if more to the Weſt. There are other caſes of this Problem, according to the various

ous ſituation of the points *A* and *S*, but they are ſolved after the ſame manner.

And after the ſame manner, if the ſituation of the Stars *A* and *B* in reſpect of any other Circle, for inſtance the Equator, be given; that is, if their Right Aſcenſions and Declinations, and Diſtance of the third from both be given; the ſituation of it, in regard of that Circle, will alſo be given. And theſe things are done on the Earth after the like manner.

PROPOSITION XXIX.

TO deſcribe the method of making a Catalogue of the Fix'd Stars; and to give an account of the chief Perſons who have done ſo, and the methods they uſed; and to make a Celeſtial Globe.

Let every ſingle Star of a Conſtellation be deſcribed in order, according to the place it has in the Conſtellation; and over againſt each ſet down the Longitude and Latitude thereof, found by *Prop.* 27, and then annex its Magnitude. If there be ſeveral Stars in the ſame part of the Conſtellation, (for inſtance, in the head or the hand) let them in the deſcription be diſtinguiſhed into Northern and Southern, and into ſuch as precede, are in the middle or follow; where ſuch as are ſituated toward the Weſt are ſaid, by ſuch as follow *Ptolemy*, to precede; or ſuch as poſſeſs the preceding parts of the Zodiac, *i. e.* nearer to the beginning of Aries; that is, ſuch as precede in the Diurnal motion: They are alſo diſtinguiſhed by ſome by the Letters of the Alphabet annex'd. If there be any remarkable Star in the Conſtellation, which has a peculiar name, this name alſo is put down; as *Arcturus* in Bootes, *Regulus* in Leo, *Syrius* in the Great Dog, *&c.* After the Stars proper to each Conſtellation, the unformed Stars about it

it are annexed, described after the like manner by the nearness of this or that part (about which they stand,) together with their Longitude, Latitude and Magnitude.

The first that attempted to reduce all the Fixed Stars into a Catalogue, and determine their Places, was *Hipparchus of Rhodes*, about 120 Years before Christ, who, in *Pliny*'s judgment, *dared to do a thing that God himself did not approve of, to tell the number of the Stars for Posterity, and reduce them to a Standard.* Tho' 'tis certain from *Ptolemy* that *Tymocharis* and *Aristyllus* left several observations about this matter, made 180 Years before *Hipparchus*. *Menelaus* also the Geometrician, in the first Year of *Trajan*, observed some of the places of the Fix'd Stars at *Rome*. After these, *Claudius Ptolemy*, about the 140th Year of Christ, began to observe and commit to Writing, some things about the progress of the Fixed Stars *in consequentia*; the Catalogue of *Hipparchus*, which rises to 1026 Fix'd being still retained in regard of the place as to Longitude and Latitude; nor have we any Observations of the Ancients besides these, which are delivered to us by *Ptolemy*. *Albategnius* the *Syrian* has done the same about the 880th Year after Christ: For in the Book that he has given the title of the *Science of the Stars*, Chap. 52. Pag. 202. he says that he has added 11 degrees and half and a third part to the places in which he found them to be described in *Ptolemy*'s Book. For neither the *Persian* nor *Alphonsine* Astronomers, nor *Copernicus*, who is later than any of them, observed all the Fix'd Stars, but rested satisfied with the Catalogue of *Hipparchus*, only adding in the respective Catalogues the portion of the Motion, which was made in the mean while from them all; of which we shall speak here-

hereafter. *Copernicus* reckons the Longitudes of all the Stars fromthe preceding of the two in the Ram's Horn, which he calls the *First of all*, and which, by consequence, are the same in all Ages, tho the Longitude of this first Star from the Equinoctial point continually increases.

All these Astronomers in the determining the Places of the Fix'd Stars, for the most part, made use of an Armillary Sphere, or a part of it, (which they called an *Astrolabe.*) Its description may be seen in Chap. 1. Book 5. of *the Great Construction.* This Instrument being fitted to the Latitude of the Place and the Cardinal points of the World, they placed the Armilla or Hoop representing the Ecliptic, in the day time, by the help of theSun's Rays, (whose place they knew by the Theory of it,) in such a situation as the Ecliptic had then, in regard of that Horizon; then they observed the place of the Moon in the Ecliptic, by the help of a movable Circle of Latitude in the Astrolabe. The next Night, by the help of the Moon, (whose place found before, they corrected according to its motion performed in the mean while,) they placed this Zodiacal Armilla in such a situation as was agreeable to the present moment of time, (as before they had done by the mediation of the Sun;) thus they observed the Longitude of each Star reckon'd according to the Ecliptic, as they had before observed in the day time the Longitude of the Moon (looking along the moveable Circle of Latitude,) and its Latitude by the help of a moveable Sight in the said Circle. You may see this method of Observing, fully described in Chap. 4. Book 7. of the *Great Construction of Ptolemy*, and in *Chap.* 14. *B.* 2. *of Copernic. of the Revolut.*

Tho'

Tho' the Ancients thought the Moon was absolutely neceſſary to determine the places of the Stars in reſpect of the Equinoctial and Solſtitial points, and together with theſe the moſt ſagacious *Copernicus*, Chap. 14. B. 2. ſays thus: *For without her there would be no way to comprehend the places of the Stars, becauſe ſhe alone of them all partakes of Day and Night*: But *Cardan*, in a Book, which he entitles *Supplementum Almanack*, (after he had told us that the Fix'd Stars were to be compared with the Moon during a Lunar Eclipſe, becauſe the Moon's Place in reſpect of the Sun, is at ſuch a time beſt known, becauſe oppoſite to the Sun's,) very happily ſubſtitutes Venus inſtead of the Moon in this affair, becauſe ſhe is both a day and a night Star, and has ſome other advantages, mention'd in the *Schol.* of *Prop.* 25.

The next after *Hipparchus*, that obſerved the Fixed Stars themſelves a-new, was *Ulugh Beigh*, the Grandſon of *Tamerlane the Great*. He ſays in the Preface, that he obſerved all that could be obſerved, beſides 27 in the South. This Prince died about 1449, and ſet down in his Catalogue (tranſlated into Latin by Mr. *Tho. Hyde*) the Stars places for the Year of Chriſt 1437.

At length in the 16th Century appear'd the *Atlas* of that Age, the immortal *Tycho Brahe*, who by indefatigable labour and pains obſerved a-freſh all the Fix'd Stars that are eaſily ſeen above the Horizon of *Uraniburgh*. And at the ſame time *William* the moſt illuſtrious *Landgrave* of *Heſſe*, attempted the ſame great Work at *Caſſel*, and with his Mathematicians *Rothman* and *Byrgius*, continued in it for above 30 Years. He ſought for the Longitude and Latitude of the Fix'd Stars by Calculation, from the Right Aſcenſion and Declination firſt obſerved by the method in

Prop. 27; as *Tycho* assures us in *Chap.* 2. *Part* 1 *Progymn. pag.* 147; tho' he does not intirely approve of it, because of the difficulty of reckoning the time exactly from Noon, which tho often times tried by himself, by making Clocks and other ways described in the before quoted Ch. of the *Progymnasmata*, yet never succeeded according to his wishes. But tho' the Observations of the Prince don't extend to above 400 Fix'd Stars, and were made by smaller Instruments, yet *Hevelius* in *Prod. Astro.* prefers them to the labours of *Tycho* himself, because in his opinion, the Prince and his Observers used more diligence than *Tycho* and his fellow Observers. The Prince afterwards performed this business by the help of Venus; but *Tycho*, tho in *pag.* 146. *B.* 1. *Progymnasm.* he charges *Cardan* with an error in settling the place of the more Southern and Bright Star of the Scales of Libra by Venus, greater than it is in the *Alphonsine Tables*; neverthelefs confesses, that it arose from hence, that the situation of Venus was not at that time well determined by Numbers. And he himself uses Venus for the same purpose after the manner described in *Prop.* 25. Instead of a Zodiacal Armilla used by the Ancients in observing the different Longitudes of the Fix'd Stars and their Latitudes, *Tycho* for very good reasons, substituted the Æquatorial Armilla by which he might observe the differences of the Right Ascensions and the declinations out of the Meridian, the Meridian Altitude being always made use of to confirm the others. All the other cautions which this most diligent Observer took in this Affair, may be seen in *Chap.* 2 *B.* 2. *Progymn.* The Places of some of the Fix'd Stars being once laid down, found by the methods already delivered, he determined the Pl-

ces of the otherStars which were near the Ecliptic or Equator, after the manner related in *Prop.24.* He sought the Right Ascension of any unknown Star by two, whose places were already verified, and if the Right Ascensions of the same Star thus found, were a little different from one another, he always took the mean for the true one without any hesitation: Afterwards from the Right Ascension and Declination of a Star he computed its Longitude and Latitude. As for the Stars about the Poles he deduced them from the former by the the Praxis described in *Prop.* 28; tho' he verified the Place thus found by observing its distance from a third Fix'd Star first determined. And universally, he never rested satisfied with the Place of a Fix'd Star determined only by one observation; but each of them by the methods described before, by his Equatorial Armilla, and by the Praxis laid down in *Prop.* 23, making use of the best Clocks for measuring the time, Quicksilver and Lead being Chymically reduced into powder, and shewing the time by its running thro' a little hole; all which you may see accurately describ'd in the *Progymnasmata.*

Tycho's Catalogue of 777 Fix'd Stars made with such care and diligence as this for the Year 1600, and taken from the Heavens themselves, at last came forth, in the Year 1610, in *Astron. Instaur. Progymnasm.* And afterwards in the Year 1627, copied into the *Rudolphine Tables,* and increased by 223 Stars from some other observations of *Tycho*; so that this Catalogue in the *Rudolphine Tables* rises to the number of a thousand Stars. *Kepler* has added a second Class of Fixed Stars, that *Tycho* left out of the ancient Catalogue of *Hipparchus,* repeated and corrected by *Ptolemy,* and calls them *Semi-Tychonic*: He

ſubjoins a third containing 12 Celeſtial Images which are inviſible in our Northern temperat Zone. Theſe he has put into his Catalogue being named by the *Portugueſe*, and corrected by *Peter Theodori.* 'Tis generally reported that theſ Southern Conſtellations were afterwards ob ſerved by one *Frederick Houtman, a Dutchman* and the *Celeſtial Globe of Bleau* corrected accord ing to thoſe Obſervations. But the Celebrate Mr. *Edmund Halley* was the firſt that took tha pains about them as became an Aſtronomer while he was at St. *Helena*, and publiſh'd a Ca talogue of 350 Southern Fix'd Stars to the Yea 1677 compleat, which is a true *Supplement* to *Tycho* *Catalogue.* He obſerv'd the Diſtances of th unknown Southern Stars from ſome of *Tycho's* and calculated their places from the aſſume places of ſome of the *Tychonic* ones (by *Prop.* 28. adding alſo the diſtances themſelves, which h had obſerved.

The next after *Tycho*, that preſented th World with an intire Catalogue of the Fix' Stars of his own, was *R. P. Ricciolus*, in *Aſtron Reform. Lib.* 4. to the Year of Chriſt 1700 com pleat. But this Catalogue containing as man Stars as *Tycho's*, is taken from *Tycho's*, exceptin in 101 Stars, which are all that he had obſer ved in the Heavens. In the others, he recede a little from *Tycho*, and has copied *Tycho* manifeſt errors into his own, retaining at th ſame time ſome Stars as were really obſerved b *Tycho*, but were not viſible in the time o *Ricciolus*, as *Hevelius* has obſerved in *Prodr. Aſt.* *p.* 133. and the following.

Laſt of all came out a *Catalogue of the Fix' Stars*, by the Illuſtrious *Jo. Hevelius* of *Dantzick* containing 1888 in all; namely 950 know by the Ancients, 603 which he calls his own

becau

because they were determined by none before himſelf as they ſhould be by proper Inſtruments, and the 335 of Mr. *Halley*, inviſible in the Horizon of *Dantzick*. In the making of this he ſolemnly profeſſes that he had no regard to the obſervations of any Author, but entered upon the work, as if there were no Catalogue at all in being. Nay, he intirely laid aſide all other foundations, and compoſed a-new the Theory of the Sun and Solar Tables, founded intirely upon his own obſervations. And laſtly, he ſincerely declares, that he has borrow'd nothing of any body, or put into his Catalogue the Place of the ſmalleſt Star, that he himſelf had not obtained by his different Diſtances and Meridian Altitudes.

Hevelius drew up two ſuch Catalogues, the firſt or larger, containing as well the Longitudes and Latitudes, as the Right Aſcenſions and Declinations, to the Year of Chriſt 1660 compleat; to which are annexed the Places as to Longitude and Latitude, reduced to the ſame Year, of the Fix'd Stars obſerv'd formerly by *Tycho Brahe*, the Prince of *Heſſe*, *Ricciolus*, *Ulugh Beigh* and *Ptolemy*; the ſecond or leſs, exhibiting the Places of the ſame Fix'd Stars as to Longitude and Latitude, to the Year of Chriſt, 1700 compleat; to which he has annexed Mr. *Halley's Catalogue of the Southern Stars*, partly corrected as to their Longitude, from obſervations at *Dantzick*, and reduced to the ſame time: So that the larger Catalogue preſents us at the ſame time with the Catalogues of all his Predeceſſors; by which means it is evident at ſight, how far the obſervations of them all, agree or diſagree. To theſe he ſubjoins a Table of the Right Aſcenſion and Declination of ſeveral Fix'd Stars

to the Years of Chriſt 1660 and 1760, with their differences for that hundred Years.

By the help of ſuch a Catalogue of the Fix'd Stars as this,a CeleſtialGlobe may be eaſily made: For the Ecliptic and its Poles being marked out upon the ſurface of the Globe, let the places of the Fix'd Stars taken out of the Catalogue be in like manner marked out according to their Longitude and Latitude in regard of that Ecliptic, draw alſo the tract of the Milky Way, thro' the places of the Fix'd Stars, thro' which it paſſes; and add the Equinoctial Circle, cutting the Ecliptic in the point aſſumed for the beginning of the the reckoning, and the oppoſite point, and duly inclined to it; as alſo let the Polar Circles and Tropics, and the other neceſſaries be added; of which we have ſpoken before in *Prop.* 13. And to this very purpoſe did *Hevelius* deſign his leſſer Catalogue, and from hence did he deduce his *Uranography* or Tables of all the Stars drawn upon a Plane, which was publiſh'd in theYear 1690,under the Ti le of the *Firmament of Sobieski.* For they differ no more from a Celeſtial Globe than Maps do from the Terreſtrial. Theſe *Uranographic* Schemes of the Conſtellations are delineated *in Plano*, juſt as if the convex part of the Heavens were ſeen by us, as it is in the Artificial Globe, as was uſual among ſuch as went before him, and made Catalogues of the Fix'd Stars. He ſays that *Bayer*, who in his *Uranometry* attempted the contrary, *thoughtleſly inverted all the Stars*; and that, *by his Figures he did but diſorder the whole buſineſs of the Heavens*; and that, *all the Stars which are placed on the left hand in the Globes, appear on the right hand.*

The making of a Celeſtial Globe by *Hevelius's Firmament* is much eaſier,becauſe the Figures of

of the Conſtellations are very aptly delineated in it, together with the Milky Way, and its different Streams as it were. But we are to expect the moſt ample Catalogue of the Fix'd Stars from Mr. *Flamſteed*'s Obſervations, which have been continued for ſeveral Years with this view, and made with large Inſtruments, and at the ſame time very fit for that purpoſe.

PROPOSITION XXX.

To give an account of ſome of the more remarkable Changes as have happen'd among the Fix'd Stars.

So long ago as the Age of *Hipparchus*, a new Star appeared among the Fix'd ones. And this, as *Pliny* informs us, *was the reaſon why* Hipparchus *ſet upon numbering the Fix'd Stars for the uſe of Poſterity, and reducing them to ſome Rule, having invented ſome Inſtruments for obſerving the Places and Magnitudes of each; ſo that it might eaſily be known by this, not only whether they periſhed or had a beginning, but whether any at all paſſed away or had any Motion; as alſo whether they increaſed or decreaſed, the Heavens being left in common to them all.*

And tho' *Ricciolus* reckons no leſs than ſix new Stars between this and that famous one, which appeared in *Caſſiopeia* in the Year 1572; yet ſince they were either ſcarce obſerved at all, or by Perſons not over-skill'd, there hardly remains any thing to be ſaid of them.

This new Star in *Caſſiopeia*, a rival to the biggeſt in the Heavens, firſt appeared about the beginning of *November*, in the Year 1572, and continued until *March* in the Year 1574. This, juſt as that of *Hipparchus*, was the occaſion of *Tycho*'s obſerving the Fix'd Stars again, with new and large Inſtruments; as he informs us himſelf in the *Progymnaſm.* where he treats expreſly of this Fix'd Star.

In *August* 1596, *David Fabricius* discover'd a New one of the 3^d^ magnitude in Cetus, which disappeared about two months after.

In the Year 1600 a New Star appeared in the breast of Cygnus, first seen by others, but afterwards by *Kepler* himself, who has written an Astronomical account about it. This continu'd of the same magnitude *viz.* the 3^d^ until the Year 1659, as *Hevelius* assures us; (who had observed it from the Year 1638, with large Instruments,) but from the Year 1659 it sensibly decreased; and about the latter end of the Year 1661, it intirely disappeared. Yet five Years after, namely in *September* 1666, *Hevelius* observ'd it again with his naked Eyes, of the 7^th^ or 6^th^ magnitude, retaining the same place in the Heavens precisely.

In the Year 1604, about the beginning of *October*, a New Star was seen in the right Foot of Serpentarius, which appeared bigger than Jupiter and almost equal to Venus: It continued a whole Year; because it was not seen after *October* in 1605. Concerning this *Kepler* has written a Book full of Astrological and Physical disputes.

It was the Milky Way that presented us with these three New Stars, namely in Cassiopeia, Cygnus, and Serpentarius; it has therefore on that account been called by some, the Fund of new Stars.

There have been some other New Stars observed, namely in the Year 1612. that in the belt of Andromeda, by *Simon Marius*, who is the Author of *the World of Jupiter*, and another in Antinous by *Justus Byrgius* and others.

In the Year 1638, *John Phocyllides Holuuarda* of *Franequer* observed a New Star in the Whale's neck of the 3^d^ magnitude, or bigger. How it disap-

disappeared every Year, and then appear'd, but not always exactly at the same time, (tho' *Bullialdus* thinks otherwise,) and afterwards was invisible for four Years, is fully evident, from the History of it written by *Hevelius* and annexed to his *Mercury*, and from his *Climacteric Year*. Mr. *Cassini* has found by Observation, that the same Phases of this Star return at the end of 330 days; but so as that this return is sometimes retarded or anticipated by 15 days. This Star seems to be the same with that, which *Fabricius* observed in the Year 1612; for their Places are the same.

In the Year 1670 in *July*, *Hevelius* discovered a New Star of the 3^d magnitude under the Swan's head, and in his Catalogue reduces it to his Vulpecula; but it was lessen'd so much in a short space of time, that about the end of *August* 1671 it was intirely extinguished: But next *March* it appeared again, at first like a very small Star, but by degrees it increased to one of the third magnitude, and afterwards grew less again sensibly as it did before; and in *September* 1672, totally disappeared, and was seen no more.

Not only New Fix'd Stars now-a-days make their first appearance, but others of the sixth, fifth and fourth magnitude, known by the Ancients and observed by *Tycho*, have intirely vanished, instances of which may be frequently met with in the *Catalogue of Hevelius*. He himself mentions four in *Prod. Astr.* namely a Star in the left thigh of Aquarius, the contiguous one preceding in the Tail of Capricorn, the second of the Belly of the Whale, and the first of the unformed ones after the Scales of Libra. There are also other Observations which have been made about them by the *French* Astronomers.

mers. And even ſome conſiderable Fix'd Stars of the firſt, ſecond and third magnitude are found to alter their brightneſs and magnitude ſenſibly: as is evident from the different ſentiments of Authors about Stars of the firſt and ſecond, or of the ſecond and third magnitude, That Poſterity may be more capable of judging about the change of the more conſiderable Fix'd Stars with greater eaſe and leſs miſtake; *Hevelius* in his *Prodromus* has drawn up ſeveral things tending to that purpoſe obſerved by himſelf.

I paſs over other changes in the Fix'd Stars obſerved by the help of a Teleſcope, ſuch as are thoſe ſaid to be diſcovered lately by Mr *Caſſini*; namely that ſome, as for inſtance, the firſt of Aries and the preceding head of Gemini, ſometimes appear divided each into two equal to one another, and diſtant from one another the length of the diameter of either of them; others, as ſome of the Pleiades and the middle in Orion's Sword, appear triple, nay quadruple ſometimes. For theſe Phænomena, beſides that they are not to be ſeen by the naked Eyes (of which we only treat at preſent,) ſeem to have another original, and conſequently are to be conſidered in another place.

PROPOSITION XXXI.

TO *define the true preceſſion of the Equinoctial Points, or the motion of the Fix'd Stars* in conſequentia; *and to determine from a given Catalogue of the Fix'd Stars, the place of any Fix'd Star at a given time.*

Hipparchus of old, as *Ptolemy* informs us, Ch. 1. Book 7. *of the Great Conſtruction,* from a compariſon which he made of his own Obſervations with thoſe of *Ariſtyllus* and *Tymocharis,* ſuſpected that the Stars of the Zodiac had a motion

tion *in consequentia*: But *Ptolemy* from a comparison of the Observations of *Hipparchus* and of others, with his own, has expresly asserted it of them all, and that it is made in Circles parallel to the Ecliptic; which he proves in several of the more considerable Fix'd Stars, Chap. 2, and 3, comparing their places in regard of the Equator observed by such as lived before him and by himself. And he finds that this progressive motion is one degree in an hundred Years. *Albategnius* from the places of *Regulus* observed by *Menelaus* and himself, at the distance of 785 Years, gathers in Chap. 52 of his *Book of the Knowledge of the Stars*, that the motion of the Fix'd Stars is one degree in 66 Years. *Ulugh Beigh*, comparing his Observations with such as were made a great number of Years before, determines in the *Preface to his Tables*, that the Fix'd Stars move one degree in every seventy Solar Years. Later Astronomers comparing the Observations of the Ancients and Moderns, and the intermediate ones of *Albategnius* about this matter, find that this motion is equable, quicker than *Ptolemy* believed, but slower than *Albategnius*, and nearly the same with what *Ulugh Beigh* assigns. For *Tycho* in his *Progymn.* Book 1. gathers, that it is one degree and 25′ in a hundred Years, or 51″ in one Year. *Copernicus*, tho' he looked upon this motion to be unequal, made the mean to be, from the Observations of the Ancients and his own compar'd, to be about 1°. 23′. 40″. 12‴ in 100 Years. *Ricciolus* makes it something less, namely 1°. 23′. 20″. in that time, which Mr. *Flamsteed* retains. *Street* in his *Astronomia Carolina* makes it still less, as not above 1°. 20′. *Bullialdus* in his *Astronomia Philolaica* makes it 1°. 24′. 54″ in 100 Years; whilst *Hevelius* lately in his *Solar. Tab.* in *Prodrom.* will have it to be only 1°. 24′. 46″. 50‴.

For

For every Year therefore, forward from the the Year to which the Catalogue is fitted, add 50″ (the middle round number) to the Longitude of the Fix'd Star found by the Tables, and you will have the Longitnde ſought: The Latitude in the mean while continues invariable, ſince both the Fix'd Stars and the Ecliptic in the Heavens are immovable, and only the Equator (originally a Terreſtrial Circle) being looſen'd as it were leaves its ancient ſeat. But for every ſingle Year, backwards in reſpect of the given Catalogue, ſubtract 50″ from the Longitude in the Tables, if you would find the Longitude for ſuch a given time. But if the order of the given Catalogue be ſuch, as that the Longitudes of each of the Fix'd Stars are reckon'd from the firſt Star of Aries, (as it is in *Copernicus*'s Catalogue of the Fix'd Stars, as alſo *Clavius*'s, and of ſome annexed to the *Caroline Tables*,) to the number of the Longitude in the Tables, (becauſe continuing always the ſame) add the Longitude of the firſt Star of Aries (which at the beginning of the Year 1701, according to *Hevelius* is 29°. 00′. 58″, and it is to be increaſed by 50″ every Year afterwards, but leſſened by as many for every Year before, as was ſaid before) for the time given, and you will have the Longitude ſought from the Vernal Equinox, caſting away 360 degr. if the ſum exceed that. But if the Right Aſcenſion and Declination at a time given be required, what is propoſed may be done by the converſe of *Prop.* 27. from the Longitude and Latitude found for the given time, as before.

SECTI-

SECTION VI.

Concerning the resolution of the more considerable Problems of the First Motion by Calculation.

PROPOSITION XXXII.

TO determine by observation the difference of the Meridians or Longitudes of two Places given upon the Surface of the Earth, and to make a Catalogue of the most noted Places of the Earth, with their Latitudes and the Differences of their Meridians, and to make a Terrestrial Globe.

Let any instantaneous Phænomenon (as the beginning or end of a Lunar Eclipse, or of an Eclipse of any of Jupiter's Satellites,) be observed by Observers in the given Places, noting the hour in both Places: The difference of the hours converted into an Arc of the Equator gives the Difference of the Longitudes or Meridians sought. And that of the Places proposed is more toward the East, that reckons more hours after Noon; But if they reckon the same, they are under the same Meridian. For since the same moment of time has differing names as to the hour in these two Places, the Noons of these Places or any other hours of the same name, will be as distant from one another, as these different hours in the same Place; and consequently the Arc of the Equator intercepted between the Meridians of the Places is rightly found by allowing 15° for every hour. This interval of Time is called the *Difference of the Meridians*, appositely enough, because it is so of the Noons.

In

In like manner, if a Clock that ſhews exactly the hours at one of the Places be carried to the other, to be compared with another ſhewing the hours there; the Difference of the hours reckoned by the Clocks, converted into an Arc of the Equator, gives the Difference of the Longitudes ſought: For thus, as in the former caſe, you know the interval between the hours reckoned at the ſame moment in the two Places. For the Clock, that is tranſported, is ſuppoſed to ſhew the hours after the ſame manner, as it would have done, had it continued in its place.

To each of the more noted places that are put into a Catalogue, add the Latitude proper to it, expreſſed by the ſame number of degrees (by *Prop.* 7,) as is the Altitude of the Pole obſerved there by *Prop.* 17. Add alſo in another column the Difference of the Meridians expreſſed by hours and their parts (found by what has been ſaid before) of the Place propoſed and of the Place aſſumed, together with the mark of Addition or Subtraction, according as the Meridian of the one lies to the Eaſt or to the Weſt of the Meridian of the other.

The Latitudes of the Places and the Differences of the Meridians being given, 'tis eaſy to make by them a Terreſtrial Globe: For the Poles and a great Circle upon them being drawn upon a Sphere made for that purpoſe, which will repreſent the Poles of the Earth and the Equator, thro' the Poles draw a Circle repreſenting the Meridian of the Place (to which the Meridians of the other Places in the Catalogue are referred,) and in it mark out a point diſtant from this feigned Equator, as many degrees as there are in the Latitude of the Place ſet down in the Table; and that point will

there-

therefore represent the said Place. Any other Place is to be put in a Circle passing thro' the Poles, between which and the Meridians already settled, there are as many degrees of the feigned Equator (and that according as the second Place is, towards the East or West,) as there are in the tabular difference of Longitudes found above; and as many degrees from the feigned Equator towards either Pole, as there are in the tabular Latitude of that Place. In like manner, the other more noted Places are to be marked upon the surface of the Globe. The intermediate or less noted Places are supplied by the common Chorography, and Itineraries, making use of the Latitude of some for correction; for this is more easily observed than the Difference of Longitude. The Globe being thus finished, either the Meridian of our own place, or any other passing thro' the Azores or any of the Fortunate Islands, may be reckoned for the first; that is, the degrees of the Equator are to be reckoned from the intersection of it with the said Meridian (or beginning) towards the East. Then the other Circles and necessary appurtenances described in *Prop.* 14. are to be added according to art.

PROPOSITION XXXIII.

THE Elevation of the Pole, the Right Ascension and Declination of a Star, or any other point in the Heavens being given, to find its Ascensional Difference, and by that means its Oblique Ascension, Continuance above the Horizon, and Eastern Amplitude.

Having laid down in what goes before, the method of determining the situation of the Circles of the Sphere for any habitation or place, and of finding by observation the places of the

Stars

Stars in regard of the ſaid Circles in the Celeſtial Sphere, and the ſituations of Places on the Earth in reſpect of the ſame, conſidered on the ſurface of the Earth; we ought now, by the rules of Method, to ſhew how to reſolve by calculation ſome of the chief remaining Problems, about what has been determined and found out already, and according to their Example may any others relating to the Firſt Motion, be eaſily reſolved after the ſame manner.

In the preſent caſe, let *E Q* (*Fig.* 12.) repreſent the Equator, whoſe Pole is *P*; *HOR* the Horizon, whoſe common interſection's, O and the oppoſite point, are the points of true Eaſt and Weſt, namely the Poles of the Meridian *PEQ*. Let *S* be a Star riſing or ſetting, and thro' it imagine a Circle of Declination *PSA* drawn. In the Triangle *OAS* there are given, beſides the right at *A*, the Angle *SOA*, whoſe meaſure is *HE*, the Altitude of the Equator; the ſide *AS*, the Declination of the Star given; conſequently *OA*, the Aſcenſional Difference ſought, may be found: For *O* is the point of the Equator that arrives at the Horizon together with the Star; *A* is a point of the ſame that arrives with it at the Meridian. This Aſcenſional difference therefore ſubtracted from the Right Aſcenſion of the Star declining towards the elevated Pole, but added to the Right Aſcenſion of the Star declining towards the depreſſed Pole, gives its Oblique Aſcenſion; and ſo on the contrary in the Oblique Deſcenſion. But this ſame Aſcenſional Difference added to the Quadrant, or ſubtracted from the ſame, gives the ſemidiurnal Arc of the Star, according as it declines towards the elevated or depreſſed Pole: And that turned into Time, gives half the continuance of the Star above the Horizon. In the Triangle *OAS* the ſide *OS* is alſo found, the Eaſtern or Weſtern Ampl[illegible]

By

By reſolving the ſame Triangle OAS after the ſame manner may the converſe of this Problem be ſolved; that is, the Right Aſcenſion, Declination and Continuance above the Horizon of any Star or Point being given, to find the Elevation of the Pole.

PROPOSITION XXXIV.

THE Elevation of the Pole being given, if the Altitude of a Star be known by obſervation, whoſe Right Aſcenſion and Declination are known, to find the moment of Time, and the Azimuth of the Star.

The ſame Circles of the Sphere being expreſſed by the ſame letters, let the Vertical ZSA (Fig. 9.) be ſuppoſed to be drawn thorough the Star S. In the Triangle PZS, all the ſides are given; namely PZ, the complement of the Elevation of the Pole; ZS, the complement of the Altitude of the Star above the Horizon; and PS, the diſtance of the ſame Star from the Pole, or the complement of the Declination: The Angle PZS therefore is found, which is meaſured by OA the Azimuth ſought; as alſo the Angle ZPS, whoſe meaſure is the Arc of the Equator ER. But the Sun's place being given, its Right Aſcenſion is alſo given: Conſequently the difference between this and the Right Aſcenſion of the Star, known alſo by ſuppoſition, is alſo given: Therefore the ſum or difference of theſe Arcs will be given; namely the Arc of the Equator intercepted between the Meridian PEH and the Circle of Declination drawn thro' the Sun, which converted into time, gives the time before or after Noon, according as the Sun is to Eaſt or Weſt of it.

You will have the ſame, if inſtead of the Altitude of a Star its Azimuth be known by obſervation.

PROPOSITION XXXV.

THE Longitude of the Place, the Hour of the day, and the Sun's Place being given, to determine the situation of the Ecliptic in respect of the Horizon; that is, to find the Angle of the Ecliptic and Horizon, (or the Altitude of the Nonagesim degree of the Ecliptic from the rising point above the Horizon,) the rising point of the Ecliptic (and from thence the Nonagesim degree) and the point of the Horizon, in which the Ecliptic intersects it.

Things remaining as before, let *S* be the Sun in the Ecliptic *MSEA* [*Fig.* 13.] Let the right Ascension for the given hour be found (by *Prop.* 20.) or (making *PSB* the Circle of Declination,) the point *B* in the Equator, or consequently its distance from *E* the next Equinoctial point, namely *EB*. And since the Hour is given, the Arc of the Equator *B*♋ intercepted between *B* and the Meridian, and consequently *BO* its complement to the Quadrant of the Equator ♋*O*: But *BE* is given, from whence the Arc *EO* is known. Therefore in the Triangle *AEO* comprehended between the Ecliptic, Equator and Horizon, there are given the Angle *AEO*, the obliquity of the Ecliptic, and *AOE* the inclination of the Equator to the Horizon, and the side *EO* just found: From whence the rest will be known; of which *EAO* or its complement to two right *MAH* is the Angle that the Ecliptic makes with the Horizon, measured by the Altitude of the Nonagesim degree; *EA* the distance of the rising point of the Ecliptic from the Equinoctial point *E*, and from thence the highest point, or the Nonagesim degree from the rising point *A* becomes known; and *OA* the distance of the rising point *A* from the true East *O*, computed in the Horizon. *Q. E. F.*

PRO-

Plate 3. Book 2.

[Fi]g. 13.
[p.] 322.

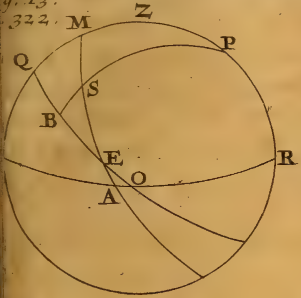
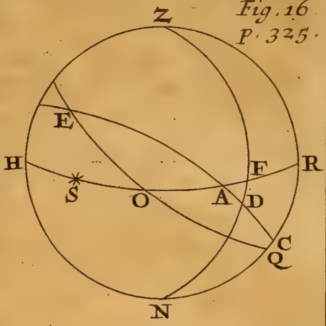


Fig. 16.
p. 325.



[Fi]g. 14.
[p.] 323.

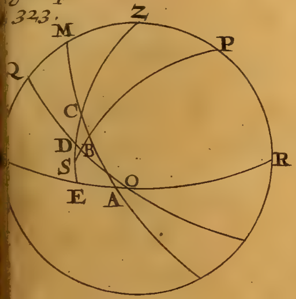
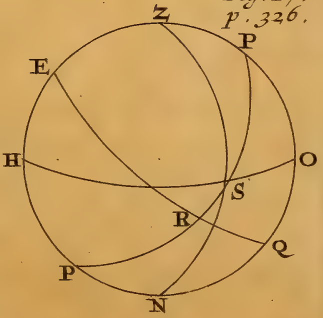
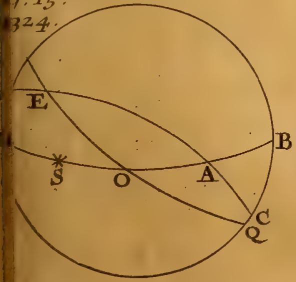


Fig. 17.
p. 326.



[Fi]g. 15.
[p.] 324.



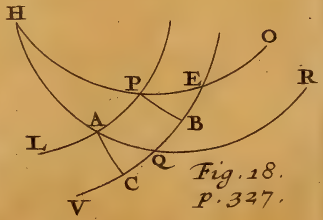


Fig. 18.
p. 327.

PROPOSITION XXXVI.

THE *same things being given, to find the situation of a Star, whose Longitude and Latitude are given, in regard of the Horizon, or the Altitude and Azimuth; as also to determine the point and Angle, in which the Vertical thorough the Star intersects the Ecliptic.*

Let the place of the Star *S* given in regard of Longitude and Latitude (by the converse of *Prop.* 27) be reduced to the Equator; is, find its Right Ascension and Declination. And since the Sun's Right Ascension is given, the difference of these Ascensions will also be given. But from the Hour of the day given, the Arc of the Equator intercepted between the Meridian and Circle of declination drawn thro' the Sun is known: The Arc therefore (*Fig.* 14.) *♋B* of the Equator, between the Meridian and *P S* the Circle of declination passing thro' *S*, is also known; and consequently the Angle *♋P B*, which it measures. In the Triangle *Z P S*, there are given the sides *P Z*, *P S*, (the one the complement of the Pole's Elevation, the other the distance of the given Star from the Pole, known by its Declination,) together with the Angle *Z P S* just now found; by which means there will be known the side *Z S*, and the Angle *P Z S*: *Z S* is the complement of the Altitude *E S*, and *R E* the Arc of the Horizon, which is called the Azimuth, measures the Angle *PZS*; and since *R O* is a Quadrant, the Arc *O E* the distance between *O* the point of the true East and the Vertical *Z E* drawn thro' *S*, is also found. Besides, in the Triangle *E C A*, comprehended between the Vertical *Z S E*, Horizon *HOR*, and Ecliptic *ACM*, the Angles at E and *A* are given, (the one being found above,

 and

and the other a right angle,) together with the intermediate ſide *E A*, the ſum or difference of the known Arcs *O E*, *O A*: Therefore the other ſides are alſo given, namely *A C*, the diſtance of the point *C* from the riſing point *A*, which is known, the ſide *E C*, and the Angle *E C A*, which the Vertical *Z S E* drawn thro' the Star *S*, contains with the Ecliptic. *Q. E. F.*

Tho' this Problem be no more than the converſe of *Prop.* 26, yet becauſe of its frequent uſe, I thought fit to inſert it.

COROLLARY.

The ſame things being given, by the ſame labour may the Place on the Earth, to which the Star *S* is Vertical, be determined. For in the Triangle *Z P S*, in which all the Sides and Angles are known, the ſide *P S* denotes the diſtance of the Place on the Earth from the Pole *P*, from whence its Latitude is known; and the Angle *Z P S* is the diſtance of the Meridians of the Place given and ſought. But if *S* be in the Ecliptic, the Problem becomes more ſimple; and is ſtill the more ſo, if it be the Sun, or the Point oppoſite to it.

PROPOSITION XXXVII.

THE Elevation of the Pole and the Oblique Aſcenſion of the Star being given, to find its Coſmical and Achronical Riſing.

The Circles of the Sphere [*Fig.* 15.] being marked by letters, as oftentimes before, in the Triangle *E O A*, there is given the Angle at *E*, namely the Obliquity of the Ecliptic; the ſide *EO* the Oblique Aſcenſion of the Star *S* (at leaſt the diſtance of the point of the Equator that riſes together with it from the next Equinox, from the Oblique Aſcenſion given) and the Angle *E O A* the inclination of the Equator to the Horizon, or its complement to two

right:

right: Therefore the ſide *E A* will alſo be given; and conſequently that point in the Ecliptic that riſes with the Sun. Therefore when the Sun is in this point of the Ecliptic, the Star *S* riſes Coſmically; and the Sun being in the oppoſite point, the ſame Star riſes Achronically. And after the like manner may the Coſmical and Achronical Riſing of a Star be found from the Oblique Deſcenſion of the Star (found by *Prop.* 33.)

But if the Coſmical Riſing and Setting ſignify the ſame thing with an Aſcent above Horizon or Deſcent below it; the time of the Coſmical Riſing and Setting is determined by the preceding: For the Altitude of the Star (being nothing in its Riſing or Setting) is given, and therefore the moment of time will be known. And then the Achronical Riſing will ſignify what it commonly does; namely the Evening one. And the Achronical ſetting all one with the Coſmical ſetting, as it is commonly called, or Morning one.

PROPOSITION XXXVIII.

TO *determine the Heliacal Riſing and Setting of a given Star.*

Things remaining as they were, let *D* [*Fig.* 16.] be the place the Sun is in when the Star *S* riſes Heliacally. Therefore in the Triangle *A F D* there is given, beſide the right angle at *F*, the Side *F D*, *viz.* the Arc of the Sun's depreſſion requiſite to permit the Star of a given magnitude to appear; alſo the Angle *D A F*, which the Ecliptic comprehends with the Horizon, when the Star *S* riſes, found by Prop. 35; therefore the Side *A D* will alſo be found. But this Arc added to the Arc *E A* (found by the preceding) gives *ED* the diſtance

the Sun from the Equinoctial point *E*, when *S* rises Heliacally. Therefore the time of the Heliacal Rising of a given Star *S* will be known; namely the same as that wherein the Sun possesses *D* the point of the Ecliptic found. And after the like manner the Heliacal setting of it may be found.

PROPOSITION XXXIX.

THE *Latitude of the Place and the Sun's Place being given, to find the beginning of the Morning and end of the Evening Twilight.*

Let *ZEPN* [*Fig.* 17.] express the Meridian; *EQ* the Equator; *HO* the Horizon, whose Poles are *Z* the Zenith and *N* the Nadir; *S* the Sun depressed below the Horizon: Thro' which let the Circle of Declination *PSR* and Vertical *ZSN* be supposed to be drawn. In the Triangle *ZPS* all the Sides are given; namely *PZ* the complement of the Height of the Pole; *PS* the distance of the Sun from the Pole, known by its Place; and the side *ZS*, made up of the Quadrant and depression of the Sun, necessary for the beginning or ending of the Twilight: Therefore the Angle *ZPS* will be known, whose measure is the Arc *RE*, which converted into Time, gives the Hours sought, to be reckoned after Noon to the end of the Evening Twilight; and their complement to 12 Hours, or the Arc *RQ*, the Hours to be reckoned from Midnight to the beginning of the Morning Twilight.

PRO-

PROPOSITION XL. LEMMA.

If two great Circles of the Sphere cut another great Circle at equal Angles, the Arcs of all Circles parallel to this intercepted by the two cutting the third, are similar to one another; and the Arcs of the intersecants contain'd between two Parallels are equal.

Let two great Circles *HEO*, *HQR* [*Fig.* 18.] cut the great Circle *EQV* in *E* and *Q*, so as that the Angles *HEQ*, *HQV* may be equal. I say, the Arc *AP* of any Circle *PAL* parallel to *EQV*, intercepted between *HEO* and *HQR* is similar to *EQ*; and the Arcs *PE*, *AQ*, of *HEO*, *HQR* intercepted between the two Parallels, are equal. Thro' the Poles of *EQV*, draw the great Circles *PB*, *AC*. And the Sine of the arc *AC*, is to the sine of the arc *AQ*, so is (the Sine *AQC* to Radius, so is Sine *PEB* to Radius so is) Sine *PEB* to Sine *PE*. But the arcs *PB*, *AC*, are equal (by *Prop.* 10. *B.* 2. *Spher. Theodos.*) and therefore their Sines; consequently their Arcs *AQ*, *PE* are equal. Besides, in the Triangles *PEB*, *AQC*, in which the angles *B* and *C* (because right) and *E* and *Q* (by supposition) are equal, and the sides *PE*, *AQ* and *PB*, *AC* are already shewn to be equal, the sides *EB*, *QC*, will also be equal, and consequently the Arcs *EQ*, *BC* are equal. But *AP*, *BC* (by *Prop.* 10. *B.* 2. *of the Sphæric, of Theod.*) are similar Arcs; therefore *QE* and *AP* are also similar. And because the Circle *PAL* is taken at pleasure, the Arcs of all the Circles parallel to *EQV*, comprehended by the Circles *HEO*, *HQR*, are similar to *EQ*, and consequently to one another. *Q. E. D.*

PROPOSITION XLI.

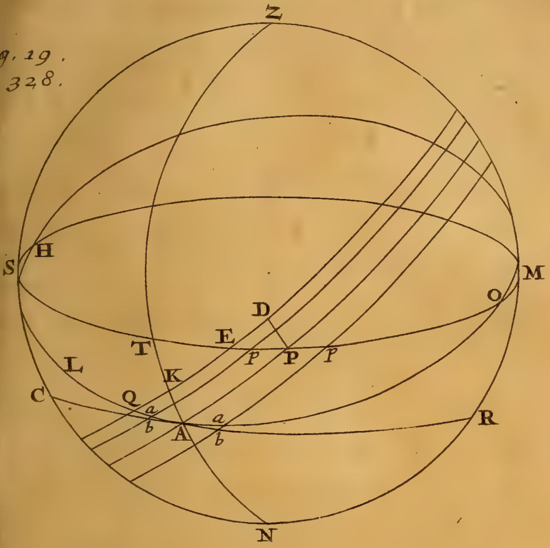
TO *find the Parallel the Sun is in, at the time of the Least Twilight.*

It has been long ago obſerved, that the proportion of the increaſe and decreaſe of the Twilight is very much different from the increaſe and decreaſe of the Artificial Days. For the Days are upon the increaſe all the while the Sun returns from the Winter Tropic to the Summer one; but the Twilights do not. For theſe are leſſen'd after the Winter Solſtice to a certain limit before the Vernal Equinox, where they are ſmalleſt; and are aftewards increaſing while the Sun paſſes the Equator and moves towards the Summer Solſtice: And from thence they are upon the decreaſe to a certain limit after the Autumnal Equinox; and then again they increaſe to the middle of Winter. Let that Parallel then be propoſed to be ſought, that is deſcribed by the Sun in its Diurnal Motion, when the Twilight is ſmalleſt at a given Place.

Let *Z S N M* [*Fig.* 19.] be the Meridian, *S E M* the Horizon, to which let a leſſer *Circle C A R* called that of the *Twilights*, be parallel, namely that wherein the Sun is at the beginning of the Morning and end of the Evening Twilight. Let *A P* be the Parallel ſought. Thro' the point *A*, where it interſects *C R*, ſuppoſe the great Circle *A Q H O* to be drawn meeting the Horizon *H O* in *O* and *H*, which cuts the Equator *E Q* in *Q*, ſo as that the angle *A Q E* is equal to the angle *P E D* or *T E K*. And therefore (by the preced. Lemma) the arc *E Q* is ſimilar to the arc *P A*; and the arc *a p* of any other Circle parallel to the Equator, intercepted between *O E S* and *O A L*, will alſo be ſimilar to the ſame *A P* or *Q E*; and each of them

Plate 4. Book 2.

ig. 19.
.328.



g. 20.
331.

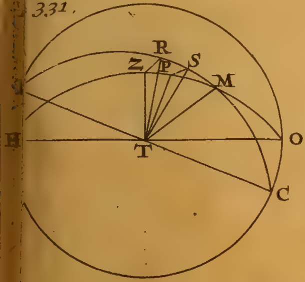
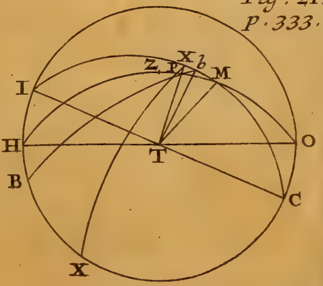


Fig. 21.
p. 333.



them will be the meaſure of the duration of the Twilight, while the Sun deſcribes the Parallel *P A*. But becauſe (by hypotheſis) it is the leaſt Twilight, none of theſe arcs, beſides *P A*, will reach to *C R*. (For while the Sun deſcribes in its Diurnal motion any other Parallel, the meaſure of the duration of the Twilight is its arc *p b*, intercepted between the Horizon *SEM* and the Circle of Twilights *C A R*; which ſince it is (by ſuppoſition) greater than the meaſure *P A* of the Leaſt Twilight, 'tis evident that *p a* does not reach to *C R*, becauſe *p b* which is greater, only reaches ſo far.) And therefore the Circle of Twilights *C A R* touches the great Circle *O A H*: and becauſe part of it under the Horizon, (namely *O A H*) is a ſemicircle, and *C A R* is parallel to the Horizon, 'tis evident that *H L A* or *O A*, is a quarter of a Circle. And therefore the Circle *N A T Z*, deſcribed upon the Poles *O* and *H* thro' *A*, interſecting the Equator at *K*, is a great Circle, and paſſes thro' the Zenith and Nadir, and meets the Horizon at right angles in *T*. Beſides, in the Spherical Triangles *E T K*, *♋ A K*, the angles *E T K*, *♋ A K*, (by *Prop.* 15. *B.* 1. *Theod.*) are right and conſequently equal, as alſo the angles *T E K*, *A ♋ K*, (by conſtruction) are equal, and the reſt *T K E*, *A K ♋* are likewiſe equal, (becauſe vertical:) From whence (by Spherical Trigonom.) theſe Triangles are mutually equilateral; that is, the arc *T K* is equal to *K A*, and *T E* to *♋ A*: But (by the preced. Lemma) *♋ H* is equal to *E P*, therefore *T E* is equal to *E P*. Beſides, in the Spherical Triangle *E T K* right angled at *T*, (by a known Prop. in Spheric. Trigonom.) the Radius is to the Co-Tangent of the angle *T E K* of inclination of the Equator to the Horizon, as the Tangent

gent of the arc *TK* to the Sine of the arc *ET*: And by permut. the Radius is to the Tangent of half the Arc of the depreſſion of the Circle of Twilights below the Horizon, as the Tangent of the Pole's Elevation to the Sine of the arc *ET* or *EP*. Again, in the Triangle *EDP*, (making *DP* the Circle of Declination) right angled at *D*, the Sine of the arc *EP* is to the Sine of the Arc *DP*, as the radius to the Sine of the Angle *DEP*; that is, to the Co-ſine of the Elevation of the Pole: But in all arcs, the Tangent is to the Sine as the Radius to the Co-ſine; wherefore the Sine of the Arc *EP* is to the Sine of the Arc *DP* as the Tangent of the Height of the Pole to the Sine of it. And by inverting and permutation, the Tangent of the Height of the Pole is to the Sine of the arc *EP*, as the Sine of the ſaid Height is to the Sine of the arc *DP*. But (by what has been ſhewn before) in the caſe of the Leaſt Twilight, as the Radius to the Tangent of half the arc of the depreſſion of the Circle of Twilights, ſo is the Tangent of the Height of the Pole to the Sine of the arc *EP*; and therefore the Radius is to the Tangent of half the arc of the depreſſion of the Circle of Twilights, as the Sine of the Height of the Pole to the Sine of the arc *PD*; namely of the declination of the Parallel that the Sun deſcribes when the Twilight is Leaſt. *Q. E. I.*

SCHOLIUM.

If the depreſſion of the Circle of Twilights below the Horizon be 18^d (as it is commonly determined,) the Radius will be to the Tangent of 9^d as the Sine of the Latitude to the Sine of Declination of the Sun towards the depreſſed Pole at the time of the Leaſt Twilight. But the Decli-

Declination of the Parallel and Obliquity of the Ecliptic being given, the two points of the Ecliptic will be found (by *Prop.* 20.) and (by the Theory of the Sun) the times of the Year, when the Sun is in the given Places; that is, when the Twilight is Leaſt. Tho' at *London* the Leaſt Twilight is when the Sun declines from the Equator towards the South 6 d 7′, and in ♎ or ♓ 17 d 30′, which happens in the preſent Age, about 22d of *Febr.* and 27th of *September* of the *Julian* Year. This was noted by *Blancanus* in his *Sphere*, who ſays that the ſhorteſt Twilights happen about the middle of *Libra* and the middle of *Piſces*. *Peter Nonius* has propoſed this Problem and ſolved it (yet by two Proportions) in a *Book* concerning the *Twilights*, publiſhed in the Year 1541; who is followed by *Ambroſius Rhodius* and *Clavius*.

PROPOSITION XLII.

IN a Plane given by poſition, on which a Sun-Dial is to be drawn, to determine the Meridian Line, Angle of the Style, and the Subſtyle line.

Every Plane upon which a Sun-Dial is drawn, is ſuppoſed to paſs thro' the Center of the Sphere, (becauſe in reſpect of the diſtance of the Sun, its diſtance from the Center is inſenſible,) and conſequently its interſection with the ſurface of the Sphere is a great Circle.

Let *IRSMCT* [*Fig.* 20.] be ſuch a Plane paſſing thro' *T*, the Center of the World which produced to the ſurface, forms the Circle *IRS MC*. Let *HIO* be the Horizon of the Place, *HZO* the Meridian (in which is the Pole *P*, and the Zenith *Z*) interſecting the former Circle in *M*. The poſition of the Plane *IRSC* being given, its *Declination* is given, meaſured by the Angle *ITH*, or Arc *IH*, namely that whereby the

the Horizontal Line *IC* of the Plane *IRSC* declines from *HO* the Meridian line drawn on the Horizontal Plane, by *Prop.* 16. *ZR* is alſo given, namely the arc of the Vertical Circle perpendicular to *IRSC*, intercepted between the Zenith and the ſaid Circle, the meaſure of the *Inclination* or *Reclination* of the ſaid Plane. In the Spherical Triangle *MZR* rightangled at *R*, there is given *ZR*, and the the angle *RZM*, equal to the complement of the angle *ITH* to a right one: (For the Vertical Circle, *ZR* perpendicular to the Circle *IRSC*, declines from the prime Vertical (drawn thro' the Equinoctial Eaſt and Weſt) as much as *IT* does from *HT*; and conſequently the complement of the one, namely *RZM*, is equal to the complement of the other, namely *ITH*.) Therefore the ſide *RM* will be found, or the Angle (which it meaſures) in the plane of the Dial *RTM*. But *TR* is given by poſition; (for it is ſuccedaneous to a perpendicular on the ſame Plane, or a right line perpendicular to the Horizontal line *ITC*;) wherefore the Meridian line *TM* ſought in the Plane of the Dial, or the common interſection of the plane of the Meridian with the plane of the Dial, will be given. In the ſame Triangle *MZR*, are alſo found the ſide *ZM* and Angle *RMZ*, which will be of uſe by and by. Again, thro' the Pole of the World *P*, draw an arc of a great Circle *PS*, perpendicular to *IRSC*: And in the Triangle *RSM* rightangled at *S*, there is given the Angle *PMS* found juſt now, and the ſide *PM*, the ſum or difference of the Arc *IM* found before, and of the diſtance of the Pole from the Zenith, namely *ZP*: Conſequently there will be known both the arc *PS*, or the Angle, which it meaſures, *PTS*, the inclination of the Axis

PT

PT to the Plane *IRSC*, that is, the Angle of the Style ſought; and the ſide *MS*, or the rectilineal Angle *STM*, which it meaſures in the plane of the Dial, contained between *TM* the Meridian determined before, and the Subſtyle *TS*, namely that upon which the plane of the Gnomon *PTS* ſtands, whoſe other edge is the Axis of the World *PT* caſting the ſhadow, and is perpendicular to the plane of the Dial. *Q. E. F.*

PROPOSITION XLIII.

TO find the Angle, that the line of a given Hour makes with the Meridian in the plane of the Dial; and thence to deſcribe a Sun-Dial upon a given plane.

The ſame things remaining as before, let *BPb* [*Fig.* 21.] be the Hour Circle, the interſection *Tb* of whoſe Plane with the Plane *IMC* given by poſition is ſought. The Hour of this Circle being given, the Angle *HPB* or *MPb* is given, contained by it and the Meridian. Therefore in the Spherical Triangle *PMb*, there are given (by preced. *Prop.*) the ſide *PM*, and the angle *PMb*, as alſo the angle *MPb*: Therefore the arc *Mb* is found; and the rectilineal angle, which it meaſures, *MTb*, which the Hour line *Tb* makes with the Meridian *TM*. Therefore the right line *TM* being given (by *Prop.* preced.) the Hour line ſought *Tb*, in the plane of the Dial given, will be determined. *Q. E. F.*

And after this manner a Sun-Dial may be made, by drawing a Meridian and Subſtyle from a point aſſumed at pleaſure for a Center, (by preced. *Prop.*) and erecting upon the Subſtyle a Style in an Angle found alſo by the preced. and drawing the other Hour lines in Angles

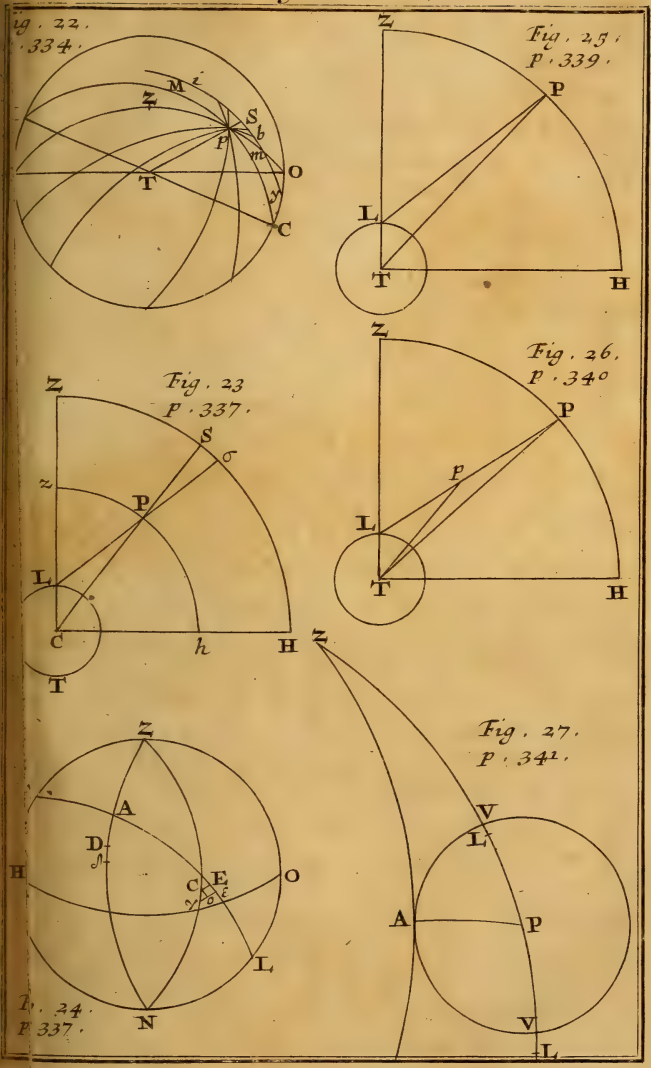
Angles with the Meridian *TM*, determined by this Propoſition, and marking them with their proper numbers. Thus *Tb* is the eleven a clock Hour line after Midnight, if *PB* be a Circle to the Eaſt of the Meridian *PZH*, making with it an Angle of 15 ^{d}; that is, if *Tb* be to the Weſt of the Meridian line *TM*, and the Angle *MPb* in the ſurface of the Sphere of 15 ^{d}. And after the like manner *TX*, ſituated on the ſame ſide, will be the ten a clock Hour line, if *MPX* be 30 ^{d}, and ſo on.

PROPOSITION XLIV.

TO *draw a given Hour line upon a Sun-Dial, whoſe plane paſſes thro' the Poles; and ſo to deſcribe a Polar Dial.*

Becauſe all the Hour lines interſect one another in the Axis of the World, and this Axis is always in a Polar plane; 'tis evident that all the Hour lines, in ſuch a plane, are to be found in that Right line: But that they may be diſtinguiſhed, and not confounded with the Subſtyle, the Dial is not to be conceived as deſcribed in the plane *IMC* itſelf, [*Fig.* 22.] paſſing thro' the Center (as it is in all other caſes,) but in another plane *imγ*, parallel and near to the former; ſo that the opake right line that caſts the ſhadow and coincides with the Axis of the World, is diſtant from the plane *imγ*, by the right line *PS*: And ſince the diſtance of the Sun is look'd upon as Infinite, any given Right line as *PS*, vaniſhes in reſpect of it, and the parallel Planes paſſing thro' *IMC*, *imγ*, are eſteem'd as very near one another; and the portions *Pm*, *Pb*, &c. of the Hour circles *HPm*, *BPb*, &c. croſſing one another in the Pole *P*, intercepted between the Pole and the Circle *imγ*, are to be looked upon as Right lines; and the

Plate 5. Book 2.



the part of the Spherical ſurface which theſe portions take up (being infinitely ſmall in compariſon of the whole) is to be look'd upon as a Plane.

In the Polar plane then, having found (by *Prop.* 42.) the Subſtyle, erect the Style of any breadth taken at pleaſure, as *P S*, whoſe other edge plac'd in the Axis of the World is parallel to the Subſtyle; for in this caſe there is no Angle of the Style. To find the Meridian line, obſerve that in the rectilinear Triangle *P S m*, beſides the right angle at *S*, there is given the ſide *P S*, namely the height of the Style; and the angle *S m P* equal to the angle *M P Z*, found at *Prop.* 42; hence you may find *S m* the diſtance of the Meridian line from the Subſtyle in the right line *S m*, which is perpendicular to the Subſtyle, becauſe *M P* (parallel to *i γ*,) is perpendicular to *P T*; and you may alſo find the ſide *P m*: A right line therefore drawn thro' the point *m*, parallel to the Subſtyle (becauſe, as was ſaid before, the Hour lines and Subſtyle, which alſo is one of them, on this ſort of Planes are parallel) will be the Meridian line in the Sun-Dial propoſed.

If any other Hour line given is to be drawn, whoſe Circle for inſtance is *B P*; in the Triangle *P b m* there are given the angles *P m b*, and *m P b*, the one found before, and the other equal to the vertical angle *Z P B*, given becauſe the Hour of the circle *P B* is given;) as alſo the ſide *P m* determined before: Hence therefore may *m b* be found, whoſe extremity *m* is given, conſequently *b* alſo will be given. A right line therefore drawn thro' it parallel to the Subſtyle, will be the Hour line of the circle *P B*. And after the like manner may all the other

Hour

Hour lines be drawn, and a Sun-Dial made upon a Polar plane. *Q. E. F.*

There are other eaſy methods of making Sun-Dials; but we have taken the moſt obvious, and one that is the fitteſt for the illuſtration of the doctrine of the Firſt Motion: And after this manner 'tis eaſy to deſcribe upon a Sun-Dial any other ſuch furniture as may be ſhewn by the ſhaddow of a Gnomon.

SECTION VII.

Concerning the Parallax of the Stars.

PROPOSITION XLV.

T*O deſcribe the nature of the Parallax, and to enumerate the various ſorts of it.*

Thro' all this Book we have ſuppoſed the Phænomena, whoſe places we have been treating of, to be ſituated among the Fixed Stars; or (if they are placed between the center or the Earth and ſurface or Fix'd Stars, that they are referr'd to the Fix'd Stars by the help of right lines drawn from the center thro' the Phænomena; that is, that they are ſeen from the Center. And tho' the Earth in reſpect of the Sphere of the Fix'd Stars, is taken for a center, and puts on the nature of one, yet in reſpect of the diſtances of the other Celeſtial Bodies it does not happen ſo; but an intermediate Phænomenon, ſeen from the ſurface of the Earth, will have a place among the Fix'd Stars (or in an infinite diſtant ſurface of a concave Sphere concentric with the Earth,) different from that, which it ſeems to have when ſeen from the center; and this diverſity of ap-

pearance

pearance is very fitly call'd the *Parallax*: Thus for inſtance, if *L T* [*Fig.* 23.] repreſent the Earth, whoſe Center is *C*; *L* a Place on the ſurface, whoſe Zenith is *Z*; *Z S H* the Sphere of the Fix'd Stars; the Phænomenon *P* ſeen from *L* will be placed among the Fix'd Stars at σ, but from the Center *C*, at *S* its true place. The difference between its true place *S* and the apparent one σ, or the Arc $S\sigma$, is the Parallax of the Phænomenon *P*: And it may be expreſſed by the Angle $SP\sigma$, or *L P C*; for ſince *L T* in reſpect of the circumference *Z S H* vaniſhes, *z P h* alſo (to which *LT* has a ſenſible proportion) is as a Center in reſpect of *ZSH*; that is, the Arc $S\sigma$ may be conſidered as deſcribed upon the Center *P*. From whence it is evident that by reaſon of the Parallax, any Phænomenon appears nearer to the Horizon than it is, in a Vertical Circle, and that ſo much the more as (other circumſtances being alike) it is leſs elevated; and that a Phænomenon ſeen from the ſurface in L, in the Vertex, as in *z*, is referred to the ſame place among the Fix'd Stars, as if it had been ſeen from the Center *C*, or that there is no Parallax in the Zenith.

By reaſon of the Parallax before deſcribed (which is call'd either *the Parallax of Altitude*, or alſo ſimply the Parallax) the Place of a Phænomenon is alter'd as to Longitude and Latitude: For let the true place of a Phænomenon be *C*, [*Fig.* 24.] the apparent γ, ſo that the Parallax is $C\gamma$; if you imagine the Circles of Latitude *C E*, $\gamma\epsilon$ drawn thro' *C* and γ, meeting the Ecliptic *E L* in *E* and ϵ, the apparent Place in the Ecliptic will be ϵ the true *E*; and therefore the change of Longitude made by reaſon of the Parallax, is $E\epsilon$, which is alſo called the *Parallax of Longitude.* And the true Latitude *C E*

Z is

is changed into the apparent one γ *e*, the difference of which γ *o* (ſuppoſing *C o* parallel to the Ecliptic) is the *Parallax of Latitude.* But if it happen that the Vertical, as *Z D N*, paſſing thro the Phænomenon *D*, is alſo perpendicular to the Ecliptic (which comes to paſs when it paſſes thro' the higheſt point of the Ecliptic *A*, or Nonageſim degree from the Eaſt) then the Parallax affects only the Latitude. For by the Parallax the Place is only changed in the Vertical, and becauſe (in the preſent caſe) this coincides with the Circle of Latitude, the Place is only changed in this, that point of the Ecliptic continuing unchanged, to which it is referr'd by the ſaid circle of Latitude: and in like manner only the Longitude is ſometimes affected the Latitude continuing unchang'd; namely when the Phænomenon is in the Ecliptic paſſing thro' the Zenith. From whence it is evident, that the Longitude of a Phænomenon ſituated to the Eaſt of the Nonageſim degree of the Ecliptic, is increaſed by the Parallax, becauſe it is removed more to the Eaſt upon this account; but that the Longitude of one ſituated to the Weſt from thence is leſſen'd: Wherefore beſides all other motion which the Phænomenon may have, it will ſeem to be moved by reaſon of the Parallax, becauſe change the Parallax and its apparent place will be changed. And this motion, becauſe not real but apparent, is called by ſome *the Parallax of Motion.* Now what is to be underſtood of a Phænomenon in reſpect of the Ecliptic and its Secondaries, is to be underſtood after the like manner concerning the ſame in regard of the Equator.

PRO

PROPOSITION XLVI.

THE distance of a Phænomenon from the Earth is to the Semidiameter of the Earth, as the Sine of the apparent distance from the Vertex is to the Sine of the Parallax.

Let *T* be the center of the Earth; [*Fig.* 25.] the Place on the Surface *L*, its Zenith *Z*, Horizon *TH*; and let the Phænomenon be in *P*. Now (from the known property of a rectilineal Triangle) *TP* is to *TL* as the Sine of the angle *TLP* (or of its complement to two right *ZLP*) to the Sine of the angle *TPL*; that is, the distance of the Phænomenon from the center of the Earth is to the semidiameter of the Earth, as the Sine of the apparent distance from the Vertex to the Sine of the Parallax. *Q. E. D.*

Consequently the Parallax agreeing to a given Altitude being given, the distance of that Phænomenon from the Earth will also be given; and so *vice versa*.

PROPOSITION XLVII.

THE Sines of the apparent distances of a Phænomenon from the Vertex are as the Sines of the Parallaxes.

For (by preced. *Prop.*) the Sine of the apparent distance from the Vertex is to the Sine of the Parallax, as the distance of the Phænomenon from the center of the Earth is to the semidiameter of the Earth; that is (in the same Phænomenon) in a constant Ratio. Wherefore, as the Sine of the apparent distance from the Vertex in the first Observation is to the Sine of the apparent distance from the Vertex in any other Observation, so is the Sine of the Parallax in the first observation to the Sine of the Parallax in the second Observation. *Q. E. D.*

PROPOSITION XLVIII.

THE Parallaxes of two Phænomena at unequal diſtances from the Center of the Earth, but whoſe apparent diſtance from the Vertex is equal, are reciprocally as the diſtances from the Center of the Earth.

Things being expreſſed as before, let the two Phænomena be *P* and *p* [*Fig.* 26.] in an equal apparent diſtance from the Vertex *Z*, meaſured by the Angle *ZLP*. 'Tis evident that the Sine of the Angle *LPT*, is to the Sine of the Angle *PpT*, or *LpT*, that is, that the Sine of the Parallax of the Phænomenon *P*, is to the Sine of the Parallax of the Phænomenon *p* as *Tp* to *TP*, that is, reciprocally as the diſtances of the Phænomena from the Center *T*.

PROPOSITION XLIX.

THE ratio of the Sine of the Parallax of one Phænomenon to the Sine of the Parallax of another Phænomenon is compounded of the inverſe ratio of the diſtances from the Center of the Earth, and of the direct ratio of the Sines of the apparent diſtances from the Vertex.

For when the diſtance from the Center of the Earth is given, the Sine of the Parallax is as the Sine of the apparent diſtance from the Vertex, (by *Prop.* 47;) and the apparant diſtance from the Vertex being given, the Sine of the Parallax is reciprocally as the diſtance from the Center (by preced.) And therefore when neither is given, the Sine of the Parallax is as the apparent diſtance from the Vertex directly, and the diſtance from the Center of the Earth reciprocally, conjunctly. *Q. E. D.*

PRO-

Plate 6, Book 2.

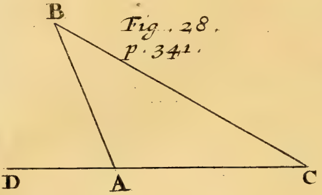


Fig. 28. p. 341.

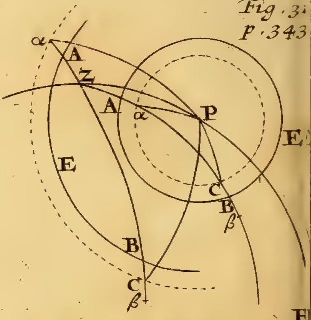


Fig. 31. p. 343

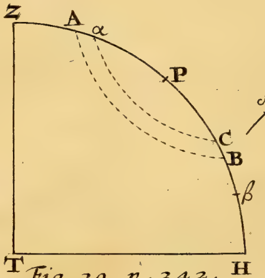


Fig. 29. p. 342.

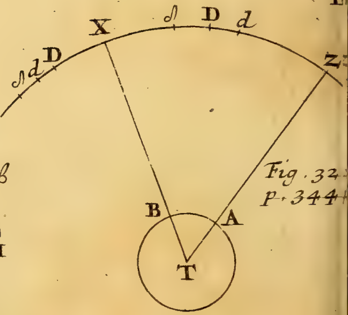


Fig. 32. p. 344

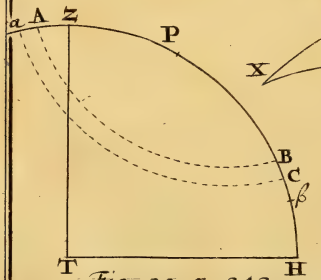


Fig. 30. p. 343.

Fig. 33. p. 346.

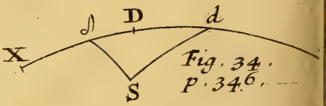


Fig. 34. p. 346.

PROPOSITION L.

TO *find the Parallax of a Phænomenon that does not change its Declination, passing between the Pole and Vertex.*

Let *ZP* [*Fig.* 27.] be the Meridian, and in it *Z* the Zenith, *P* the Pole; the true Place *V*, and *L* the apparent Place of the Phænomenon, whose Parallel described by the first Motion is *VA*; the Vertical farthest off from the Meridian *ZA*, touching the Parallel *VA*, *PA* an Arc let fall perpendicularly upon it, from the Pole *P*. In the Spherical Triangle *ZAP*, right angled at *A*, by the Side *ZP*, the Complement of the Height of the Pole, and the Angle *PZA*, or Azimuth known by Observation being given, you will find the Side *PA*, or *PV* equal to it, the true distance of the Parallel from the Pole *P*; the sum or difference of this and *ZP* is *ZV* the true distance (or that seen from the Center of the Earth) from the Vertex *Z*: And the difference between this and the apparent one *ZL*, namely *VL*, is the Parallax of the Phænomenon agreeing to the Altitude in *V*. *Q. E. I.*

PROPOSITION LI. LEMMA.

THE *sum or difference of any two Arcs, together with the Ratio of their Sines, to find the Arcs themselves.*

First, if the sum of the Arcs be given, the Problem is all one with that common Trigonometrical one, where two Sides *AB*, *AC*, [*Fig.* 28.] of a rectilineal Triangle being given, together with the Angle included *BAC*, the other two Angles *B* and *C*, are required: For when the Angle *BAC* is given, the Angle *BAD* is the sum of the Angles *B* and *C* themselves; and the ratio of the Sides *AC*, *AB*, is the same

with that of the Sines of the Angles *B* and *C*. This Problem therefore will be ſolv'd after the ſame manner as that; for 'tis known that the ſum of the Sides *AB*, *AC* is to their difference, that is, the ſum of the terms of the given ratio is to their difference, as the Tangent of half the ſum of the Angles *B* and *C*, or of the Arcs that meaſure them, to the Tangent of half the difference of them: But the ſum and difference of the terms of the given ratio are given, and the ſum of the Arcs being given, the Tangent of half the ſum is given; therefore the Tangent of half the difference will be known, and conſequently half the difference it ſelf; from hence, and half the ſum, will the Arcs themſelves be known. But if the difference of the Arcs be given, half the one and Tangent of the other will be given, and conſequently (from the former Proportion) the Tangent of half the Sum of them, and the half Sum it ſelf will be found; from whence the Arcs themſelves will become known, after the like manner as before. *Q. E. F.*

POPOSITION LII.

Both the Meridian Altitudes of a Phænomenon being given, which does neither change its declination nor ſet, to find its Parallax.

Let *ZH* be a quarter of the Meridian [*Fig.*29.] reaching from the Zenith to the Horizon, in which the Pole is *P*; the real place of the Phænomenon above the Pole *A*, the apparent α; its true place below the Pole *B*, its apparent β. If the Phænomenon paſs between the Pole and the Vertex, the difference between the obſerved diſtances from the Pole, namely $P\beta - P\alpha$, is the ſum of the Parallaxes in α and β. For a Circle deſcribed upon the Pole *P* thro' *A*, paſſes alſo thro' *B*, becauſe the true declination of the Phæ-

Phænomenon is always the ſame. If farther thro' α you deſcribe upon the ſame Pole the Circle αC, then will $PC = P\alpha$; and therefore $P\beta - P\alpha = (P\beta - PC = C\beta =)$ to the ſum of the Parallaxes in α and β. But the ſum of the Parallaxes in α and β being given, and the ratio of their Sines, (being the ſame, by *Prop.* 47. with the ratio of the Sines of the apparent diſtances from the Vertex,) the Parallaxes themſelves ſought, will alſo be found by the preced. Lemma.

If the Phænomenon paſs beyond the Vertex in reſpect of the Pole, then the difference between $P\beta$, $P\alpha$ [*Fig.* 30.] the diſtances obſerved from the Pole is equal to the difference of the Parallaxes. For ſince $B\beta$ is the Parallax in the apparent place β, and $A\alpha$ or (the ſame preparation being made uſe of as before) BC in the place α, $C\beta$ (that is, $P\beta - P\alpha$) is the difference of the Parallaxes in the apparent Places β and α. But the difference of the two Arcs being given, together with the ratio of the Sines (namely the ſame with the ratio of the Sines of the apparent diſtances $Z\alpha$, $Z\beta$ from the Vertex,) the arcs themſelves or the Parallaxes ſought will be found.

But if the Phænomenon paſs thro' the Vertex it ſelf; that is, if the points Z, A and α coincide, alſo B and C, then $B\beta$ or $C\beta$ is the Parallax it ſelf of the apparent Place β.

PROPOSITION LIII.

Both the Altitudes of a Phænomenon, that does not change its declination, being given, and obſerved in the ſame Vertical Circle, to find its Parallax.

The reſt being expreſſed as before, Let ZAB [*Fig.* 31.] be the Vertical Circle, in which the Phænomenon has been twice obſerved,

namely in α and β, but whose true Places are A and B, and AEB the Parallel of the Phænomenon described by the first Motion. Upon the same Pole P, thro' α, describe another Circle αC, and the portions $A\alpha$, BC of the Verticals intercepted between them (by *Prop.* 13. *B.* 2. *Sphær. of Theodosius*) are equal; and therefore $C\beta$ is either the sum or difference of the Parallaxes $A\alpha$, $B\beta$. And to find it let the great Circles $P\alpha$, PC, be imagined to be drawn. In the Triangle $Z\alpha P$, ZP the complement of the Height of the Pole, $Z\alpha$ the observed distance of the Phænomenon in α from the Vertex Z, and the Angle contained αZP, from the Azimuth, known by observation, being given, you will find αP, to which the arc CP is equal. Again in the Triangle PZC, two Sides PZ, PC being given, together with the Angle PZC, ZC will also be known, whose defect or what it wants of the observed Arc $Z\beta$, is $C\beta$ the sum of the Parallaxes in the places α and β, when PZ exceeds $P\alpha$; or their difference, when PZ is less than $P\alpha$. But in both cases, when over and above, the ratio of the Sines of the said Parallaxes is given, (namely the same, by *Prop.* 47. with the ratio of the Sines of the apparent distances from the Vertex,) both the Parallaxes agreeing to the apparent places α and β will be found.

PROPOSITION LIV.

TWO Altitudes of the same Phænomenon observed at the same moment of time in the common Azimuth of two given Places upon the Earth, to find the Parallax of the Phænomenon.

Let the two Places on the Earth be A and B, [*Fig.* 32.] whose Vertices are Z and X; the common Azimuth ZDX; and let the true Place of the

the Phænomenon be D, δ the place obſerved at A, d its place ſeen at B: And firſt let theſe points be between the Vertices Z and X. The ſum of the apparent diſtances from the Vertex, (namely of the arcs $Z\delta$ and Xd,) known by Obſervation, exceeds ZX the diſtance of the Vertices (found by *Prop.* 24) by an exceſs equal to $d\delta$, the ſum of the Parallaxes. But if the points D, d, δ, be without the arc ZX, the difference of the apparent diſtances from the Vertex (namely $Z\delta$ leſſen'd by Xd,) exceeds the diſtance of the Vertices XZ by exceſs equal to the difference δd of the Parallaxes ſought.

Beſides the ſum or difference of the Parallaxes found after the manner deſcribed before, there is given the ratio of the Sines of the ſaid Parallaxes; namely the ſame (by *Prop.* 47.) with the ratio of the Sines of the arcs $Z\delta$, Xd of the apparent diſtances from the Vertex. From whence (by *Prop.* 51.) in both caſes the Parallaxes themſelves will be found. *Q. E. F.*

PROPOSITION LV.

Given two altitudes of a Phænomenon obſerved at given Places on the Earth, the ſame moment of time, even without the common Azimuth, to find the Parallax of the Phænomenon.

The Longitudes of the Places being given, the ſame moment of time neceſſary for the making the Obſervation in the two Places, is expreſſible by the number of Hours reckon'd from Noon or Midnight reſpectively, and their difference is equal to the difference of Longitudes turn'd into time, the more Eaſtern Place reckoning more hours; which is determined during the Obſervation in the preced. when the Phænomenon is found in the common Azimuth, unleſs both Places be in the Equator. Let XZ

[*Fig.*

[*Fig.* 33.] be the common Azimuth of the two Places upon the Earth, whofe Vertices are *X* and *Z*; from which at the fame moment of time (tho' expreffed differently,) let the Phænomenon be obferved, whofe true place let be *D*; and its place feen from that Place whofe Vertex is *X* be *d*, from the Place whofe Vertex is *Z* be *δ*. In the Triangle *XDZ*, the angles *X* and *Z* being given (contained between the common Azimuth of the Places and the obferved Azimuths,) and the Side *XZ* the diftance of the Vertices (found by *Prop.* 24.) the Arcs *XD*, *ZD* may be found, and they fubtracted from *Xd*, *Zδ* obferved, leave *Dd*, *Dδ* the Parallaxes fought. *Q. E. F.*

PROPOSITION LVI.

To find very exactly the Parallax of a Phænomenon, by a method not much different from the two foregoing, by the help of fome neighbouring Fix'd Star.

Since the Phænomenon whofe Parallax is fought, is in the common Azimuth *XDZ* [*Fig.* 34.] of the two Places on the Earth pitched upon for this purpofe, whofe Vertices are *X* and *Z*; let it be obferved with fome neighbouring Fix'd Star; and let *S* be the Place of the Fix'd Star, *d* the Place of the Phænomenon obferved at the place below *X*; confequently in this Obfervation there are given the arc *Xd* the apparent diftance of the Phænomenon from the Vertex, the arc *Sd* the diftance between the Phænomenon and the Fix'd Star, and the angle *XdS* contained between it and the common Azimuth. At the fame moment of time from the place below *Z*, where the apparent place of the Phænomenon is *δ*, let *Zδ*, *Sδ* and the angle *SδZ* be obferved: And tho' the arcs *Xd*, *Zδ* can't

can't be observed exactly, (because there is need of a Quadrant to measure them;) yet they may do for our purpose: But if they could with perfect accurateness, the Parallax of the Phænomenon would (by *Prop.* 54.) immediately appear. Notwithstanding the arcs $d\,S$, $\delta\,S$ and the angles at d and δ, may be observed very accurately, by projecting the image of the Phænomenon and neighbouring Fix'd Star S, by the help of Speculums or Lens upon a Plane, upon which a right line is drawn representing the common Azimuth $X\,Z$; and being in the place of its image, (if it should project any;) as is evident to any one acquainted with Optics and Astronomical Observations. Therefore in the Triangle $d\,S\,\delta$, the sides $S\,\delta$, $S\,d$ with the angles subtended by them d and δ being given, the Base $d\,\delta$ will be found, which is the sum of the Parallaxes; from whence and the ratio of the Sines of the Segments $\delta\,D$, $d\,D$, (D being supposed to be the true Place of the Phænomenon,) namely the same with the ratio of the Sines $Z\,\delta$, $X\,d$ of the apparent distance from the Vertex (by *Prop.* 47,) the Segments themselves Dd, $D\delta$ the Parallaxes sought, will (by *Prop.* 51.) become known. Much after the same manner the differences of the Parallaxes may be found, and from thence the Parallaxes themselves (by the said Lemma,) if D, d, δ, be without $X\,Z$, but yet in the continuation of it. If the true Place D [*Fig.* 35.] of the Phænomenon be without the common Azimuth $X\,Z$ of the Places, the same things being expressed the same way as before, thro' the Vertices X and Z and the respective apparent Places, let the Vertical Circles $X\,d$, $Z\,\delta$ be drawn, whose intersection is the true Place D of the Phænomenon. Draw also thro' S, the place of the Fix'd Star, the Ver-

Verticals *XS*, Z *S*, and join the points *S*, *δ*, *d* by the great Circles *S d*, *S δ*, *d δ*. Let the Obſerver below *X*, obſerve accurately (beſides the arc *X d*) the angles *XdS*, *XS d* and the Side *SD*, either by the method inſinuated above, or any other more exact; in like manner let the Obſerver below Z (beſide *Z δ*) do to the Angles *Z δ S*, *Z S δ* and the Arc *S δ*. Beſides, the habitations being given (by *Prop.* 24.) *Z X* is given, and from the ſituation of this, by the Obſervation of the Fix'd Star *S*, the angles *Z XS*, *XZ S*, are had; from whence the Angle *XS Z* will be known, which taken from the ſum of *XS d*, *Z S δ*, will leave *d S δ*, from which and the ſides *S d*, *S δ*, being known, the angles *S d δ*, *S δ d* and the ſide *d δ* are found. But *S d X*, *S δ Z*, were known by Obſervation; conſequently the others *D d δ*, *D δ d* are known, by which and the ſide *d δ* known in the Triangle *D d δ*, the ſides *D d*, *D δ*, the Parallaxes ſought, are found.

PROPOSITION LVII.

TWO *Obſervations of a fix'd Phænomenon, that is, one that has only a diurnal motion, being given, to find its Parallax.*

Let *ECLI* [*Fig.* 36.] repreſent the Ecliptic, *B* the true place of the Phænomenon, intirely fix'd in regard of the Ecliptic. And in the one Obſervation let *Z B L* be a Vertical Circle, in which *Z* is the Zenith, and *β* the apparent place of the Phænomenon; in the other, let *XBL* be a Vertical, *X* the Zenith, (for the Vertical point may be conſidered as moved in regard of the Ecliptic unmoved, as well as the Ecliptic in reſpect of the Vertex unmoved, and indeed more properly;) and *b* the apparent place of the Phænomenon. From the firſt Obſervation (by the Problems of the Firſt motion con-

Plate 7. Book 2.

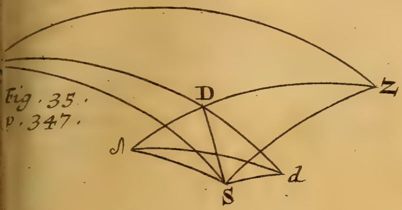


Fig. 35. p. 347.

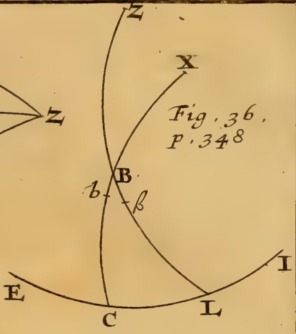


Fig. 36. p. 348

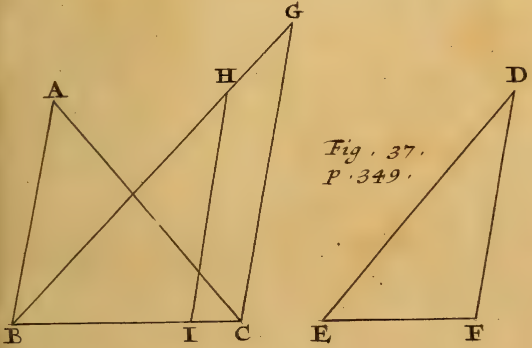


Fig. 37. p. 349.

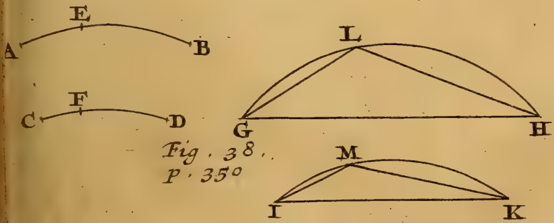


Fig. 38. p. 350

constructed before,) you may determine the point L of the Ecliptic, where the Vertical passing thro' B meets the Ecliptic; and the arc $Z L$, namely a portion of a vertical Circle, intercepted between the Zenith and the Ecliptic; and the angle $Z L E$ contained between the said Vertical and the Ecliptic. And in like manner in the second Observation of the Phænomenon in β, the point of the Ecliptic C, the arc $X C$, and the angle $X C L$ will be known. Therefore in the Triangle $B C L$, the angles $B C L$, $B L C$ being given, and the side $C L$ that lies between them (namely the distance of the known points in the Ecliptic,) the sides $B L$, $B C$ will be known, which taken from the arcs found before $Z L$, $X C$, will leave $Z B$, $X B$. And these again taken from $Z\beta$, $X b$, the known apparent distances from the Vertex, leave $B\beta$, $B b$ the Parallaxes of the Phænomenon agreeing to the moments of the observations.

PROPOSITION LVIII. LEMMA.

LET there be two rectilineal Triangles, ABC, DEF [Fig. 37.] *having the Angles* ABC, DEF *given; and let the ratios of* AB *to* DE, AC *to* DF, *and* BC *to* EF, *be given: To find both Triangles from the side* AB *given.*

Make the angle $G B C$ equal to $D E F$, and take $B G$ such, as that the ratio of $B A$ to $B G$ may be equal to the ratio compounded of the given ratios of $B A$ to $E D$, and $E F$ to $B C$, which is done by taking $B G$ equal to a fourth proportional to $E F$, $B C$ and $E D$. Then in $B C$ given by position, chuse such a point C, as that the ratio of the lines connected $A C$, $G C$ is equal to that compounded of the given ratios of $A C$ to $D F$, and $E F$ to $B C$, which may be done as before. Then let $B G$ be to $B H$ in the

the ratio of *BC* to *EF*, and let *HI* be parallel to *GC*: I ſay the Triangles *ABC*, *BHI*, are thoſe ſought.

'Tis manifeſt from the Conſtruction, that the ratio of *BC* to *BI*, is equal to the ratio of *BC* to *EF*; and that the Angles *ABC*, *HBI*, are equal to the given ones. Beſides, the ratio of *AB* to *BG* is compounded (by conſtruction) of the ratios of *AB* to *ED* and *EF* to *BC*: But the ratio of *AB* to *BG*, is compounded of the ratio of *AB* to *BH*, and of the ratio of *BH* to *BG*, or *EF* to *BC*; and therefore *AB* has the ſame ratio to *BH* as it has to *ED*. Again, the ratio of *AC* to *CG* (by conſtruction) is compounded of the ratio of *AC* to *DF*, and of the ratio of *EF* to *BC*, or *HI* to *GC*: But the ratio of *AC* to *CG* is compounded alſo of the ratio of *AC* to *HI* and *HI* to *GC*; and therefore *AC* has the ſame ratio to *HI* as it has to *DF*. Therefore the Triangles *ABC*, *BHI*, have the conditions required.

PROPOSITION LIX. LEMMA.

THE *ratio of the Sines of the Arcs* AB, CD, [Fig. 38.] *belonging to a given Circle, and the ratio of the Sines of the Arcs taken away* AE, CF, *and the arcs remaining* EB, FD *being given; to find the whole arcs and thoſe taken away,* AB, CD, AE, CF.

In Circles equal to the given ones, let there be ſuppoſed to be taken the arcs *GL*, *LH*, *IM*, *MK* reſpectively the doubles of *AE*, *EB*, *CF*, *FD*: Wherefore *GLH*, *IMK* will be reſpectively the doubles of *AB*, *CD*. Compleat the rectilineal Triangles *LGH*, *MIK*, in which are given the ratio of *GL* to *IM*, the ſame with the given ratio of the Sine of the arc *AE*, to

to the Sine of the arc *C F*; the ratio of *L H* to *MK*, the ſame with the ratio of the Sines of the arcs given *E B*, *F B*; the ratio of *G H* to *IK*, which is the ſame with the given ratio of the Sines of the arcs *A B*, *C D*. There are alſo given the angles *L G H*, *M I K* inſiſting upon the known arcs, and therefore (by the prec. *Prop.*) the angles *L H G*, *M K I* will be known, and from thence the arcs *G L*, *I M*, and their halves *A E*, *C F*: But the arcs themſelves *E B*, *F D* are given; therefore the wholes *A B*, *C D* are alſo known.

And after the like manner, the ratio of the Sine of the whole arc to the Sine of the arc taken away, and the remaining arc itſelf being given, the whole arc and the arc taken away may be found. For ſuppoſe *GLH* the double of the whole arc, and *G L* of the arc taken away; from whence you have *L H* double the remaining arc. Connect *G H*, *G L*, *L H*, and in the Triangle *L G H*, there is given the ratio of the ſides *G L*, *G H*; the ſame with the given ratio of the Sines of the whole arc and that taken away: The angle *L G H* is alſo given, inſiſting upon the given arc *L H*. And therefore (by 41. *of the Data of Euclid*,) the angles *G L H*, *G H L*, are given; and therefore the arcs on which they ſtand, *viz.* the complement of the arc *G L H* to the whole circumference, and the arc *G L*; and conſequently *G L H* and *G L* themſelves, and their halves, namely the arcs ſought.

PRO-

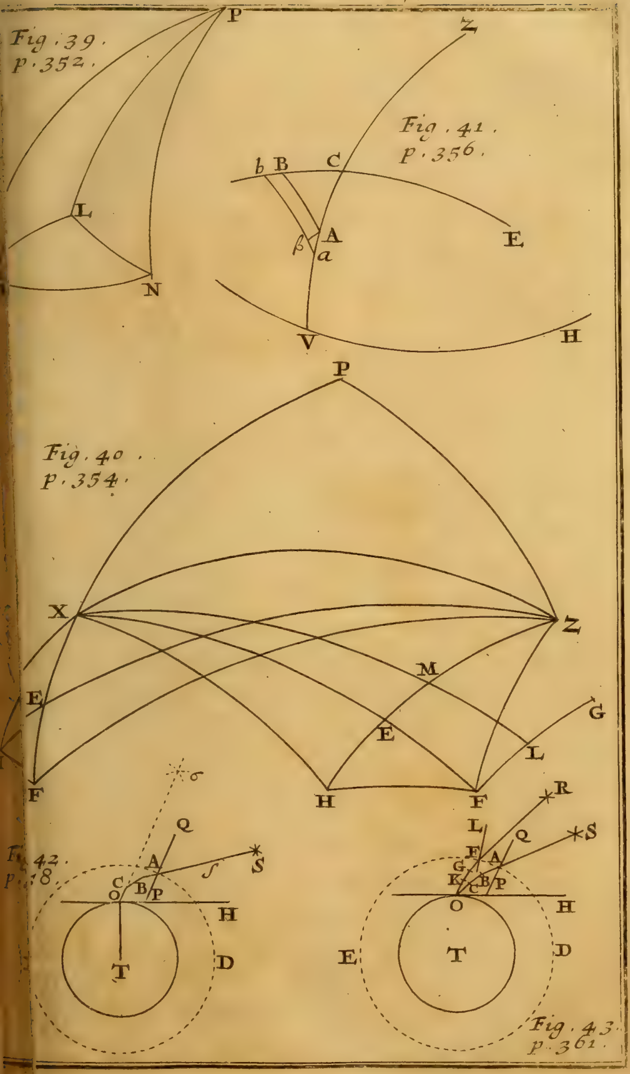
PROPOSITION LX.

TWO *Altitudes of a Phænomenon not altering its Declination, and the correſponding Azimuths being given, to find its Parallax.*

In two obſervations of a Fix'd Star five things may be given; namely the two Altitudes, two Azimuths, and the Time between the obſervations; from any four of which the Parallax may be found, from whence ariſe theſe three following Problems.

For the ſolution of the firſt, let *P* [*Fig.* 39.] repreſent the Pole; *L* the true place of the Phænomenon; and in one of the obſervations let *LM* be the Vertical Circle, and *M* the place of the Phænomenon obſerved in it, and therefore *LM* the Parallax; in the other let *LN* be the Vertical Circle paſſing thro' *L*, and *LN* the Parallax of this obſervation. Let the great Circles *PM*, *PN*, *NM* be ſuppos'd to be drawn. From the Altitudes and Azimuths obſerved, and the Height of the Pole, the Arcs *PM*, *PN* the apparent diſtances from the *P*, and the Angles *PML*, *PNL* contained between the Circles of Declination and the Verticals are known: There is alſo given the ratio of the Sine of the Parallax *LM* to the Sine of the Parallax *LN*; namely, the ſame (by *Prop.* 47.) with the ratio of the Sines of the diſtances obſerved from the Vertex. There are therefore given the ratio of the Sines of the whole Angles *PMN*, *PNM*, namely the ſame with the ratio of the Sines of the known Arcs *PN*, *PM*; alſo the ratio of the Sines of the Angles *LMN*, *LNM* taken away, the ſame with the ratio found before of the arcs *LN*, *LM*; as alſo the remaining Angles *PML* *PML* found before by Calculation. Therefore the Angles themſelves *PMN*, *PNM*, *LMN*, *LNM*.

Plate 8, Book 2.



LNM will be known, (by the preced. Lemma.) Again, in the Triangle *MPN*, the Sides *PM*, *PN* being given, with the Angles ſubtended by them, the Baſe *MN* is found. And laſtly, in the Triangle *LMN*, the Baſe *MN*, and the Angles adjacent *LMN*, *LNM* being given, the Sides *LM*, *LN* the Parallaxes ſought will be found.

If one of the Obſervations was made when the Phænomenon was in the Meridian, then one of the Triangles *PML*, or *PNL* is changed into an Arc, and the Calculation becomes much ſimpler.

But if the Phænomenon *L* change its Declination, and that equably, (which will be done in one revolution of the Earth, or thereabouts,) we may ſtill find its Parallax, by correcting the places of the Phænomenon, by a proportional part of the Declination thus: When the Phænomenon firſt, next after the firſt Obſervation, arrives at the Azimuth *LM*, obſerve its apparent Altitude, which will be different from that obſerved the Day before, otherwiſe it would retain the ſame declination. Let the difference be *A*. Call the diſtance of time between the two firſt Obſervations made in *M* and *N*, *B*; and the time between the two Obſervations of the Phænomenon in the Azimuth *LM*, *C*. And if the difference of the aforeſaid Altitudes *A*, be an exceſs, to the apparent Altitude in *M* add $\frac{BA}{C}$; but if this difference be a defect, ſubtract it from the ſame. This ſum or difference will be the corrected apparent Altitude, which put in the preceding Calculation (inſtead of the former Altitude in *LM*) gives the Parallaxes ſought of a Phænomenon that does not change its declination.

PROPOSITION LXI.

GIven two Altitudes of a Phænomenon, with th Azimuth of one of them, and the Time be tween the two obſervations, to find the Parallax o the Phænomenon.

Let *P* be the Pole, (*Fig.* 40.) *E* the true plac of the Phænomenon, *F* the apparent plac when *X* is the Zenith, and *H* when *Z* is th Zenith. There are given the two apparent di ſtances from the Vertex, namely *XF*, *ZH* and the Angle *PXF*, ſince one Azimuth is gi ven, and the Azimuth *XPZ* is known by th time. In the Iſoſceles Triangle *PXZ*, the e qual Sides *PX*, *PZ*, and the Angle *XPZ* be ing given, *PXZ*, *XZ* are found: And ſinc *PXF* is given, *ZXF* is alſo found. Therefor in the Triangle *XZF*, *XZ*, *XF*, *ZXF* are gi ven; and therefore *ZF*, *ZFX* become know Therefore the ratio of the Sine of the angl *ZFH*, to the Sine of the angle *ZHF* is given namely the ſame with that of the Sine of th arc *ZH*, to the Sine of the arc *ZF*; and th ratio of the Sine of the angle *ZHF* to the Sin of the angle *EFH*, or the Sine of the arc *E* to the Sine of the arc *HE*, or the Sine of th arc *XF* to the Sine of the arc *ZH*. There fore the ratio of the Sine of the angle *ZFH* t the Sine of the Angle *EFH* is given; and the dif ference of thoſe Angles *ZFE* is alſo given and therefore (by *Prop.* 59.) the angles themſelve *ZFH*, *EFH* become known; and conſequentl *EHF* alſo. And laſtly in the Triangle *EFH* *HF*, *EHF*, *EFH* being given, the Parallaxe ſought *EF*, *EH* become known. *Q. E. F.*

PROPOSITION LXII.

Given two Azimuths of a Phænomenon, together with the Altitude of one of them, and the Time between the two obſervations of the Azimuth, to find the Parallax of the Phænomenon.

The ſame things being ſuppoſed in the ſame Diagram, after the ſame manner may *PXZ* equal to *PZX*, and *XZ* be found. The two Azimuths being given, the Angles *PXE*, *PZE* are given, and therefore the angles *XZE*, *ZXE* become known; by which, together with *XZ*, may *XE* be found. But becauſe the Altitude is given, its complement *XF* is alſo given, and therefore *EF* the Parallax ſought will become known. *Q. E. F.*

If, in the caſe of either of theſe Problems, the Phænomenon has any motion after an intire revolution of the Earth, when the Zenith *X* returns to the ſame point of the Heavens, the Phænomenon will be obſerved, not in *F* (as before) but in ſome other place, for inſtance, *G*; and *FG*, *XFG* become known by obſervation. Make therefore, as a whole Circle is to *FG*, ſo is the Angle *XPZ* to *FL*, and draw a great Circle *XL* cutting *ZH* in *M*. In the Triangle *XFL*, *XF*, *FL*, *XFL* being given, *XL*, *FXL* are alſo given; and therefore *ZXL* is alſo given. Laſtly, taking the points *L*, *M* inſtead of *F* and *E*, *HM* the Parallax of the Phænomenon may be found as before. *Q. E. F.*

There are other ſorts of Obſervations, whereby the Parallax of a Phænomenon may be Geometrically determined; as its ingreſs into the ſhadow of a Planet; and a Conjunction of Bodies ſeen between the Phænomena. But ſince theſe things do not belong to the Parallax conſidered in general, they more properly belong to another place in the following Books.

PROPOSITION LXIII.

THE *place of a Phænomenon in respect of the Ecliptic, and the Parallax of a given Altitude, as also the place of the Sun, and the hour of the Day being given, to find the Parallax of Longitude and Latitude of the Phænomenon.*

Let *VH* be the Horizon, (*Fig.* 41.) *E C B* the Ecliptic, and *Z C V* a Vertical Circle passing thro' the Phænomenon. Let *A* be the true place of the Phænomenon in it, *a* the apparent: and therefore *A a* the Parallax of Altitude Thro' *A* and *a* draw the Circles of Latitude *A B* *ab*, meeting the Ecliptic in *B* and *b*. Now (by *Prop.* 36.) there are given the Angle *V C B*, and arcs *VA*, *VC*, and therefore their difference *A C* But because the Parallax of a certain given Altitude is given, the Parallax of the given Altitude *VA* will also be given (by *Prop.* 47.) namely the arc *A a*. Consequently the sum or difference *C a* of these arcs *C A* and *A a* are given. Therefore in the Triangle *a C b*, besides the right Angle at *b*, there are given the Side *a C*, and Angle *a C b* from whence the Sides *C b*, *a b* will become known. But because the place of the Phænomenon *A* in respect of the Ecliptic is known and the point of the Ecliptic (by *Prop.* 36.) also known, the arcs *A B* and *C B* are known Therefore the difference between *a b* and *A B* namely the Parallax of Latitude become known; and the difference between *C b* and *C B* namely the Parallax of Longitude also. *Q. E. I*

The converse of this Problem is solved after the same manner; for if the Parallax of Longitude *Bb*, or the Parallax of Latitude, the difference between *b a* and *B A*, be given, *C b* or *a b*, (from the Place of the Phænomenon in respect of the Ecliptic already given, and from

thenc

thence the arcs CB and BA,) will be given. And since besides the Angle at C in the right angled Triangle Cba being given, Ca will be given, and consequently Va, since VC (by *Prop.* 36.) is already given: But VA is known; therefore the Parallax of Altitude Aa will not be unknown. If the Phænomenon, as the Sun, be in the Ecliptic, the case is much more simple.

But because the Parallax of the Celestial Bodies, even the nearest, is a pretty small Arc, therefore some construct this Problem of finding the Parallax of Longitude or Latitude from the given Parallax of Altitude, more expeditiously thus: Thro' A imagine the line $A\beta$ to be drawn parallel to Bb. Therefore in the small Triangle $A\beta a$, which is looked upon as rectilineal, there being given, besides the right angle at β, the angle βAa equal to ACB, and one of the Sides Aa the Parallax of Altitude, the other Sides $A\beta$ the Parallax of Longitude and $a\beta$ the Parallax of Latitude are found, by the bare resolution of a plane Triangle.

But if not the absolute Parallax of the Phænomenon be given, but the relative in respect of another more remote Phænomenon, that is, the excess of the Parallax of the nearer Phænomenon above the Parallax of the more remote; such an excess in a vertical Circle being given, the excess in regard of any other Circle will also be given.

For the absolute Parallax of a Phænomenon, is the excess of the Parallax of the Phænomenon itself above the Parallax of the Circle of a Sphere, which is nothing, because it is supposed infinitely distant.

SECTION VIII.

Concerning the Refraction of the Stars.

PROPOSITION LXIV.

ON the account of an Atmoſphere, which is denſer than the Æther, circumfuſed about the Earth, each Star appears more elevated above the Horizon in the ſame Vertical Circle, than it would appear were there no Atmoſphere at all.

Let *T* repreſent the Earth (in *Fig.* 42,) round which the Atmoſphere *AED* is diffuſed; *S* any Star; O the Spectator, placed on the Surface of the Earth. If there were no Atmoſphere at all, or if it were equal in denſity to the circumfuſed Æther, the Rays of Light would reach directly from *S* to *O*, nor would they ever be bent into a Curve line; becauſe in an homogeneous Medium. But if the Rays, after their paſſage thro' the Æther *SQ*, enter into a denſer Atmoſphere in *A*, they will be refracted towards the Right line *QAP*, perpendicular to the Surface of the Atmoſphere at *A*; as is evident from *Dioptrics.* But it is not probable that an Atmoſphere very conſiderably denſer than the Æther is immediately under it, and then continued all along to the Earth of the ſame denſity; or that all the Refraction of the Rays, that proceed from any Star, is perform'd in that ſingle Surface which ſeparates the Æther from the Atmoſphere, (as it is commonly believed,) the portions of the Ray from *S* to the outmoſt Surface of the Atmoſphere, and from thence to the Earth, continuing all the while Right lines; as with us a Ray paſſing out of Air into Water is only

only bent at the Surface of the latter, being ſtrait both before and afterwards. The Atmoſphere rather ſeems to extend itſelf to a very great height, and to be rarer in its higher and more remote parts, and denſer in the lower; on which account a Ray proceeding from a Star *S*, ſuffers ſome Refraction towards the perpendicular, in a diſtance from the Earth much greater than is commonly believed; and in its progreſs (as at *B*, *C* &c.) it is refracted the ſame way, becauſe the medium of the Atmoſphere, that it enters into, is more and more denſe, the nearer it comes to the Earth: And therefore a Ray after its entrance into the Atmoſphere (by reaſon of the perpetual refraction at the Spherical ſuperficies concentric to the Earth of a continually denſer medium) is curved in all its parts, and concave towards *T*. But an Eye plac'd at O ſees a Star in the laſt part of the Ray *ABCO* from O produc'd, namely in σ; for this part only affects the Sight. But the part OC of the Ray produced, or the Right line touching the Curve *ABCO* in O, namely, Oσ, is leſs inclin'd to the Horizon of the Spectator *HO*, than the Right line connecting *S* and O; becauſe the line *ABCO* is concave towards *T*: And therefore, to an Eye at O a Star appears in σ, higher above the Horizon, by reaſon of the circumfuſed Atmoſphere, than if there were no Atmoſphere at all.

Beſides, the Curve that the Ray propagated oblique thro' the Atmoſphere, puts on, is in the ſame Plane produced thro' the Star and the Center of the Earth. For in *Dioptrics* 'tis demonſtrated that the former part *AB* of the Curve (conſiſting, as it were, of Right lines) is in the Plane that the Right lines *AS*, *A♀*, (becauſe in the Plane of Inflection;) that is, in the Plane

produced and passing thro' the Star *S* and Center of the Earth *T*; for (by *Prop.* 5. *B.* 1. *Spher. Theodos.*) the Right line *Q A P* produced, passes thro' the Center *T*; and its second part *B C* is in like manner in the Plane passing thro' *B A*, and the perpendicular thro' *B*; that is, in the same Plane with the former: and so on in the other parts of the Ray *A B C O*. Therefore that intire Curve is in the same Plane produced, passing thro' the Star and the Center of the Earth; and therefore the Right line *O σ* touching the Curve, (because a small particle of it produced in a Right line) is in the same Plane: But this Plane is vertical to the Observer O, (for it passes thro' the point O, and the Center of the Earth) therefore the Star, by reason of the Refraction made in the Atmosphere, appears in the same Vertical, but higher above the Horizon. *Q. E. D.*

SCHOLIUM.

It appears, from the demonstration of the preced. *Prop.* that the Refraction is the same, whether the Star be nearer the Earth, or farther off (for instance, in *s* or in *S*,) provided their apparent Altitude above the Horizon be the same: For in this case the Right line *σ O* revolving above the Axe *T O*, reaches to the apparent place of *s*. Besides, 'tis probable that the Atmosphere spread about the Earth does not extend it self to the Moon, much less to the more distant Stars; and therefore the strait part *s A* of the line *S s A B C O*, (before it begins to be bent into a Curve by the Atmosphere) will graze upon the next Star *s*. And consequently the Ray proceeding from it (namely *s A*) is bent in the Atmosphere into *A B C O*, after the like, nay, after the very same manner, as *S A* propagated from the more distant Star; since it is

all

all one (as to the Refraction made in the Atmosphere) whether the Ray begin from *S* or *ſ*. Therefore 'tis contrary to the nature of Light, that the Rays of a nearer Star should be refracted otherwise than (all other circumstances remaining the same) that of a more distant one. 'Tis true that the Angle $\sigma O S$ is greater than the Angle $\sigma O ſ$, but the difference of them $ſ O S$ is owing to the excess of the Parallax of the nearer Star above the Parallax of the more distant Star, and not to the Refraction.

POPOSITION LXV.

THE *lower Stars are more elevated by reason of the Refraction at the Atmosphere, than the higher, other circumstances remaining the same.*

In *Dioptrics* 'tis demonstrated, that in the same Mediums, the more oblique the incidence of the Ray is, the greater the Angle between the refracted Ray and the incident Ray produced; that is, in the present case [*Fig.* 43.] that the part *AB* of the Ray *ABCO*, proceeding from the lower Star *S*, contains a greater Angle with *SA* produced, than the part *FG* of the Ray *FGKO* proceeding from the higher *R* does with *RF* produced; because *SA*♁ is a greater Angle than *RFL*, supposing that *LF* produced falls upon *T*. After the like manner *BC* will contain a greater Angle with *AB* produced, than *GK* with *FG* produced, and so on; that is, the line *ABCO* is more curved in all its parts than *FGKO* in its corresponding parts, or such as are equally distant from the Earth. And therefore a Right line touching the Curve *ABCO* in *O* contains a greater Angle with the Right line *OS*, than a Right line at the same place, touching *FGKO* with the line *OR*. The lower

lower Star therefore is more elevated by the Refraction than an higher. *Q. E. D.*

COROLLARY 1.

The Place of a Phænomenon when it is in the Zenith is not changed by the Refraction: For the Rays in this case falling perpendicularly upon all the Surfaces of the Mediums, of which the Atmosphere consists, are not refracted; but going directly to the Eye in O, make the Phænomenon appear in the same Right line as it would appear were the Atmosphere away. And for the same reason the Eye being placed in the Center *T*, the Refraction made at the Atmosphere would have no power to change the apparent place of the Star; for every point in the Heavens would be a Zenith to it.

COROLLARY 2.

An account may from hence be given, why the Sun or Moon appears of an Oval Figure near the Horizon. For since the upper Limb appear but a little more elevated than it should be, and the lower a great deal, the latter by the Refraction, will appear to approach the former. And therefore the erect or vertical Diameter of the Luminary seems contracted or shortened, while the transverse or horizontal one is not so, because its extremities are equally elevated by the Refraction. For the same reason the distance of two Fix'd Stars by observation is sensibly less, (if it be measured by an Instrument,) when, both being in the same Vertical, one of them is nearer the Horizon, than when both of them, by reason of their greater Altitude, are free from all Refraction.

COROLLARY 3.

On this account it is also, that the Sun and the other Stars rise sooner above the Horizon, and set later, than they ought, according to their

their places determined by the aforeſaid Methods. For the Ray flowing from ſuch points of the Heaven as are but a little below the Horizon, are ſo incurvated in the Atmoſphere, that the Right lines touching them at their extremities, where they enter the Eye, being produced, are above the Horizon. For which reaſon the Moon has been ſeen to be eclipſed, tho' the Sun was above the Horizon; whereas one of the Luminaries at that time being oppoſite (by *Prop.* 18. *B.* 1.) to the other, and above the Horizon, the other muſt neceſſarily be below it. 'Tis to the ſame Atmoſphere diffuſed round the Earth, that its Inhabitants owe the Sun's ſhining to them longer than it would do were there no Atmoſphere; and that when the Sun is kept from inlightening any given Inhabitant by reaſon of the Spherical Surface of the Earth, thick darkneſs does not immediately cover him, but he enjoys a Twilight for a conſiderable time, the Light being gradually extinguiſhed.

PROPOSITION LXVI.

TO define how much a Star appearing in a given Altitude is elevated by the Refraction; and to make a Table of the Refraction of the Stars.

Chuſe a Star that has no ſenſible Parallax; namely ſome one of the Fix'd Stars that is elevated much above the Horizon of the Obſerver. Let its Place be determined (by *Prop.*26.) by obſervations made when it was elevated pretty much, and therefore when it was out of the Effects of a ſenſible Refraction. Let the time be marked of its coming afterwards to the given Altitude, either aſcending or deſcending, and find then by Calculation, the Altitude, for the time marked, of the Star (whoſe place is known,

known,) and that will want of the obſerved Altitude, as much as the Star is elevated by the Refraction in that given apparent Altitude.

After this manner may a Table of the Refraction of all the Stars be beſt conſtructed, if the true Altitudes of the Fix'd Stars, agreeing to the times exactly reckoned by a Pendulum Clock, be compared with the Altitudes of the ſame Star, in the ſame apparent points of time: For the difference between the true and apparent Altitude is the Refraction agreeing to that Altitude. And ſince the Refraction of a Fix'd Star at the greateſt diſtance, and of any other Star at a leſs diſtance, is the ſame, in the ſame apparent Altitude (by *Schol. Prop.* 64.) 'tis evident that a Table of the Refraction of all the Stars is made. *Q. E. F.*

SCHOLIUM.

If the *Dioptric* denſity of the Atmoſphere were known in any given diſtance from the Earth, or the ratio of the Sine of the Angle of incidence to the Sine of the refracted Angle, in the paſſage of a Ray thro' its Surface concentric with the Earth, whoſe diſtance from the Center is known; then by Geometry you may find out the nature of the Line, into which the Ray is bent by reaſon of the continued Refraction of the Atmoſphere. Conſequently, the Angle being given by obſervation, by which a Star in a given Altitude is elevated more than it ſhould be on the account of the Refraction, the Angle may be defined, by which it is elevated in any other Altitude propoſed more than it ſhould be, the Atmoſphere continuing the ſame; that is, a Table of Refraction may be made for every degree of a Star's Elevation. But becauſe our ignorance of the Law of Refraction obſerved in the Atmoſphere hinders all

this

this, 'tis better to make the Table by Obſervation, as in the preceding. But beſides, the diverſity of the Atmoſphere in different places, and the change of it in the ſame place, on the account of which it happens that the ſame Table is not fitted for all places, is an impediment to both methods of making a Table of Refraction. For in places near either Poles the Refraction is greater in the ſame diſtance from the Horizon, (as is well known from the obſervation of ſome *Dutch Men* who Wintered in *Nova Zembla*;) and in the ſame place at different times of the Year, 'tis certain, not only from the obſervation of the *Dutch*, but of other conſiderable Obſervers, who make the Winter Refraction greater than the Æquinoctial, and that again greater than the Summer one, and form intire Tables of Refraction built upon this change. But the hourly change of the Refraction is demonſtrated by the Tops of Mountains being ſometimes viſible at a given place, and ſometimes inviſible, tho' there be no darkneſs or miſt between: To this purpoſe alſo do the Experiments mention'd by *Hugens*, in his *Treatiſe of Light*, tend and ſerve; namely that the lower part of a Tower or any other thing that is immovable is ſeen in the Morning and Evening, but the upper at Noon thro' a Teleſcope continuing fix'd all the while; and that at the ſame place, ſometimes the top of a more remote Tower has been viſible, riſing above the top of one nearer, and ſometimes it is inviſible. Now all theſe things, beſides the hourly change of the Refraction of the Rays in the Atmoſphere, demonſtrate the Curvity of them, (from their nature, as was ſhewn above,) ſince they are propagated obliquely from an higher to a lower place, and *vice verſa*.

It

It muſt be confeſſed that all this change of the Refraction is made within pretty narrow limits: For the beforeſaid obſervations of the change of the apparent Altitude of the Sun, Mountains and Towers are made only in a ſmall Elevation above the Horizon, nor do they reach much beyond three degrees above it. The viciſſitude and changeableneſs near the Earth, of Vapours ariſing from it, as alſo of Heat and Cold, by which they are rarified and condenſed, is ſo great, that nothing certain can be laid down about the denſity of the Medium. Beſides, the inclination of the Rays of Light is ſo very oblique at their incidence on the Atmoſphere, when the Phænomenon is not elevated above three or four degrees, that its place is changed very much by the Refraction. Nay, ſo denſe yet pellucid a Cloud as it were of Vapours lies ſometimes upon the Surface of the Earth, that to an Eye placed above without it, the viſible Objects below it, namely the very low ones, continue unſeen, becauſe the Rays do not penetrate the Surface of the upper and rarer Medium, but are reflected by it downwards; as it is evident from *Dioptrics*, that it ought to be: But ſuch things as are immediately above and without it, namely ſuch as are a little higher, beſides that they are ſeen by the direct Ray, appear alſo inverted by a reflection made at the Surface of the abovementioned Cloud, juſt as if it were an Horizontal Speculum. But theſe things neither belong to this place, nor directly to Aſtronomy. They ſhow in the mean while, that no credit is to be given to any Table of Refraction of the Stars below the third degree of Altitude; nor conſequently to Obſervations made in that

Alti-

Altitude for determining the place of the Phænomenon.

If a Perſon has a mind to make a Table of the Refraction of the Fix'd Stars, not from Obſervation, but upon the Principles of *Dioptrics*, he may proceed the following way very ſafely. Let the Atmoſphere, which is extended 40 or 50 Miles high, be ſuppoſed to be divided by 8 or 10 parallel Superficies, into as many Mediums of different denſities; ſo as that the Medium which lies between the two next Superficies, may be of the ſame denſity, that at the ſaid Superficies may be changed inſtantly into a rarer above, and a denſer below: Not that it is really thus in nature; for no doubt the denſity of the Atmoſphere in its deſcent, is increaſed by the ſmalleſt degrees. Let theſe 8 or 10 Mediums be ſo tempered in regard of one another by trials, that in one, two or more Altitudes (for the more they are, the more accurate will the Table be) the total Refractions made at all theſe Mediums, may be the ſame with the Refractions that are found by the accurateſt Obſervations, to agree to thoſe reſpective Altitudes. In which caſe the Refraction made in any aſſumed Altitude at the ſaid Mediums, and known by Calculation, according to the Laws of *Dioptrics*, is very nearly equal to the Refraction in the Heavens, agreeing to the ſame Altitude. After this manner alſo will the Line be determined very nearly, into which the Ray is bent in its paſſage thro' the Atmoſphere. But in finding, by this method, the Refraction correſponding to an Elevation three or four degrees leſs, the lower, or two or three lower, ought to be ſuppoſed a little rarer; for they really are ſuch, in the vicinity of the Earth, which is heated.

SEC-

SECTION IX.

Of the Tables of the First Motion, and of the Fix'd Stars, depending upon the Second Book.

THO' all the Particulars that belong to the First Motion may be found, by the Methods demonstrated in this Book, as occasion requires; yet because there are some things of more frequent use, *Tables* are made for them, that they might be more expeditiously taken out of them, which would otherwise require a new Calculation every time they are wanted. And besides Tables taken from other disciplines, as of Right lines inscribed and circumscribed in a Circle, of Logarithms, *&c.* Astronomical ones are also made; and such of them as depend upon this Book are of two sorts; namely such as are the same to the Inhabitants of the whole Earth; which regard either the First Motion, or the Fix'd Stars; or such as only hold in some certain place of the Earth.

PROPOSITION LXVII.

TO *describe Tables common to all the Inhabitants of the Earth, relating to the First Motion.*

The Equator and Ecliptic are the same to all the Inhabitants of the Earth, and they are of considerable and frequent use in Astronomy: The former, because it is the mean, and as it were the Standard of the First or apparent Diurnal Motion of the Heavens; the latter, because it is the Way and Path of the Sun, (who is the governor of the Days and Seasons.) The mutual

mutual habitude and reſpect of theſe Circles to one another in all their points is diſpoſed into Tables: That is, to every degree of the Ecliptic, marked with its proper name, (that is, the number it bears in the Sign) are annexed to the correſponding Declination from the Equator, Right Aſcenſion, and Angle that the Ecliptic comprehends with the Meridian; as is ſhewn in *Prop.* 20. And therefore theſe three Tables are put together in one very commodiouſly, under the univerſal Title of *The Declination, and Right Aſcenſion of every degree of the Ecliptic, and of the Angles of the Ecliptic with the Meridian.* And becauſe in ſuch Tables, either the ſame number recur, or the ſame increaſed by a Quadrant, a Semicircle or three Quadrants of a Circle; in the making of them there will be no need of carrying the Calculation beyond one Quadrant, ſince the remaining part of the Table flows naturally from the former, either by the change of the firſt Column or by Addition only.

Beſides, ſince the intire apparent revolution of the Celeſtial Equator makes a natural Day, and proportionally in the parts of this revolution, by reaſon of the equability of the diurnal Motion of the Earth; they make a Table, out of which the time correſponding to each part of the revolution of the Equator, and on the contrary, the part of the revolution correſponding to any time, may be taken. And becauſe among politer Nations, where Aſtronomy is cultivated, the natural Day is divided into 24 Hours, and the Hour in 60 Minutes, every one of which again is divided into 60 Seconds, and ſo on; the aforeſaid Tables are accomodated to this diviſion of the Time.

PROPOSITION LXVIII.

To describe the Tables which relate to the Fix'd Stars, that are common to all the Inhabitants of the Earth.

The Places of the Fix'd Stars, in regard of Longitude and Latitude, are disposed of by the methods explain'd in *Prop.* 29. Such a Table (or Catalogue) as this, is the foundation of the whole Science of the Stars: And because the places of the Planets are found, by knowing the places of the Fix'd Stars, Astronomers are very careful in noting the places of the Fix'd Stars, as are in the Zodiac; not only those as are only seen by the naked Eye, but such also as are seen by a small Telescope.

Because the Longitude of the Fix'd Stars continually increases, by reason of the regress of the Equinoctial Points, the *Table of this Regress* is annexed to the Catalogue of the Fix'd Stars; that is, 'tis shewn how much it is in a Year, any parts of a Year, or collection of Years, so that the Tabular Longitude of a Fix'd Star may be very easily corrected by this means. *Copernicus* and others following him, reckon the distance of the Fix'd Stars in the Ecliptic toward the East, from the preceding of the two in the Horn of Aries, as was said before: By adding to this Arc (in any constant Fix'd Star) the distance of the Vernal Equinoctial point, perpetually augmented from the said first Star of Aries, to be taken out of that other Table, you make the Longitude of the Fix'd Star from the Vernal Equinox.

Because the Right Ascension and Declination of some of the more considerable Fix'd Stars are of more frequent use; and to find them out by the Longitude and Latitude given (by Conversf. *Prop.* 27.) as often as there is occasion of them, would

would be very tedious; ſuch as have a mind to ſpare others ſo much trouble, form to a given time, *a Table of the Right Aſcenſion and Declination of the more conſiderable Fix'd Stars*, adding the difference of each for a certain number of Years, in which they ſuppoſe their change, as to ſenſe, to be equable.

To the abovementioned Tables of the Motion of the Fix'd Stars is alſo added *a Table of the Refraction of the Stars*, made for the different Altitude of them above the Horizon (by *Prop.* 66.) The Altitude of any Star obſerved is to be leſſened by the correſponding Refraction, that it may become true; or the true Altitude (found from the true place, at a given time, by Calculation) is to be increaſed by the ſaid Refraction, that it may become the apparent Altitude. This different Altitude of the Star from Refraction induces a diverſity of its Place in regard of Longitude or Latitude, or in any other Circle, after the ſame manner with that ſhewn before in the Parallax. We have reckoned this Table of Refraction here among the Tables of the Fix'd Stars, tho' it might more properly be refer'd to the preceding, as what is common to every Celeſtial Phænomenon: Others in the mean while will think it ought to be reckoned among the Tables that are to be deſcrib'd in the following Propoſition, ſince different Tables of Refraction are requiſite for the different denſity of the Atmoſphere in different places.

PROPOSITION LXIX.

TO enumerate the more common Tables, wherein the ſituation of the more conſiderable Places upon the Earth, in regard of one another, and in reſpect of the Equator, is deſcribed, and thoſe wherein the Time reckoned in any of them is reduced to the Time reckoned according to the cuſtom of any other.

In the Catalogue of the moſt conſiderable Places, you have the Elevation of the Pole in that place ſet down by it, (determined by *Prop.* 17;) to which the Latitude of the Place is equal. There is alſo ſet down the difference (found by *Prop.* 32.) of the Firſt Meridian (whether it be ſuppoſed to paſs thro' the place where the Aſtronomer lives, or through the *Azores* or *Fortunate Iſlands*) and of the Meridian of that place, and it is expreſſed in parts of the Equator, or in Hours and parts of an Hour. Thus we have the ſituation of theſe places to one another, and to the Equator ſettled, and may make uſe of it as occaſion requires in Geography or Hydrography. The Hour reckoned in one of the Places, with the difference of the Meridians, expreſſed in Hours and parts of an Hour, added to it, gives the Hour reckoned in the more Eaſtern place; but ſubtracted from it, ſhews the Hour reckoned in a more Weſtern place, provided both places make uſe of the ſame ſort of Hours; but if they uſe a different ſort, they are to be reduced to the ſame by a proper reduction.

You have alſo Tables for the more expeditious reduction of any Time in one of the places expreſſed by any Calendar, and reckoned from any known Epocha, into Time of any given form in the other place. For ſince (by *Prop.* 11. and 12.) both the Epochas may be compared together, and the various forms of the Years among different Nations, the grounds and method of conſtructing theſe Tables (which are as neceſſary for Men of Buſineſs and Hiſtorians, as Aſtronomers) is evident.

Beſides, the Circles diſtinguiſhing the Climates and Parallels are reduced into Tables by ſome. For if you begin at the Equator, the quantity of the longeſt Day is had, at the end of each Climate

mate and Parallel, or the beginning of the next; that is, the continuance of the Solſtitial Point towards the viſible Pole above the Horizon. There are alſo given the Declination of the Solſtitial Point (namely the Obliquity of the Ecliptic) and the Right Aſcenſion: The Elevation of the Pole in that place therefore is alſo given; that is, the diſtance of the Circle bounding the Climate or Parallel propoſed from the Equator.

POPOSITION LXX.

TO *deſcribe the more common Tables, by which the Problems of the Firſt Motion, in different places of the Earth, are more readily ſolved.*

For the finding out the places and times of the Riſing and Setting of the Fix'd Stars and other Celeſtial Bodies, whoſe Places are known, and ſolving other Problems about them, Calculation is commonly uſed, as occaſion requires. But becauſe the uſe of the Sun's Motion is more conſiderable and frequent, and the Sun itſelf in its Annual courſe, is in different degrees of the Ecliptic; Tables are made for all Latitudes, to ſhew the reſpect of theſe points to the Horizon: that is, for every degree of Latitude from the Equator, the Difference between the right and oblique Aſcenſion of every degree of the Ecliptic (found by *Prop.* 33.) is added to that degree of the Ecliptic expreſſed by its name, out of all which a Table is made, or rather a heap of Tables; for indeed every degree of Latitude ſhould have a Table, extending to the ſeveral degrees of the Ecliptic. Theſe ſort of Tables are called *Tables of the Aſcenſional Differences.*

The Aſcenſional differences being thus reduced into Tables, the oblique Aſcenſions and diurnal Arcs of every one of the degrees of the Ecliptic are in like manner eaſily reduced into Tables by

bare Addition and Subſtraction; and from thence the Hours of the Riſing and Setting of the Sun, when in thoſe points. But theſe Operations are ſo eaſy, as that they are uſually left to the pains of every one that makes the Calculation; excepting, that generally an Aſtronomer, that is pretty much taken up in ſupputations, has them already calculated for the place of his own Habitation. And ſometimes alſo he has by him Tables (made by *Prop.* 33.) of the *Ortive Amplitudes* of the degrees of the Ecliptic, either for ſeveral Latitudes, or at leaſt for his own.

There is another ſort of *Table* (to be made by *Prop.* 35.) of the *Angle of the Eaſt*, namely that which the Ecliptic makes with the Horizon, for the different points of the Ecliptic riſing in a given Latitude, or (which comes to the ſame) of the Altitude above the Horizon of the Nonageſim Degree from the Eaſtern point. And becauſe in Calculating Eclipſes of the Sun and other things, there is very great uſe of this Angle, Tables of it are made for every degree of Latitude from the Equator to the Pole.

And ſometimes Tables of the Beginning and Ending of the Morning and Evening Twilight are made (by *Prop.* 39.) for the degree of the Ecliptic that the Sun is in, and for different Latitudes, or at leaſt for the Latitude of their own Habitation.

In making all theſe ſorts of Tables, there are ſeveral compendiums of the calculation and pains that occur, ſuch as are ſhewn in *Prop.* 67; and a ſhorter and more elegant way of ordering theſe Tables may be had from thence.

THE

THE
ELEMENTS
OF
Aſtronomy,
Phyſical and Geometrical.

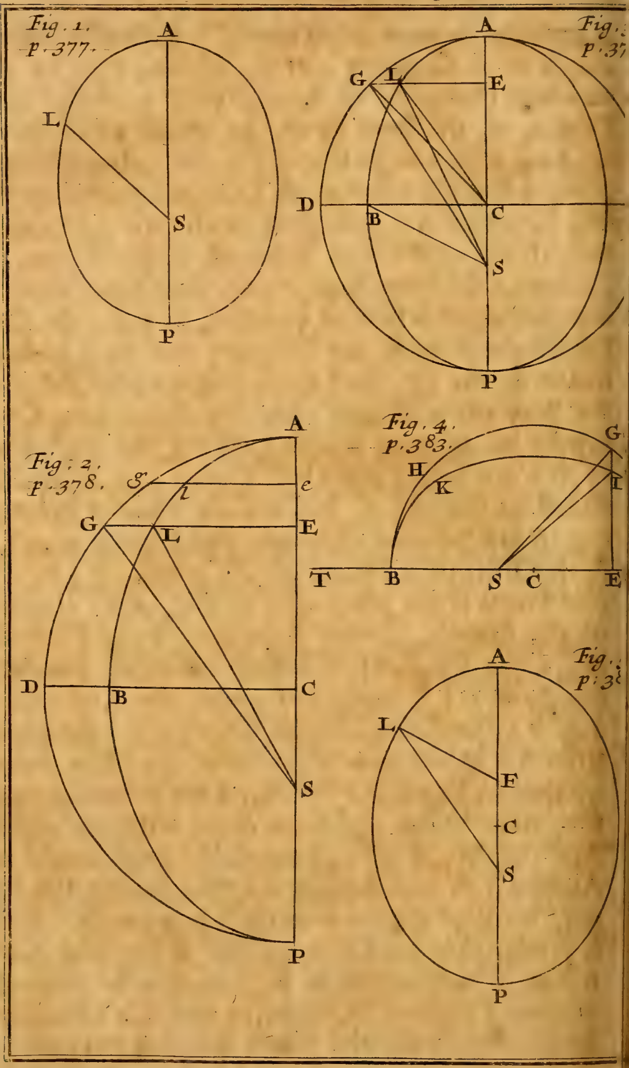
The THIRD BOOK.

Of the Theory of the Primary Planets.

IN the Firſt Book, we have ſhewn what Laws a Planet muſt move by, to deſcribe an Ellipſe about the Sun placed in one of its Foci, which Figure only the Phænomena will admit of, as *Kepler* has ſhewn at large, in his *Commentaries of the Motions of Mars*, and all Aſtronomers now allow of. The ancient Aſtronomers themſelves, together with *Ptolemy*, (who were for admitting nothing but perfect Circles in the Heavens, which ſeem'd the moſt proper for Bodies that were

not ſubject to generation or corruption, ſuch as they took the Heavenly Bodies to be,) almoſt come into this opinion, being compell'd by the Phænomena themſelves, ſince they have imagined a motion really unequal in the Excentric, but equal in the Equant Circle, whoſe Center was as far beyond the Center of the Excentric, as the Center of the true Motion was on this ſide of it; that is, ſince they biſſected the Excentricity, and deſcribed the Orbit of the Planet, from the point of biſſection as a Center. For ſuch an Excentric circular Orbit of a Planet does not differ from an Elliptic Orbit, whoſe Foci are the ſaid Centers, excepting that the former is ſomething thicker about the middle, but the latter ſomething ſmaller; which difference of the thickneſs of the Orbit is almoſt inſenſible in moſt of the Planets, by reaſon of the ſmallneſs of the Excentricity, and not diſtinguiſhable by the obſervations of the Ancients. Tho' *Kepler*, in *Chap.* 44. of the ſaid *Commentaries*, has made it out very clearly and unqueſtionably, from the obſervations of *Tycho*. 'Tis true *Ptolemy* uſes an Excentric in the ſimple Theory of the Sun, and does not think that the Excentricity of the Orbit is to be biſected; but in every one of the other Planets, beſides the Epicycle (by which he explains the Phænomena really ariſing from the motion of the Earth, which he did not acknowledge) he uſes (*Ch.* 5. *B.* 9.) an Excentric deſcribed upon the Center biſecting the Excentricity. Nay, *Copernicus* himſelf took the perfection of the motion and way of the Planets to be ſo firm, that upon this account he thinks (*Ch.* 2. *B.* 5.) the biſection of the Excentricity ought to be rejected, becauſe it is abſurd to grant *that an equality of circular motion can be made upon a ſtrange Center and not upon its own*, (which that it is was once *Ptolemy*'s opinion, is evident

Plate 1, Book 3.



dent from *Chap.* 3. *Book* 3. where he ſays *that all the motions of the Stars are equal and circular by nature*; that is, (as he himſelf explains it) *that all the lines, which are underſtood to carry about the Stars or their Circles in all ſimply equal Times, intercept equal Angles at the Centers of each Circulation*, tho' he afterwards changed, in *B.* 9. already quoted,) and *Chap.* 4. of the ſame Book, he would rather have an Epicycle and a ſimple Excentric introduced, performing the ſame as an Excentric and Equant do; or even a Concentric with two Epicycles. But thoſe exact and repeated obſervations of *Tycho* had not then been made, that *Kepler* wholly depends upon when he demonſtrates that the Orbits of the Planets are Elliptic; tho' *Ricciolus* inſinuates by the by, that *Kepler* began to ſuſpect that the Way of a Planet thro' the Æther was Oviform; his natural ſagacity taking the firſt hint of it from ſeeing the Oval Figure that *Reinholdus* added in the end of *Purbachius*'s *Theorics* belonging to the Moon. Therefore in ſuch a true Elliptic Orbit of a Planet *ALP*, where *S* [*Fig.* 1.] one Focus is the place of the Sun, *ASP* the line of the Apſides, *A* the Aphelion, *P* the Perihelion, *L* the Place of a Planet taken at pleaſure, the Area *ALS* contained under the Right lines *SA*, *SL*, and the Elliptic Curve *AL* (computed from *SA in conſequentia*) is called the *Mean Anomaly*, that is, Equable; for this Area increaſes equably, namely in the ſame ratio with the time: And the Angle *ASL*, contain'd by the line of the Apſides *SA*, and Radius *SL* that carries the Planet, is called the *Coequate* or *True Anomaly*. For it is proper that the Cultivators of a Phyſical Aſtronomy, ſhould recede as little as may be from the ancient Terms of Art; being not only eſtabliſhed by uſe, but really the fitteſt for that purpoſe.

SEC-

SECTION I.

Some General Things belonging to the Theory of all the Planets.

PROPOSITION I. LEMMA.

IF *upon the greater axe* PA, *of the Ellipse* PBA [Fig. 2.] *whose Center is* C, *as a Diameter, you describe the Circle* PDA, *and from a point* G *taken at pleasure in the Circumference of the Circle, a Perpendicular* GE *be let fall on* PA, *intersecting the Ellipse at* L; *and from some point in the Axe* AP (*for instance from one of the Foci as* S) *to* G *and* L, *the Right lines* SG, SL *be drawn; the trilinear Figure* AGS *contained under the Right lines* SA, SG, *and the Circular Curve* AG, *will be to the whole Circle, as the trilinear* ALS *contained under the Right lines* SA, SL *and the Elliptic Curve* AL *is to the whole Ellipse.*

Draw eg parallel to EG, meeting the Diameter in e, the Ellipse in l, and the Circle in g. Then (by *Prop.* 21. *Book* 1. *of the Conics of* Apollonius) el will be to CB in the subduplicate ratio of the rectangle under Pe, eA, to the Rectangle under PC, CA, or CDq: And eg to CD is also in the subduplicate ratio of $Pe \times eA$ to CDq; therefore (by 11 *Elem.* 5.) el is to CB, as eg to CD; and therefore $el : eg :: CB : CD$; that is, in a given ratio. And since this is always done, all the el insisting upon EA, will be to as many eg; that is, the Elliptic Semi-segment ALE to the Circular Semi-segment AGE, in the same given ratio of CB to CD. Again, (by *Prop.* 1. *Elem.* 6.) the Triangle LSE, is to the Triangle GSE as LE to GE; that is, (as has been already shewn) as CB to CD; from whence (by *Prop.* 12. *Elem.* 5.) ALE and LSE together, *i. e.* ALS is to AGE and

GSE

GSE together, or *AGS*, in the ratio of *CB* to *CD*: And after the like manner, in that ſame ratio of *CB* to *CD* are all the *el* inſiſting upon the whole *PA* to as many *eg*; that is, the Semi-ellipſe *PBA* to the Semicircle *PDA*. And therefore *ALS* is to *AGS*, as the Semi-ellipſe *PBA* to the Semicircle *PDA*, or as the Ellipſe to the Circle. And on the contrary the trilinear *ALS* is to the whole Ellipſe, as the trilinear *AGS* to the whole Circle. *Q. E. D.*

Conſequently, if the Semi-ellipſe *PBA* were propoſed to be divided into two Segments having a given ratio, by a right line *SL* proceeding from a given point *S* in the Axe, it is ſufficient to divide the Semicircle *PDA* (in ſome reſpect more known than an Ellipſe) by a right line *SG* in the ſaid ratio, and to let fall a perpendicular *GE* from the point *G*, on the Axis *PA* meeting the Ellipſe in *L*: For the right line *SL* divides the Semi-ellipſe *PBA* in the ſame ratio as the Semicircle *PDA* is divided by the right line *SG*: Or if *ABP* be an Elliptic Orbit of a Planet, in one of whoſe Focus *S*, the Sun is; as the Mean Anomaly of the Planet in *L* is repreſented by the trilinear figure *ALS*, ſo it may alſo be repreſented by the trilinear figure *AGS*.

PROPOSITION II.

THE *Orbit of a Planet being given, to determine by calculation, as many Mean Anomalies and correſponding True ones, as you pleaſe.*

Let the Elliptic Orbit given be *ALBP*, [*Fig.* 3.] in one of whoſe Foci *S* the Sun is placed. Let *A* be the Aphelion, *P* the Perihelion, *C* the Center, *CA* or *CP* the Mean diſtance of the Planet from the Sun, *CBD* perpendicular to *AP*. In the Circle *ADP*, deſcribed about the greater axe as diameter, take at pleaſure from *A*, the arc *AG*, or angle *ACG*, expreſſed by

by the number of Degrees and Minutes (as is uſual.) And becauſe (by 23 *Elem.* 6.) the arc *AG* is to the whole circumference, as a ſector *AGC* to the whole circle, if the Circle as well as the Circumference be ſuppoſed to be divided into 360 equal parts, or circular ſectors correſponding to the degrees of the circumference; the ſame number that expreſſes the arc *AG*, expreſſes alſo the circular ſector *AGC*; the latter in ſmall ſectors, the former in ſmall arcs, either of theſe *Kepler* calls the *Anomaly of the Excentric.* But from the dimenſions of a Circle, (accurate enough for Calculation,) the ratio of the ſquare of the Semidiameter to the whole Circle is given; and from the ſpecies of the Orbit given, the ratio of the mean diſtance *BS* or *CA*, and alſo of the leſſer Semi-axe *BC* to the Excentricity *SC* is given; and from thence the ratio of the Triangle *BCS* to the ſquare of the Radius; and from the ratio of the arc aſſumed are *AG* to the quarter of the Circumference *AD*, is given the ratio of the Sines of them, *viz.* *GE* to *DC*, and conſequently of *BC* to *GE*, that is, (by 1. *El.* 6.) the ratio of the Triangle *BCS* to *GCS*. In the parts therefore, of which the Circle contains 360, there are given the ſector *AGC* and the Triangle *GCS*, (whoſe analogous one *LCS* is call'd the *Phyſical part of the Equation*, as the angle *SLC* the *Optical part*;) and therefore the trilinear aggregate of them *AGS*, which in the preceding *Prop.* is ſhewn to be proportional to the trilinear *ALS*, and conſequently may juſtly repreſent the Mean Anomaly. For if the whole area *ABP* of the Elliptic Orbit be imagined to be divided into 360 equal parts, the part of it *ALS*, namely the Mean Anomaly of the Planet in *L*, will contain as many parts of it, as *AGS* contains ſimilar parts of the circular Area *ADP*; and ſo will be expreſſed by the ſame number.

Plate 2. Book 3.

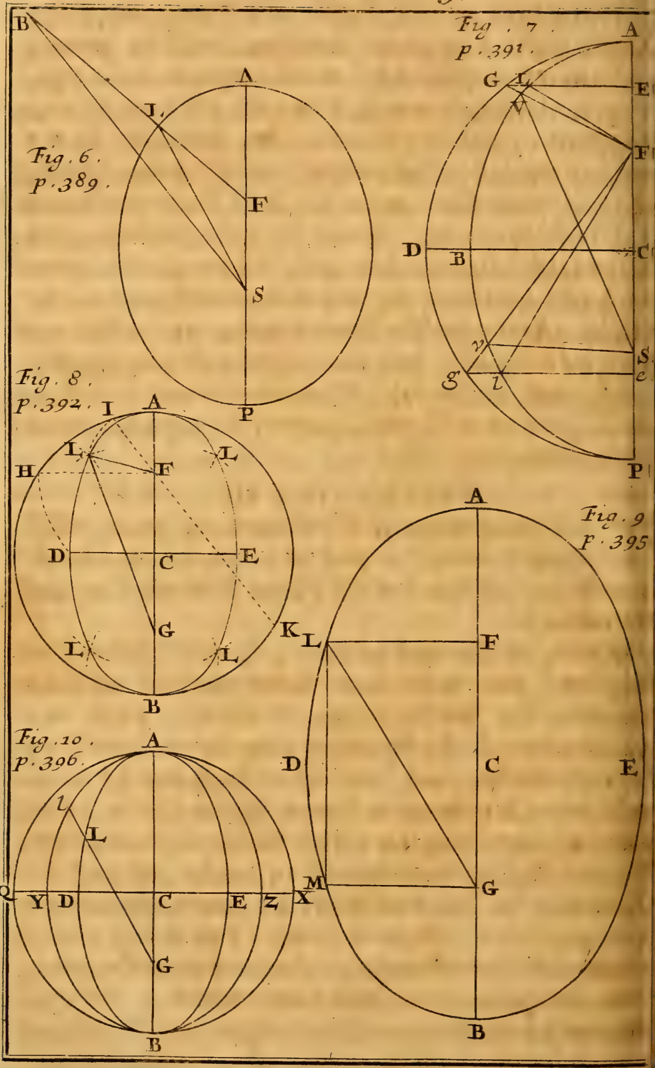


Fig. 6. p. 389.

Fig. 7. p. 391.

Fig. 8. p. 392.

Fig. 9 p. 395

Fig. 10. p. 396.

ber. Again, in the triangle GCS, the ratio of the Sides GC, CS being given,(the Mean diſtance of the Planet from the Sun to the Excentricity) and the contained Angle GCS, (the complement to two Right of the Angle ACG aſſumed,) the Angle GSC (by 41 of the *Data*) will be given; and therefore alſo ASL the Coequate Anomaly, namely the Angle whoſe Tangent EL is to EG the Tangent of the Angle ASG, found before, in the ratio of the leſſer ſemi-axe CB of the given Orbit, to the greater ſemi-axe CD. And ſince the mean and true Anomaly, found by calculation, agree to the ſame aſſumed Arc AG, they will alſo agree with one another; or one will correſpond to the other. And after the ſame manner any other arc may be aſſumed, and the agreeable Anomalies, (*viz.* mean and true) determined thence by calculation, which will conſequently be correſpondent to one another.

PROPOSITION III.

THE mean Anomaly of a Planet being given, whoſe Orbit is given, to find by calculation the true Anomaly and Diſtance of the Planet from the Sun, and the converſe.

Kepler, at the end of *Part* 4 of his *Celeſtial Phyſics*, ſays, that there is no direct method of Calculation for ſolving this Problem. How the calculation of the Problem may be perform'd by an indirect method, and fit enough for Aſtronomical uſes,is now to be ſhewn. In the Circle ADP, (*Fig.* 3.) touching the Orbit in the Aphelion and Perihelion, let the arcs AG or angles ACG of the Anomaly of the Excenter, be taken, exceeding one another by ſingle degrees. For every one of them (by the preceding) find the correſponding Anomalies, *viz.* the mean and the true. When a Canon of the Mean and correſponding Coequate Anomaly, for a given Orbit of a Planet, is made

to

to each particular Arc of the Anomaly of the Excentric, look among them for the mean Anomaly given; the correſponding true Anomaly is that which is ſought. But if the mean Anomaly propoſed, is not found among thoſe in the Canon, the true Anomaly ſought will be found (after the manner commonly known) by a proportional part. If you are to work without Tables, *Kepler* adviſes the uſe of the Rule of Poſitions (call'd alſo the Rule of Falſe;) by ſuppoſing the Anomaly of the Excentric *AG* an arc, or *ACG* an angle, or *AGC* a Sector, (for theſe three are expreſſed by the ſame number) to be of as many parts as you pleaſe, and by adding or ſubtracting from it, ſo aſſum'd as there is need, an analogous Triangle congruous to the Phyſical part of the Equation, namely *GSC*, to make the mean Anomaly *AGS*, for if it comes out as great as that propoſed, the Anomaly of the Excenter was well put, and the true Anomaly that agrees to it is the Anomaly ſought; becauſe fitting the given mean Anomaly; but if it does not come out ſo great, the Poſition is to be corrected by that which does come out, and the operation is to be repeated. For finding the Diſtance of the Planet from the Sun, you have in the Triangle *LCS*, the Angle *LSC* given, namely the above found coequated Anomaly; alſo the Angle *LCS*, the Supplement of the Angle *LCA*, whoſe Tangent is to the Tangent of the Angle *ACG* of the Anomaly of the Excentric, as the leſſer axe of the Orbit to the greater; the Excentricity *SC* is alſo given, from whence the diſtance *SL* of the Planet from the Sun becomes known.

And after the like manner, may the Mean Anomaly be determined from the given True one, either by Tables already made, or by the Rule of Falſe Poſition.

PRO-

PROPOSITION IV.

TO *divide a Semicircle in a given ratio, by a right line drawn from a given point of the diameter.*

Let AGB [*Fig.* 4.] be a Semicircle, C its Center, AB its Diameter, and S a point given in the Diameter; from whence a right line as SG is to be drawn, cutting the circumference in G, ſo as that the Area AGS may be to the Area $BHGS$ in a given ratio, namely as p to q. Suppoſe it done, and from G on AC let fall the Perpendicular GE. Let the Semidiameter AC or CB be called r; AE, x; let m be a fourth proportional to the Right lines SC, CA, and the Semicircumference AHB. Then Semcircumf. $\times$ Radius $=$ twice the Semicircle $= SC \times m$, or $m = \frac{2AHB}{SC}$. Again, ſince by ſuppoſition, area AGS : area $BHGS$ :: $p : q$, th. Semicircle AHB : area $AGS :: p+q : p$; th. $p \times AHB = \overline{p+q} \times AGS$, and conſeq. $2AGS$, $= \frac{p \times 2AHB}{p+q}$. Put a for $\frac{pm}{p+q}$; then is $a = \frac{p \times 2AHB}{\overline{p+q} \times SC} = \frac{2AGS}{SC}$; Or, $a \times SC = 2AGS$. Towards B take CT, ſo to CB, as CB is to CS; and call AT, b, then is $CT = b - r$, and th. $SC = \frac{rr}{b-r}$; and $SC \times a = 2AGS = \frac{arr}{b-r}$. And ſince $CE = r - x$, th. $SE = r - x + \frac{rr}{b-r}$; Alſo $2AGS = 2AGE + 2GSE = 2AGE + GE \times ES$. But, by the Doctrine of Series, in a Circle, whoſe radius is r, the Segment whoſe verſed Sine is x, that is,

$$2AGE = \tfrac{4}{3}\sqrt{2} \times r^{\frac{1}{2}} x^{\frac{3}{2}} - \frac{2r^{-\frac{1}{2}}x^{\frac{5}{2}}}{5\sqrt{2}} - \frac{2r^{-\frac{3}{2}}x^{\frac{7}{2}}}{28\sqrt{2}} - \frac{2r^{-\frac{5}{2}}x^{\frac{9}{2}}}{72\sqrt{2}} \text{ \&c.}$$

$$\text{And } GE = \sqrt{2xx-rx} = \sqrt{2} \times r^{\frac{1}{2}}x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}}x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}}x^{\frac{5}{2}}}{16\sqrt{2}} - \frac{r^{-\frac{5}{2}}x^{\frac{7}{2}}}{64\sqrt{2}} \text{ \&c.}$$

Hence

Hence $2\,\triangle\, SEG = (SE \times EG = \overline{r + x + \frac{rr}{b-r}} \times EG)$

$= \overline{r+x} \times GE = \sqrt{2} \times r^{\frac{3}{2}} x^{\frac{1}{2}} - \frac{5 r^{\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} + \frac{7 r^{-\frac{1}{2}} x^{\frac{5}{2}}}{16\sqrt{2}}$ &c.

$+ \frac{rr}{b-r} \times GE = \frac{rr}{b-r} \times \sqrt{2} \times r^{\frac{1}{2}} x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}} x^{\frac{5}{2}}}{16\sqrt{2}}$ &c.

But $\frac{rra}{b-r} = 2\,AGS = 2\,AGE + 2\,SEG$, therefore, by reducing the Equation according to Art, you will find that

$$a = \sqrt{2} \times b r^{-\frac{1}{2}} x^{\frac{1}{2}} - \frac{2 r^{-\frac{1}{2}} x^{\frac{3}{2}}}{3\sqrt{2}} + \frac{\sqrt{2} \times b r^{-\frac{3}{2}} x^{\frac{3}{2}}}{12} - \frac{\sqrt{2} \times r^{-\frac{3}{2}} x^{\frac{5}{2}}}{20} \text{ \&c.}$$

$$\text{Th. } aa = \frac{2b^2}{r} x + \overline{\frac{b^2}{3r^2} - \frac{4b}{3r}} \times x^2 + \overline{\frac{2}{9r} + \frac{4b^2}{45r^3} - \frac{14b}{45}} \times x^3 \text{ \&c.}$$

$$\text{th. } x = AE = \frac{ra^2}{2b^2} + \overline{\frac{r^2}{6b^5} - \frac{r}{24b^4}} \times a^4 + \overline{\frac{r}{720b^6} - \frac{13r^2}{360b^7} + \frac{7r^3}{72b^8}} \times a^6 \text{ \&c}$$

Taking therefore AE equal to ſo many terms of this Series as you pleaſe, and from E erecting EG perpendicular to AB, it will meet the Circle in the point G ſought. *Q. E. F.*

And (by *Prop.* 1.) the ſame EG will meet any Ellipſe AKB, deſcrib'd upon the greater axe AB, in L, ſo as that SL being joyn'd will divide the Semi-Ellipſe AKB into two parts ALS, $BKLS$, having a given ratio to one another.

The leſs the diſtance of the points C and S (that is, the leſs the excentricity of the Ellipſe) and the ratio of p to q, the fewer terms of this Series will ſuffice: For the ratio of p to q ought to be ſmall, that the Series, expreſſing the value of the Segment AGE, may ſoon converge; that is, that few of its terms may be very near equal to the Segment.

I formerly publiſh'd this Solution of the Problem before us, in a *Geometric Exercitation concerning the Dimenſions of Figures*, Printed at *Edinburgh* in the

the Year 1684; but I thought it would not be amiss to insert it again here, as its proper place.

'Twas first thus done by Mr. James Gregory, *and communicated to Mr.* John Collins, *in a Letter dated* April *the 9th* 1672, from St. *Andrews.*

PROPOSITION V.

TO divide an Ellipse that is not very excentric, by a right line thro' one of the Foci, so as that the portion of the Elliptic area contained between the greater Axe and the Line to be drawn, may be very nearly to the whole Ellipse in a given ratio.

The method delivered in *Prop.* 3. of finding the true Anomaly by the mean, is reckoned tentative and indirect: But any Geometric one, however elegant, (as is that formerly given us by the protracted Cycloid by Dr. *Wallis,*) is not so fit for Astronomical Calculations; and that which does it by Series (as *Prop.* 4.) is too laborious; on which account Astronomers have recourse to Approximations, especially the following one.

Let the Ellipse proposed be *ALP*, [*Fig.* 5.] whose Center is *C*, one Focus *S*, the other *F*. From the Focus *S* a right line *SL* is to be drawn, so as that the Trilinear *ALS*, may have very nearly a given ratio to the whole Ellipse *ALPA*.

At one of the Foci *F*, make an angle *AFL*, having the same ratio to four right, as the trilinear Figure *ALS* ought to have to the whole Ellipse, whose Leg *FL* meets the Ellipse in *L*, and the Line *SL* will perform the thing proposed very nearly. For if the Foci *S* and *F* coincide with the Center *C*, it will be exactly, as is evident from *Prop.* 33. *Elem.* 6; the error therefore is as great as that which proceeds from the Excentricity: If that therefore be small, that is in an Ellipse not very excentric, the area *ALS* is to the whole Ellipse as the Angle *AFL* to four right angles; that is in a given ratio.

SCHOLIUM.

Kepler himſelf, who firſt ſhew'd that a Planet deſcribed an Ellipſe, ſo as that in equal times equal areas were deſcribed, being cut off by right lines drawn to the Sun placed in the Focus, took notice alſo of this Approximation, and deſcribes it in *Chap.* 3. *Part* 2. *B.* 5. *Epit.* thus, *as if* L *were the Planet, the Angle* AFL *might be inſtead of the mean Anomaly almoſt.* But he did wholly forbear uſing it, both becauſe in the Planet Mars it plainly differs from the Heavens, and eſpecially becauſe his elegant demonſtrations of the motions from their Phyſical cauſes by this means would lie wholly neglected and uncultivated, becauſe intirely unknown; he was therefore willing to perſiſt in the true Phyſical Theory of the Planets, tho' forced to uſe an indirect method, as in finding the coequated Anomaly from the true, rather than to follow one foreign from the natural one, tho' the thing could be done by it directly and very eaſily.

Afterwards the Celebrated Aſtronomer *Seth Ward* made uſe of this Theory of the Planets, as one that Nature it ſelf took into her ſervice; and aſſumes that the Orbit of any Planet is Elliptical, in one of whoſe Foci is the Sun, which he acknowledges is the true and Phyſical inſtrument of all the Planetary motions; in the mean while the motion of each Planet is ſo tempered about the other Focus as to form at it, Angles proportional to the times; and upon this Hypotheſis he has built his *Geometrical Elliptical Aſtronomy* (a very excellent work, and what all ſound Aſtronomy is very much indebted to.) For he has applied Geometry (with which he was very well acquainted) ſo happily to Aſtronomy, as not only to find directly and Geometrically the true Anomaly from the mean one given, but to determine the Orbits of all the Planets, as well in regard of

of their figure and ſituation, as in regard of their magnitude, (taking the Orbit of the Earth as a meaſure.) But ſtill he *has not wiped off that* Ἀγεωμετρησία, *which ſtill cleaves to Aſtronomy*; For he ſolves this Problem of Aſtronomy, not according to the true and genuine Syſtem, whereby *Kepler* had challenged Geometers to ſolve them; but by a fictitious one, that comes very near the true one. Notwithſtanding, the method of his Aſtronomy is of very great uſe: Since the Orbits of the Planets are not very Excentric Ellipſes, but ſuch as come pretty near to Circles, nor does *Ward*'s Hypotheſis, wherein he makes the motion about the Focus, which the Sun is not in, equable, differ much from the accurate one; and will be of very great ſervice in determining the Orbits of the Planets very nearly, which may (by repeating the Calculation, as ſhall be ſhewn below,) again be made more accurate at pleaſure.

'Tis true that *Iſmael Bullialdus*, in his *Philolaic Aſtronomy*, while he makes every Planet move in an Ellipſe, and cuts this Ellipſe out of a Cone, ſuch as that the Axis of the Cone paſſes thro' one of the Foci of the Ellipſe; and laſtly, while (*Chap.* 15. *Book* 1.) he makes *the motion of the Planet about the Cone and in Circles parallel to the Baſis of the Cone, to be equal*; he has ſuppoſed the equable increment of the Angles at the Focus in which the Sun is not; (namely that through which the Axis of the Cone paſſes,) that is, the Hypotheſis of *Ward*, without knowing of it: Nor did he underſtand it, till *Ward* inquired into the foundations of his Aſtronomy, and demonſtrates it in the firſt *Chap.* of his *Inquiry*. And *Bullialdus* himſelf, who had commiſerated *Kepler* on the account of his miſtakes and defects in Geometry, there meets with what he deſerved at the hands of *Ward*, who ſhows *Bullialdus* (beſides the ſeveral er-

rors, which he had committed and ingenuoufly confeffes in his *Found. of Phil. Aftr. afferted againft S. Ward*,) that he did not throughly underftand his own Hypothefis, while he fuppofes, without knowing it, that the motion about the Focus of the Ellipfe, in which the Sun is not, is equable; which notwithftanding had he throughly known it, he would have finifhed the greateft part of his Aftronomy much better and eafier.

And *Count Pagan* has alfo ferioufly embraced this fame Hypothefis, namely of an equable angular motion of a Planet about the other Focus of an Ellipfe, in which the Sun is not placed, for true and genuine, and has publifhed in *French* at *Paris An.* 1657, a *Theory of the Planets* built upon it; and at the end of his Theory, has boldly thrown upon the miftakes of the Obfervers, the difagreement of it with the Heavens, which is greateft in the Octants from the Apfides. And tho' this *Theory of Pagan* comes a full Year after *Ward's Geometrical Aftronomy*, and three at leaft after the *Inquiry into the Foundations of Bullialdus's Aftron. Philolaica*, (where he had laid down the foundations of all his Aftronomy, and conftructed moft of the Problems;) yet *Pagan* goes no farther than the determining the place of a Planet in its Orbit being given at an affigned time, and its diftance from the Sun; that is, the eftablifhing the principles themfelves of Aftronomy; omitting all the comparative Aftronomy, in the cultivating of which by Geometry, the Accurate *Ward* has well employed his pains.

PROPOSITION VI.

Suppofing the approximation lately defcribed to be accurate enough, (that is, in the Hypothefis of Ward) *the Mean Anomaly being given, to find the coequated Anomaly, and the Planets Diftance from the Sun in a given Orbit.*

Let

Let *ALP* [Fig. 6.] be the Elliptic Orbit of the Planet about the Sun placed in *S* as a Focus; let its Aphelion be *A*, Perihelion *P*, greater Axe *AP*: From the Mean Anomaly the angle *AFL* given, the coequated Anomaly is to be found, namely the angle *ASL*, and the Diſtance of the Planet in *L* from the Sun, *viz.* *SL*; that is, the ratio of this right line to *FS* or *AP*.

Produce *FL* to *B*, ſo that *LB* may be equal to *LS*; and join *BS*. And becauſe (by *Prop.* 52. *B.*3. *Conic of Apollonius*) *FL* and *LS* taken together are equal to the axe *AP*, *FB* will be equal to the ſame *AP*. And therefore in the Triangle *BFS*, the Sides *BF*, *FS*, and the Angle intercepted *BFS*, the complement of the Mean Anomaly given *AFL* to two right or four right angles, (according as *L* is in the firſt or ſecond Semicircle of Anomaly;) the Angles *FSB* and *FBS* or *LSB* will become known, whoſe difference *ASL* is the coequated Anomaly ſought. Beſides, in the Triangle *LFS* at firſt the angle *LFS* and ſide *FS* are given, and now the angle *FSL* is found, and the remaining one *FLS*; therefore the Side *LS* will be found in the parts of the given diſtance of the Foci *SF*; that is, the Diſtance of the Planet from the Sun. *Q. E. F.*

In finding the angles *FSB*, *FBS*, in the triangle *BFS*, when *BF*, *FS*, and the angle *BFS* are given; (by the common Rule of Trigonometry,) *BF* and *FS* together are to *BF* leſſen'd by *FS*, as the Tangent of half the Angles *B* and *FSB*, to the Tangent of half the difference of the ſame. But *BF* and *FS* taken together are equal to *AB* and *FS* taken together; and *BF* leſs *FS* is equal to *AP* leſs *FS*, or twice *SP*; and the ſum of the angles *B* and *FSB* is equal to the external *AFL*, namely to the Mean Anomaly; and their difference is equal to *FSL* the coequated Anomaly. And there-

therefore the ſum of AP and FS is to $2PS$, or the half of the one SA is to the half of the other PS; that is, the diſtance of the Planet when in its Aphelion is to its diſtance when in the Perihelion, as the Tangent of half the mean Anomaly to the Tangent of half the coequated Anomaly: In which proportion the two firſt terms are conſtant in the given Planet.

After the like manner the mean Anomaly may be found, the coequated being given. But if you don't want the coequated Anomaly ASL that anſwers the given mean one AFL, but the angle FLS (called *the Elliptic Equation*;) it will be the double of the angle FBS found before. And this angle ſubtracted from the mean Anomaly AFL, the Planet deſcending from the Aphelion to the Perihelion, or in the firſt ſix Signs of the Anomaly, leaves the coequated ASL: But if it be added to the mean Anomaly in the laſt ſix Signs of the ſame it makes the coequated Anomaly; from whence it receives the name *Proſthaphæreſis.*

PROPOSITION VII.

TO explain the correction of the preceding approximation made by Bullialdus; *and to ſhew how (that being ſuppoſed) the coequated Anomaly and the diſtance of the Planet from the Sun may be found.*

Iſmael Bullialdus, in the defence of his *Philolaic Aſtronomy* againſt *Ward*, in a Book intituled, *The Foundation of the Philolaic Aſtronomy explained, Chap.* 1. and 2. ſhews from four Obſervations made by *Tycho*, that the Hypotheſis of *Ward* lately deſcribed does not agree to Mars, that in the firſt and third Quadrant of the mean Anomaly is more forward than it ought to be by this Hypotheſis, but in the ſecond and laſt Quadrant of the mean Anomaly is leſs than this Hypotheſis requires. But even now neither has he ſo much as looked back

back upon the ſagacious *Kepler*, according to whoſe Phyſical Syſtem, the matter ought to be thus in every Ellipſe, and in Mars ſenſibly, (becauſe its Orbit is ſenſibly excentric;) and as the very numbers themſelves which *Kepler* brings do plainly ſhow. But being reſolved upon correcting *Ward*'s Hypotheſis, in requital of the like-kindneſs from him, he brings the following correction of the mean Anomaly, found as above, in *Chap.* 4. of the ſaid Book. Let *ALBP* [*Fig.*7.] be the Elliptic Orbit of a Planet, whoſe Foci, Apſides, Center, &c. are expreſſed by the ſame letters as they have been frequently before: And let the angle *AFL*, the mean Anomaly in the Hypotheſis of *Ward*, be to four right angles, as the time in which the Planet deſcribes the Elliptic Arc *AL* to the intire periodic Time. And becauſe the point *L* in the firſt and third Quadrant of the mean Anomaly (that is, while the Angle *AFL* is leſs than a right, or greater than two, or leſs than three) is not ſo forward as Obſervations require, but in the firſt and fourth Quadrant too forward; that is, becauſe the angle *AFL* is in the firſt caſe too little, in the ſecond too great; therefore *Bullialdus* corrects this angle thus. Thro' *L* draw *LE* perpendicular to the Axe *AP*; meeting it in *E*, and the Circle deſcribed upon the diameter *AP* in *G*, join *FG*; let the Angle *AFG* be taken for the mean Anomaly inſtead of *AFL*; the angle *AFG* *Bullialdus* takes the liberty of calling the *True-Mean Anomaly*, leaving the name of the *Mean* to the angle *AFL*, which is proportional to the Time: And therefore the Place of the Planet in its Orbit corrected will be the point *V*, where *FG* meets the Orbit. The angle of the True-Mean Anomaly *AFV*, correſponding to the angle of the mean Anomaly *AFL* is found very readily, by taking an angle whoſe Tangent is to the Tangent

of the mean Anomaly, as the greater axe of the Ellipſe is to the leſs; and this will be that which is ſought. For *GE*, the Tangent of the angle *GFE*, is to *LE* the Tangent of the Tangent of the Angle *LFE*, to the ſame radius *FE*, as *DC* or *CA* to *BC*. And in the Triangle *VFS* (juſt as in the preceding *Prop.* in the Triangle *LFS*) the Side *FS*, the external angle *AFV*, and ſum of the Sides *FV* and *VS* being given, the coequated Anomaly *FSV* is found, and the Side *VS* the Diſtance of the Planet from the Sun corrected. *Q.E.F.*

And this is the correction of *Ward*'s *Hypotheſis* which *Bullialdus* brings; and 'tis well enough, if it be taken only for a correction of the approximation to the true Syſtem, as it ought to be; becauſe *that by which he had effected, that at length the coequated Anomaly might be found from the mean* à priori, *and at the ſame time the calculation ſatisfy obſervation; which no one had done before him* (according to *Mercator*'s judgment) *in the Elliptic Hypotheſis.* But when *Bullialdus* puts it upon us as the true and genuine Syſtem, and derives the Phyſical cauſes of it (*Chap.* 3. and 4.) after his manner from the Cone, he not only leaves the Foundations of his *Philolaic Aſtronomy* unexplain'd, but diſputes as if there were no Phyſical foundations of Aſtronomy at all.

PROPOSITION VIII.

TO *deſcribe the* Orbit, *wherein that Celebrated Aſtronomer* Dominicus Caſſini, *thinks a Planet is carried about the Sun, and to explain its agreement and diſagreement with Natural cauſes and the Phænomena.*

Let the Diameter *AB* of the Circle *AKBH* [*Fig.* 8.] be divided into two equal parts in the point *C*, and thro' it draw an indefinite line *DE* perpendicular to *AB*. Divide *AB* again in *G*, ſo

ſo that *AG* may be to *GB* as the diſtance of the Planet (whoſe Orbit is to be deſcribed) from the Sun when it is in its Aphelion, to the diſtance from the Sun when it is in its Perihelion; and take *CF* equal to *CG*. From *F* on *AB* let a Perpendicular *FH* be erected, meeting the Circle in *H*. On the Center *F*, at the diſtance *FH*, deſcribe a Circle cutting the indefinite right line *DE*, in the points *D* and *E*, which are the extreams of the leſſer Axe of the Orbit to be deſcribed. To find its other points, draw thro' *F* any how, the right line *IFK* meeting the Circle in *I* and *K*. On the Center *F* at the diſtance *IF* or *KF*, let a Circle be deſcribed; and on the Center *G*, at a diſtance equal to one of the Segments of the right line *IFK*, namely *KF* or *IF*, another Circle: the interſections of theſe Circles, *L, L, L, L*, will belong to the Orbit. And after this manner, becauſe two Circles are drawn upon both Centers, four points of the Orbit will be determined at the ſame time. After the like manner its other points may be determined, by drawing other right lines paſſing thro' *F*, and bounded by the Circle on both ſides. Becauſe the rectangle under the ſegments of the right line drawn thro' the point *F* and bounded by the Circle is given, being equal to the rectangle under *AF*, *FB* (by *Prop.* 35. *Elem.* 3;) 'tis evident that the rectangle under the right lines *FL*, *GL*, drawn from the points *F* and *G*, to any point *L* of the Curve *ALB* is alſo given, and equal to the rectangle under *AF*, *FB* or under *AG*, *GB*; and therefore that *AG* is to *GL* as *FL* to *GB* or *FA*. As in the common Ellipſe the ſum of the right lines from the Foci inclined to any point in the Circumference is given, ſo the rectangle under the right lines drawn from the points *F* and *G* to any point in the Curve *ADBE* is given, on which account

Caſſini

Caſſini gave the name of *Foci* to theſe points *F* and *G*.

He would have this Curve to be the Orbit of a Planet, which it deſcribes about the Sun placed in the Focus *G*, ſo as that the angles at the other Focus *F* are proportional to the times; which he therefore calls the Focus or Center of the mean Motion, as he does the other *G*, of the true Motion, where he places the Center of the Sun and of the World. On which account any two diſtances of a Planet from the Focus of the true Motion are reciprocally as its diſtances from the Focus of mean Motion. In this Orbit, the true Anomaly, namely *AGL* is determined from the mean Anomaly or Angle *AFL*, if in the Triangle *LFG*, the Baſe *FG*, and one of the adjacent angles *LFG* (the complement of *AFL* the mean Anomaly,) and the rectangle under the two other ſides *FL*, *FG* being given, the other things may be found.

This *Caſſinian* Hypotheſis, briefly delivered in a *Treatiſe of the Origen and Progreſs of Aſtronomy*, has this Phyſical diſadvantage, that tho' a Planet, by the action of a centripetal Force of a certain kind toward the Sun, (whoſe Law may be known by *Cor.Prop.37.B.1.*) may deſcribe the Orbit *ALDBE* about the ſame placed in *G*, yet this force will be intirely different from that uſed in moſt of the operations of Nature, and from that whereby the common Ellipſe, having the ſame Foci and Vertices, is deſcribed about the Sun placed there; and this over and above, that a Planet deſcribing the Curve *ALD*, as *Caſſini* ſuppoſes, (namely ſo as that a line *FL* being drawn every where to the Planet, the angle *AFL* is made proportional to the time) does not all obſerve that univerſal Law demonſtrated in *Prop.* 11. *B.* 1. ſo as to make the area *ALG*, which the radius *GL* proceeding from the

the Sun, deſcribes, proportional alſo to the Time. On which account this Hypotheſis, as well as the above deſcrib'd Hypotheſis of *Ward* and *Paganus*, is Phyſically impoſſible. 'Tis utterly impoſſible that a Planet, tho' it be acted upon by any centripetal Force tending toward the Sun, ſhould deſcribe the Perimeter of the common Ellipſe, or of this *Caſſinian* Ellipſe, ſo as that the angle at the Focus where the Sun is not be alſo proportional to the Times. For this being ſuppoſed, the areas at the Sun would not be proportional to the Times, (as has been ſhewn above of the common Ellipſe, and ſhall be ſhewn hereafter of both) which notwithſtanding is done, as has been demonſtrated in *Prop.* 9. *B.* 1.

But we will demonſtrate that in the Figure above deſcribed ALDBE *(or in any other that is concave every where toward the Center, whoſe four parts made by the two axes are equal and ſimilar, and the two Foci* F *and* G *equi-diſtant from the Center,) every angle at the Focus* F *is not to four right angles as the area at the Focus* G *to the whole Figure* ADBE.

Let us ſuppoſe the angle at the Focus *F* to be to four right as the area at the Focus *G*, to the intire Figure [*Fig*.9.] Draw the right line *FL* perpendicular to *AB*; by which means the angle *AFL* will be a fourth part of four right, or half of two right; and therefore the area *ALG* will be by ſuppoſition a fourth part of the intire Figure *ADBE*, or a half of the half *ADB*, that is, equal to *LDBG* what it wants of half. From *G* draw *GM* parallel to *FL*, meeting the Orbit in *M*; and then *GM*, from the nature of the Orbit, will be equal to *FL*; and conſequently by *Prop.* 33. *El.* 1. *ML* (which intirely falls within the Curve *ADBE*, ſince by ſuppoſition it is every where concave towards the Center *C*) is equal and parallel to *GF*; and (by *Prop.* 34. *El.* 1.) the Triangle

angle *MGL* is equal to the Triangle *GFL*. And ſince the Figure *AFL* is, from the nature of the Orbit, equal to the Figure *BMG*; the Figure *ALG* is equal to the Figure *BMLG*, contained under the right lines *BG*, *GL*, *LM*, and the Curve *BM*. But it was juſt now ſhewn, that the Figure *BMDLG*, contained under the right lines *BG*, *GL* and the curve *BMDL*, is equal to the Figure *ALG*; and therefore the Figure *BMLG* is equal to the Figure *BMDLG*; that is, a part equal to the whole; which is abſurd. Therefore the Figure *ALG* is not a quarter of the whole Figure *ADBE*, when the Angle *AFL* is a quarter of four right; or every angle at the Focus *F* is not to four right angles as the area at the Focus G to the whole Figure. *Q. E. D.*

Beſides the aforeſaid Hypotheſis labours under this difficulty, that the true Anomaly, and the correſponding mean Anomaly, and the diſtance of the Planet from the Sun, do not in the leaſt agree with the Phænomena in the Heavens. For ſince the common Ellipſe does agree with theſe Phænomena, that other, the *Caſſinian* one, which is of a different nature, cannot agree with them. For the place of a Planet, at a given time, in the common Ellipſe *AYBZ*, [*Fig.* 10.] will differ from that in the *Caſſinian* Orbit *ADBE*, deſcribed with the ſame Vertices *A*, *B*, and the ſame Foci *F*, *G*, as well in regard of the true Anomaly or Angle made with *AB* at *G*, as of the diſtance from the Sun, in reſpect of the diſtance of the Aphelion *GA*. And tho' in Orbits, where the Excentricity is very ſmall, the difference is not very ſenſible; where the Excentricity is great, things are otherwiſe. In the former caſe, the Circle *AQBX*, deſcribed upon the diameter *AB*, almoſt ſatisfies the Phænomena. Yet all Aſtronomers now-a-days acknowledge that a circular Orbit is too broad

broad about the mean Longitudes, as ♌ and *X*. But this *Cassinian* Orbit *ADBE*, on the contrary is much too narrow in those places. Suppose for instance, *AYBZ* to be the common Ellipse, whose greater axe is to the less as 5 to 4; let a *Cassinian* Orbit *ADBE*, be describ'd upon the same Vertices and Foci; its lesser axe *DE* will be less than the lesser axe *YZ* of the true Elliptic Orbit, above a third part, whereas in the mean while ♌*X* the breadth of the Circular Orbit exceeds *YZ* by a lesser part; namely only by a fourth part. Therefore this Orbit, since Physical reasons don't suit with it, (because it is intirely impossible to be described after the manner that *Cassini* its Author requires, namely so as that the Angles at the other Focus, than where the Sun is, may be proportional to the Times; for thus the area described by the rapius would not be proportional to the Time: Nay, to describe it at any rate, a Centripetal force towards the Sun is necessary that is altogether different from that which is used by nature, which, like every other natural Virtue, propagated from a Center or to a Center, in right lines thro' the circum-ambient spaces, is reciprocally as the Square of the distance,) and since the Orbit does not agree with the Phænomena and Celestial Observations, by reason of the shortness of the lesser axe, and the narrowness of the middle Orb arising from thence; it ought, I say, to be thrown out of Astronomy.

Since the celebrated Mr. *Cassini*, in his before-mention'd Treatise, proposed to Astronomers this Curve, as the Orbit of a Planet, there have been various disputes about the nature thereof, and the Law of Gravity requisite to describe it. In my second thoughts upon it, its different Species, and some properties of them not sufficiently known, presented themselves to me.

'Tis

'Tis ſufficiently known that this is the nature of this Orbit. If from two given points *F* and *G* [*Fig.* 11.] to any point *H* of the Curve, the right lines *FH*, *GH* be drawn; the rectangle under *FH*, *GH* is equal to a given ſpace. The right line *FG* produced both ways till it meets the Curve, ſhows the Vertices *A* and *B*; and *AB* is the principal Axe; and the point *C* which is the middle between the Vertices, is the Center of the Figure, and *DE* thro' *C* perpendicular to *AB*, the leſſer axe; and the points *F* and *G* the Foci.

In this Figure if the leſſer axe exceed the diſtance of the Foci, the Curve terminating the Figure is every where concave towards the Center, as it is commonly taken to be. If, the principal Axe remaining the ſame, the diſtance of the Foci is leſſen'd, the leſſer Axe will be encreaſed, but it continues leſs than the Axe of the Ellipſe deſcribed upon the ſame principal axe and the ſame Foci; till at length the Foci meeting it becomes equal to the greater axe, and the Figure turns into a Circle. But if, on the contrary, the diſtance of the Foci be increaſed, the leſſer axe will be leſſened, and will become equal to the ſaid diſtance, ſince the diſtance is to the principal axe as unity to a mean proportional between unity and three.

If the diſtance of the Foci be farther encreaſed, the leſſer axe will ſtill be leſſened, and the Curve will be no longer concave towards the Center but convex, at the extremities of this axe, as in *Fig* 12; till the diſtance of the Foci being ſo far increaſed, that it is to the greater axe as the Side of a Square to the Diagonal of the ſame, the leſſer axe will become nothing, and the Curve reach to the Center on each ſide.

If the diſtance of the Foci be greater than in the ſaid ratio, the leſſer axe is impoſſible, and the Figure turns into two conjugate ones, as in *Fig.*

Fig. 13. which, upon the increaſe of the diſtance of the Foci, will be leſſened, till at length the Figure run into two conjugate points.

The diſtance of the Foci increaſing, the two conjugate Figures do again emerge, and they increaſe after the ſame manner as they decreaſed before, differing from the former in the order of the Foci and of the Vertices, and are increaſed till they become infinite. And afterwards this Syſtem will again approach to a Circle by the ſame degrees as it receded from it.

From what has been ſaid, 'tis evident at firſt ſight, that this Figure is by no means fit to be the Orbit of a Planet. Not to mention the caſes wherein it paſſes into two conjugate Figures, and lays aſide the nature of an Orbit, namely whereſoever its Excentricity is ſo great, as Comets (if they revolve round the Sun like Planets, as is moſt probable) require to deſcribe their courſe: to paſs over theſe caſes, I ſay, there are alſo in thoſe caſes, where it returns into it ſelf and makes an Orbit, ſome with an Excentricity ſo large, as that the Curve near the points *D* and *E* [*Fig.* 12.] becomes convex towards the Sun; and therefore the Planet would need a Centrifugal force from the Sun to deſcribe this part of its Orbit, whereas in the mean while, in the nearer and more remote places *B* and *A*, a Centripetal force towards the Sun is requiſite. That is, that the circumſolar Bodies may be moved by that Law, it muſt be granted, that in equal diſtances from the Sun, here the force is centripetal, but there centrifugal; which, how foreign it is to all the Laws of Nature, any body may eaſily ſee. And tho' the Excentricity of no Planet be ſo great, yet 'tis well known to Geometers, that of a Figure, all whoſe Species beyond a certain limit are unfit for the diſcharging any office in nature, the remaining

Species

Species on this ſide of that limit, can't be admitted as fit for that office. 'Tis neceſſary *that this Caſſinian Curve be thrown out of Aſtronomy*, not only upon the account of the reaſons mention'd before in this Propoſition, namely, *becauſe it does not agree with Celeſtial Obſervations, by reaſon of the ſhortneſs of the leſſer axe, and no Phyſical reaſons anſwer it, ſince to deſcribe it there would be need of a centripetal force toward the Sun, intirely different from that made uſe of by Nature*; but becauſe of its abſolute impoſſibility. For it is impoſſible that any Species of this Figure can be deſcrib'd by a Planet, ſo as that the angles at the other Focus, where the Sun is not, may be proportional to the times; for thus the area deſcribed by the radius that carries the Planet along would not be proportional to the Time. For 'tis not true that increaſing the angle at one Focus by equal increments, the increments of the area made at the other at the ſame time will alſo be equal, as I thought not long ago.

In the *Figures* 12 and 13, the greateſt breadth of the Figure is found, by deſcribing a Circle upon the Center C thro' the Foci; for it will cut the Curve in the points ſought L, L. And the greateſt Ordinate KL is a third proportional to the right lines GF and FD, in the firſt of them, or a fourth proportional to the right lines GF, GA, and AF in both.

If DE remains in being, the Ordinate from the Focus FP is equal to the leſſer Semi-axe CD, when the leſſer axe is to the diſtance of the Foci, as the Side of a Square to its Diameter. If the diſtance of the Foci be greater than in that ratio, FP will exceed CD.

SECTI-

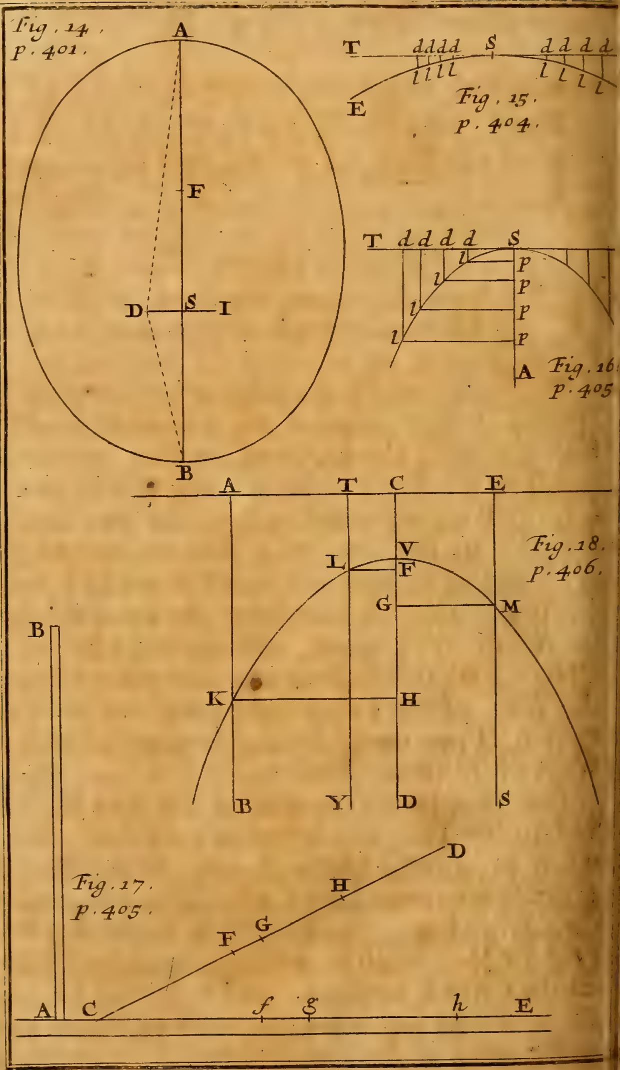


Fig. 14. p. 401.

Fig. 15. p. 404.

Fig. 16. p. 405

Fig. 17. p. 405.

Fig. 18. p. 406.

SECTION II.

Concerning the determining the Orbit of the Earth, and the Theory of the Earth ſeen from the Sun, or of the Sun ſeen from the Earth.

PROPOSITION IX.

TO *define by obſervation, the Species of the Orbit of the Earth and the Poſition of its Axe; as alſo the Periodic Time of the Earth.*

One deſign of this Book being to determine the Species of the Orbits of the Primary Planets about the Sun; the Orbit we ought to begin with is that of the Earth about the Sun, as well becauſe it is the ſame with that which the Sun ſeems to deſcribe in the ſpace of a Year about the Earth, as eſpecially becauſe the Species and Poſition of this is neceſſary to determine the ſame things in the reſt of the Planets. For ſince the Eye of the Obſerver is carried along together with the Earth, the Way of the Earth ought firſt of all to be known, if you would be well acquainted with the Ways of the other Planets.

Let the apparent motion of the Sun be obſerved in the Ecliptic, when he is ſwifteſt; that is, when in a given ſpace of time it deſcribes the greateſt arc toward the Eaſt; for in that caſe (by *Cor.2.Prop.41.B.1.)* the Earth is in the Perihelion *P* [*Fig.* 14.] but when its motion is found to be ſloweſt the Earth is in the Aphelion *A*. But the place of the Earth ſeen from the Sun is exactly oppoſite to the place of the Sun obſerved from the Earth, and therefore the place in the Ecliptic of the Aphelion and Perihelion of the Earth ſeen from

 the

the Sun is had; that is, the Poſition of the line of the Apſides of the Earth. But in Book 2[d] we ſhew'd how the places of the Fix'd Stars in reſpect of the Ecliptic, that is, the cardinal and other points of the Ecliptic might be determined in reſpect of the Fix'd Stars, from whence the Poſition of the line of the Apſides of the Earth among the Fix'd Stars will be found. The ſame may be done by biſecting the angle contained between two right lines drawn to two places, where the apparent motions of the Sun from the Earth performed in a given time are equal; for ſuch points are equidiſtant from the points of the Aphelion and Perihelion. To find the Species of the Orbit, you muſt obſerve (by *Cor.*3.*Pr.*41.*B.*1.) that the apparent motion of the Earth in the Aphelion A ſeen from the Sun in the Focus S, is to the apparent motion of the ſame in the Perihelion P, as SP^q, to SA^q. And ſince the apparent motion of the Earth ſeen from the Sun, is the ſame with the motion of the Sun ſeen from the Earth; from the obſervation of this, while the Earth is in the Aphelion and Perihelion, the ratio of SA^q to SP^q will become known, and conſequently its ſubduplicate SA to SP; by compounding and dividing them, you will have the ratio of $SA+SP$ to SP, and $SA-SP$ to SP; or of AP to SP, and (ſuppoſing F to be the other Focus) of SF to SP, and conſequently the ratio of AP to FS is alſo given.

The ſame things may be determined from the obſervation of the apparent Semidiameter of the Sun: For the Earth is then in the Aphelion, when the apparent Semidiameter of the Sun is the leaſt; and conſequently the place of the Sun oppoſite to the place then obſerved, is the place of the Aphelion ſeen from the Sun: and after the ſame manner of the Perihelion. The middle point

point alſo between the two points, where the Sun is of the ſame apparent diameter, ſhews the place of the Aphelion or Perihelion of the Earth; all which are too plain to inſiſt upon.

To determine the Species of the Ellipſe, let *DI* be the diameter of the Sun ſituated in the Focus *S*. And the angle *DAI* is to the angle *DPI*, as *SP* to *SA*; but the two firſt terms of this Proportion are given; therefore the ratio of *SP* to *SA* is known: From whence, as before, the ratio ſought of *AP* to *FS*, determining the Species of the Ellipſe, may be drawn. Inſtead of the apparent diameters or ſemidiameters of the Sun, meaſured by the Angles or Arcs, the Tangents of the angles *DAS*, *DPS*, of the apparent ſemi-diameters may be taken, as being evidently more exact.

Beſides, the ſpace of time between the moments immediately following, wherein the Earth was at the ſame Apſis, to be determined from what has been ſaid in this Propoſition, is the periodic Time of the Earth. But this will be better determined, if ſuch moments are uſed as are at the greateſt diſtance from one another as may be, and the intermediate time be divided by the number of the revolutions the Earth has made in the mean while about the Sun; for then the Time of one revolution of the Earth will come out.

PROPOSITION X.

TO *determine the Time of either Equinox by Obſervation.*

Juſt about the Equinox obſerve (by *Prop.* 18. *B.* 2.) the Sun's declination, which if it be nothing at all, will then be the Time of the Equinox itſelf: But if any thing, then from thence (by *Prop.* 20. *B.* 2.) the place of the Sun in the Ecliptic will be found. Let the ſame be repeated again, or even a third

time, at ſome diſtance of time between. And by the help of the places thus found and of the intermediate time, the Time of the Equinox itſelf will be found.

In the performing this and ſuch like Problems, the obſervations of the Sun are to be corrected, (as always) by the Parallax and the correſponding Refraction, and all other things are to be cautiouſly avoided, that may render the calculation uncertain; *v. g.* not two obſervations made on the ſame ſide of the Time which is to be determined; but rather two obſervations, whereof one precedes and the other follows the Equinox, are to be made choice of.

Proposition XI.

To *determine the Time of either Solſtice.*

Becauſe the Tropic touches the Ecliptic in the Solſticial point, the 15th Figure will repreſent the little portion of the Tropic and Ecliptic near the Solſtice; where TR is the Tropic; EC the Ecliptic touching it in the Solſticial point S; and the little right lines dl, dl perpendicular to TR, will repreſent the deviations of the points of the Ecliptic l, l from the Tropic, or the changes of their Declinations: In which caſe, the little right lines dl, dl are (by *Prop.* 24. *Book* 1.) as the ſquares of the Arcs Sl, Sl, or of the right lines Sd, Sd, being nearly equal to them reſpectively. On which account a ſmall portion of the Ecliptic near the Solſtice S, will not ſenſibly differ from the Parabola ESC; that is a Figure, whoſe property it is to have dl every where as the ſquare of the right line Sd reſpectively. Again, the Arcs Sl, Sl are as the times wherein they are deſcribed by the Sun: For, ſince the Apſides of the Elliptic Orbit, which the

the Sun deſcribes as to ſenſe, continue at this time pretty near the Solſticial points, the apparent motion of the Sun will be equable; that is, the arcs *S l*, *S l* will be as the times wherein they are deſcribed. From whence it may be fairly inferred, that a portion of the Ecliptic near the Solſtice (ſuppoſe that portion which the Sun deſcribes in about ten days, five before and as many after the Solſtice,) does not at all differ from the Parabola *S l l*, [*Fig.* 16.] in which the Abſciſſa's *S P*, *S P* are what the Declination of the Sun every moment wants of the greateſt Declination; conſequently their differences *P P*, *P P* are as the differences of their Declinations themſelves; that is, as the intervals of the Altitudes of the Sun above the Horizon, (or of the diſtances of the ſame from the Vertex) in the Meridian or any other vertical: And the Ordinates *P l*, *P l* are reſpectively as the times wherein the Sun acquires the foreſaid Declinations, and the intervals of the one are as the intervals of the other.

Theſe things being demonſtrated, let a Gnomon *A B* [*Fig.* 17.] be erected, whoſe ſhadow made by the Sun when 'tis near either of the Solſtices and received upon the Plane *C D* (to which a right line *B H*, connecting the Sun and the Vertex of the Gnomon *B*, is almoſt perpendicular) marks out the point *H*: Some few days after (for inſtance, three or four) obſerve again the point where the ſhadow of the extremity *B* falls, when the Sun is upon the ſame Vertical; and let that be *F*: And let the like obſervation be repeated again, and the point obſerved be *G*: Now from theſe three points *H*, *F*, *G* being given, the time of the Solſtice itſelf is to be found. 'Tis evident that the right lines *H F*, *G F* are the intervals of the diſtances of the Sun in the ſame Vertical from the Horizon, or from

the Vertex; (that is, by what has been ſhewn above, the changes of the Declination.) For *FGH* does not in the leaſt differ from an Arc of a Circle deſcribed upon the Center *B* with the diſtance *BG*, becauſe the Angles *FBG*, *HBG*, are very ſmall. And if inſtead of the Plane *CD*, to which *BG* is perpendicular, the ſhadow ſhould be taken upon any other Plane *CE* inclined to *CD* by the angle *DCE*, not a very great one; for inſtance, in the points *h*, *f*, *g*; the right lines *hf*, *gf* will have the ſame ratio to one another, as *HF*, *GF*, becauſe the right lines *fF*, *gG*, *hH*, are almoſt parallel; ſince they meet only in the point *B*, which is at a diſtance great enough in reſpect of the diſtances *fF*, *gG*, *hH*. The Problem therefore for finding the Solſtice from the Points *F*, *G*, *H* given, together with the moments of Time, wherein the Sun, placed in the ſame Vertical, caſts the ſhadow of the point *B* to them, is reduced to the following Problem, which is purely Geometrical.

The diſtances AT, TE, [Fig. 18.] *of three parallel right lines* AB, TY, ES, *lying in the ſame Plane being given; to deſcribe the Parabola* KLM, *whoſe Axis* CVD *is parallel to* AB, TY, ES, *that cuts them in the points* K, L *and* M, *ſo as that, drawing the Ordinates to the Axe,* KH, LF, MG, *the portions* FG *and* FH *of the axis intercepted by them, may be equal to given right lines.*

For by this means, (if *FG* and *FH* are equal to *FG*, *FH* marked out by the obſervation deſcribed above, and the ratio of *AT* to *TE* the ſame with the ratio of the Time between the obſervation of the points *H* and *F*, to the Time between the obſervation of the points *F* and *G*) *KH*, *LF*, *MG* will repreſent the Times between the obſervations of the points *H*, *F* and *G*, and the moment itſelf of the Solſtice; that is, as the points

A,

A, T, and E repreſenting the moments of the Time wherein the Shadow of the Gnomon was obſerved in the points H, F, and G, the point C will repreſent the moment of the Solſtice itſelf. Call the Line AT, a; TE, b; FH, c; FG, d; TC, x. Therefore AC or $KH = a + x$, and EC or $GM = b - x$. Call the principal *Latus rectum* of the Parabola, r. Then from the nature of the Parabola $x^2 = VF \times r$, and therefore $VF = \frac{xx}{r}$. After the like manner $VH = \frac{a^2 + 2ax + x^2}{r}$, and $VG = \frac{b^2 - 2bx + x^2}{r}$. Wherefore $c = (FH = VH - VF =) \frac{a^2 + 2ax}{r}$; and $d = (FG = VG - VF =) \frac{b^2 - 2bx}{r}$. And getting away r, by the help of theſe two Equations; you will find $\frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$. And ordering the Equation as is uſual, x will be $= \frac{b^2c - a^2d}{2ad + 2bc}$; that is, the ſpace of time between the known moment repreſented by T, (namely, when the ſhadow of the Gnomon was obſerved in F,) and the moment of the Solſtice ſought; from whence the time of the Solſtice itſelf will be given. *Q. E. F.*

But if the time between the obſervation of the Shadow in H, and the Solſtice itſelf be ſought, add AT to TC found, and the ſought $HC = (a + x =) \frac{b^2c + 2abc + a^2d}{2ad + 2bc}$. After the like manner will EC be $= (b - x =) \frac{b^2c + 2abd + a^2d}{2ad + 2bc}$. But if the order of the obſervations be ſuch, as that the obſervation of the Shadow of the Gnomon in F is exactly

actly in the middle between the obſervations of the ſame in H and G; that is, if $AT = TE$, or $a = b$, the Equation will be more ſimple; for the the diſtance of the Times of the Solſtice and of the Shadow obſerved in F will be equal to $\frac{ac - ad}{2d + 2c}$ the diſtance of the Solſtice from the time of the obſervation of the Shadow in H equal to $\frac{3ac + ad}{2c + 2d}$ and from the moment of the obſervation of the Shadow in G equal to $\frac{ac + 3ad}{2c + 2d}$; that is, in the Figure, as $2FH + 2FG$ to GH or $4HF - 2GF$ to GH, or $2HF - GH$ to $\frac{1}{2}GH$, ſo is the half of the time between the firſt obſervation of the Shadow in H and the laſt obſervation of the ſame in G, to the time between the middle obſervation of the Shadow in F and the moment of the Solſtice itſelf ſought.

The ſagacious Mr. *Edm. Halley* firſt found out and publiſhed this method of determining the times of the Solſtices, in the *Philoſph. Tranſact.* for the beginning of the Year 1695; and illuſtrated it by the example of two Solſtices determined by this, from the Obſervations of *Walther* and *Gaſſendus*; where he takes notice of ſeveral things well worth obſerving in the Practice, concerning the ſituation and advantageous form of the Gnomon, and concerning the accurate obſervation of the Shadow, and other things.

PROPOSITION XII.

TO *define the quantity of the Tropical Year.*

Since the Tropical Year is that ſpace of time wherein the ſame Seaſons of the Year return again; that is, wherein the Sun departing from any point of the Ecliptic (for inſtance that where-

in

in it touches the Tropic) returns to it again; 'tis evident that its quantity will be determined, if the Sun be twice obſerved in the ſame point of the Ecliptic, (for inſtance, twice in the ſame Equinox, by *Prop.* 10, or in the ſame Solſtice, by the 11th,) and the intermediate time be divided by the number of revolutions of the Earth about the Sun made in the mean time. If there be any error in determining the moment of the Equinox or Solſtice, that it may not diſorder the meaſure of the Tropical Year, 'tis proper to make uſe of ſuch Equinoxes or Solſtices as are at the greateſt diſtances from one another, ſo that the error divided into ſo many parts (by the diviſion made by a great number of intervening Revolutions) may become inſenſible.

But becauſe the Times of the Solſtices ſeem'd difficult to the Ancients to be diſcerned, as *Ptolemy* himſelf ſays, who therefore paſſed over the Solſtices of *Meton* and *Euctemon* and *Ariſtarchus* himſelf; which alſo ſeem'd ſo true to the Moderns, that *Ricciolus* aſſerted that *Ptolemy* was too confident in hoping that neither himſelf nor *Archimedes* had erred ſo much as a quarter of a day in their obſervation or computation in the Solſtitials; and *Hevelius*, *Chap.* 4. *Prod. Aſtron.* ſays, *That the Solſtices, tho' they be obſerved by the beſt and greateſt Inſtruments, and even the moſt experienced Obſerver, can never be determined in their ſmalleſt parts*: Aſtronomers have made uſe of the Obſervations of the Equinoxes, whoſe moments they thought more accurately determinable, on the account of the vaſt and ſenſible change of the Declination of the Sun about the times of the Solſtices. But, by the help of the preceding Method, the times of the Solſtices themſelves may hereafter be determined, at leaſt as accurately as thoſe of the Equinoxes.

Where-

Whereas the Sun ſeems to move ſometimes ſlower and ſometimes ſwifter, (according to the different velocity of the Earth about the Sun,) if inſtead of that, it be imagined to move equably, ſo as to deſcribe the Ecliptic in the time determined by this Propoſition; ſuch a motion is very fitly called its *Mean Motion*, (being really ſuch an one between the ſwifteſt and ſloweſt,) and the place the Sun is thus in, is called its *Mean Place.*

PROPOSITION XIII.

TO determine the quantity of the Sydereal Year.

By comparing the place in the Ecliptic a Fix'd Star was formerly in, with its preſent place, find the Annual preceſſion of the Equinox; that is, how much each point of the Ecliptic recedes from a given Fix'd Star towards the Weſt in the ſpace of one Year, which was done in *Book* 2. *Prop.* 31: And by the preceding Propoſition determine the ſpace of Time requiſite for the Sun to deſcribe an arc of the Ecliptic equal to the Preceſſion thus found. This therefore added to the quantity of the Tropical Year, gives the quantity of the Sydereal Year, which conſiſts of the Tropical Year, wherein the Sun departing from a point of the Ecliptic returns to the ſame, that has in the mean while moved a little way forwards to meet it, and of that beſides wherein the Sun deſcribes that arc thro' which the ſaid point of the Ecliptic moved to meet the Sun; that is, wherein the Sun departing from an immovable point (for inſtance, a Fix'd Star) ſeems to return again to it.

Plate 3. Book 3.

Fig. 19, p. 411.

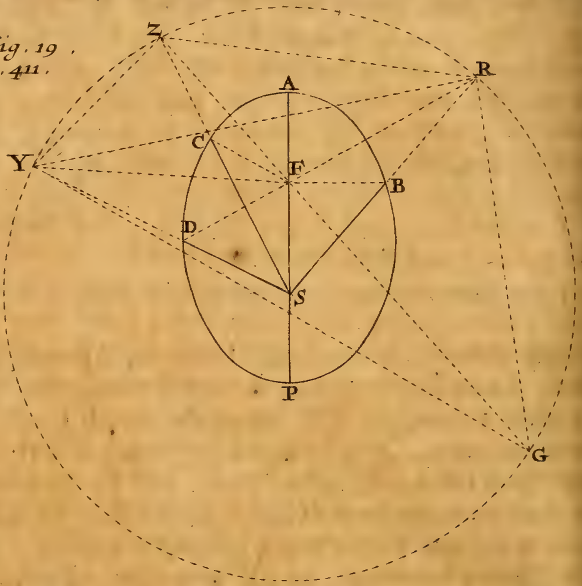


Fig. 11, p. 398.

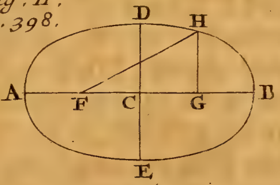
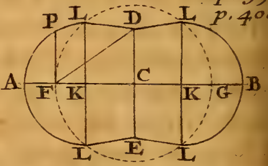


Fig. 12, p. 398, p. 400



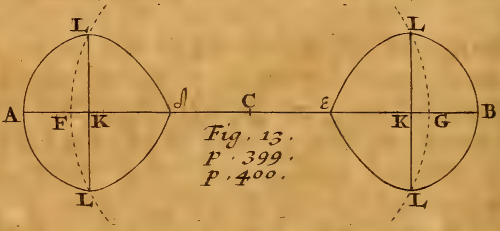


Fig. 13. p. 399. p. 400.

PROPOSITION XIV.

Three places of the Sun, determined by Prop. 20. B. 2. *and the Periodic Time of the Earth, which is very nearly equal to the Syderial Year, determined by the preced.* Prop. *being given, to find the Species of the Orbit of the Earth, the Situation of the line of the Apſides, and the Time when the Earth is in the Aphelion.*

The methods laid down in *Prop.* 9. for finding the Species and Poſition of the Orbit of the Earth, require ſuch accurate obſervations that they are hardly fit to be put in practice: We ſhall therefore add the following one better fitted for Aſtronomic uſes. Let there be given three Places of the Sun by obſervation, namely when the Earth is in the three points *B*,*C*,*D* of its Orbit [*Fig.*19.] Therefore the three lines *SB*, *SC*, *SD* are had by poſition, or the three angles *BSC*, *CSD*, *BSD*. And the Period of the Earth being given, together with the Times between the obſervations, the ratios between the whole area of the Ellipſe *ACPB* and its parts *BCS*, *CDS* and *BDS* are alſo given. The Problem therefore is reduced to this, to deſcribe ſuch an Ellipſe upon a given Focus *S*, as that its whole area ſhould be to its parts contained under the right lines given by poſition, *SB*, *SC*, *SD*, and the Curve of the Ellipſe, in given ratios.

Let *AP* then be the Orbit of the Earth, *S* and *F* its Foci, *A* its Aphelion, *P* its Perihelion. On the Center *S* imagine a Circle deſcribed by a radius equal to the greater Axe, the Circumference whereof cutting the right lines *SB*, *SC*, *SD* produced in *R*, *Z* and *Y*. Join the right lines *FB*, *FC*, *FD*, *FR*, *FZ*, *FY*; and produce one of them *ZF*, untill it meet the Circumference again in *G*, and join *RG*, *GY*, *YZ*, *ZR*.

If

If *ACDPB* be the true Orbit of the Earth, then (by *Prop.* 5.) the Angles *BFC*, *CFD*, *BFD*, will be respectively to four right angles nearly as the Times between the observations to the Periodic Time: And on the contrary, if the said ratio hold in these angles, *ACDPB* is the Orbit of the Earth sought. What remains therefore to be done is, to find the Position of the right line *SF* or *AP*, and the Species of the Ellipse, by having given, besides the angles *BSC*, *CSD* and *BSD*, the ratios also of the angles *BFC*, *CFD* and *BFD* to four right angles.

Since there are given the angles *RSZ*, *ZSY*, *RSY*, namely the observed differences of the Longitudes, and the angles *BFC*, *CFD*, *BFD*, namely the Mean motions of the Earth almost for the spaces of the times between the Observations; therefore there are also given *RFZ*, *ZFY*, *RFY* Arithmetical Proportionals between *BFC* and *RSZ*, *CFD* and *ZSY*, *BFD* and *RSY* respectively. For since *SR* is equal to *AP*, by construction, and the sum of the right lines *SB* and *BF* equal to the same, (by *Prop.* 52. *B.* 3. *of the Conics of* Apollonius) *BR* and *BF* will be equal; and after the like manner may *CZ*, *CF*, as also the right lines *DY*, *DF*, be shewn to be equal. Consequently *BRF* and *BFR*, *CZF* and *CFZ*, *DYF* and *DFY* will be equal. But the angle $BFC = RFZ + BFR + CFZ$; and $RFZ = RSZ + BRF + CZF$. Since therefore *BFC* exceeds *RFZ* as much as that does the angle *RSZ*; 'tis plain that these are Arithmetically proportional. After the same manner are *CFD*, *ZFY*, *ZSY*; also the angles *BFD*, *RFY*, *RSY* are Arithmetically proportional. Besides, in the Triangle *YFG* all the Angles are given; namely *YFG* equal to the complement of the angle *YFZ*, to two right angles a mean Proportional between *CFD* and *CSD*, which

which are known, and YGF half the obſerved Angle ZSY: And therefore ſuppoſing YF to be of any number of parts whatever, FG will be given in thoſe parts. In the Triangle RFG all the angles being given, (namely RFG the complement of ZFR to two right, and FGR half the angle ZSR) and the ſide FG, FR will be given. And in the Triangle YFR there being given FY and FR, and the angle YFR, (becauſe a mean Arithmetic proportional between BFD and BSD,) YR will be given and the angles FRY, FYR. Then, in the Iſoſceles Triangle YSR all the angles are given (becauſe YSR at the vertex of the Triangle is given) and YR, therefore YS alſo will be given. Laſtly, in the Triangle FYS the angle FYS is given (the difference between the known ones RYS, RYF) and the ſides FY, YS; therefore the angle YSF and ſide SF will be found. But the Poſition of the right line YS is given, namely the Place of the Sun in the third Obſervation, when the Earth was in D; therefore the Poſition of the line of the Apſides AP will be given, becauſe inclined in a known angle YSF to YS given by poſition. But in the ſame parts, in which YS is expreſſed, the right line SF is found; therefore the ratio between YS or what is equal to it AP the greater Axe of the Orbit, and SF the diſtance of the Foci is given; that is, the Species of the Orbit. Nor can any thing elſe be determined by the Obſervations: For every Elliptic Orbit ſimilar to $ACDP$, deſcribed upon the Focus S, whoſe greater Axe is the ſame in poſition with the right line SF, will ſatisfy the Obſervations propoſed.

In the Triangle DFS, the ſide SF, the angle DSF, and the angle SDF, the double of SYF lately found, being given; DFS is found, and conſequently DFA contiguous to it. But the

Time

Time which is to the whole Periodic Time as the angle DFA to four right, is that which happens between the moment of the obſervation in D, and the moment that the Earth is in its Aphelion; this therefore becomes known.

The method of determining the Species of the Orbit, and the Poſition of the Line of the Apſides here delivered, agrees as well to any other Planet revolving about the Sun as to the Earth, provided there are but three Obſervations, as much to the purpoſe in this other Planet, as thoſe three uſed about the Earth.

PROPOSITION XV.

To correct at pleaſure the Poſition of the Line of the Apſides, and the Species of the Orbit determined above.

Let the Poſition of the line of the Apſides and the Species of the Orbit found by the foregoing, be taken for true and accurate, (for they do not differ much from thence by *Prop.* 5.) from them and the Periodic time and the times between the obſervations being given, find the angles corrected at F, agreeable to the times of the obſervations made in the points B, C and D; and if you will uſe only the firſt correction (namely *Bullialdus*'s,) this may be done by *Prop.* 7; and you may go to what exactneſs you pleaſe by *Prop.* 4. By the help of theſe angles thus corrected, inſtead of the former, that are ſimply proportional to the times; that is, in the language of Aſtronomers, by the help of the angles of the mean Anomaly thus equated, inſtead of the angles of the mean ſimple Anomaly made uſe of before, let the calculation of the former Propoſition be repeated, and let the Poſition of the Line of the Apſides and the Species of the Orbit be found, and they will be more accurate.

COROL-

COROLLARY.

From whence the Motion of the Apſides of the Orbit of the Earth may be determined: Namely by comparing of the Place of the Aphelion at preſent with its Place ſome Years ago, (both which are found by *Prop.* 14, and corrected by *Prop.* 15,) and by dividing the arc made in the mean while by the ſaid Aphelion by number of the intervening Years, the Annual Motion of the Aphelion will come forth, which alſo will be extended to parts of a Year.

PROPOSITION XVI.

TO *find the Place of the Earth ſeen from the Sun, and its Diſtance from the Sun at any given Time.*

The ſpecies of the Orbit of the Earth, and the place of the Aphelion for any known time being defined by the two laſt Propoſitions, find the place of the Aphelion for the time given: And if it be ſince the time to which the place of the Aphelion was determined, add the motion of the Apſides made in the mean while, and defined by the Corollary of the preced. *Prop.* to the place of the Aphelion determined before, or let it be ſubtracted, if the time given be before; and you will have the Place of the Aphelion for the given time. Then let the time when the Earth once was in the Aphelion be determined (by *Prop.* 14,) and ſince by the 13th the Periodic time of the Earth may be known; from thence alſo will be known the time when the Earth was in its Aphelion next before the Time propoſed, and conſequently the ratio between the Periodic time of the Earth and the time elapſed ſince the laſt Aphelion is known; that is, in the Figure *ALP* (*Fig.*1.) repreſenting the Orbit of the Earth, the ratio of the intire Ellipſe to the area *ALS*: And conſequently (by *Prop.* 3,4, 6 and 7.) the angle *ASL* will become known.

And

And ſince the poſition of the right line *A S* is known, the poſition of the right line *S L* will alſo be known; that is, the Place of the Earth ſeen from the Sun. Beſides, by the Propoſitions above cited, the ratio of the right line *L S* to the right line *AP* becomes known; that is, the Diſtance of the Earth from the Sun in the parts of the greater Axe.

Proposition XVII.

T*O explain the inequality of the natural Days, and to ſhew the Equation of time.*

Since it is the buſineſs of an Aſtronomer to ſhew the motion of the Stars, and this is performed in Time, and can't be conceived without it; and ſince the moſt ſenſible part of Time is a natural Day, by the repetition of which a Year, and by the diviſion of which an Hour and the parts of an Hour are made; and ſince in computing their motions, all days ought to be ſuppoſed equal; as far as this ſuppoſition is from truth, ſo far even the Celeſtial motions taken out of the Tables will diſagree with the Phænomena, if no account of this inequality be taken. To underſtand this, let it be obſerved, that a Natural day is that ſpace of time, that intervenes between the Sun's leaving a given Meridian in the Heavens and returning to it again; that is, the ſpace wherein a revolution of the whole Celeſtial Equator is performed, and over and above, of that part of it, that anſwers to ſuch a portion of the Ecliptic, as the Sun deſcribes in the mean while by its Annual motion towards the Eaſt. But becauſe this portion of the Equator, added to the whole Equator, is not every where equal, as well on the account of the Obliquity of the Ecliptic, as that the apparent Annual motion of the Sun about the Earth is not equable; neither will theſe Natural days be equal: And would therefore then be equal, if the

the Sun ſeen from the Earth in its Annual motion moved both equably and in the Equator; and 'tis evident in this caſe, after that the whole Equator has paſſed the Meridian, there remains that portion ſtill to paſs, which the Sun in the mean while would deſcribe by its Annual motion towards the Eaſt, which portion would always be equal to itſelf, ſince we ſuppoſe this motion to be equable; and therefore even the Time meaſuring the paſſage by the Meridian of the whole Equator and this additional part, would be always equal to itſelf. Let us therefore ſuppoſe another Sun, as was ſaid before, moving in the Equator, and conſequently meaſuring the equal or mean Time: 'Tis evident that the difference of the Time, between the appulſe of the true and imaginary Sun to the Meridian, or any other Hour Circle, is that which makes the mean Noon of the apparent Noon, and the apparent of the mean; that is, the *Equation of Time* itſelf. And becauſe the true Sun and that point of the Equator, where its Right aſcenſion ends, arrive at the Meridian together; and the Equation of Time is that ſpace of Time, that flows while the Arc of the Equator contained between the extreme point of the Right aſcenſion of the true Sun and the place of the imaginary Sun, paſſes the Meridian; if this Arc be converted into time, you will have the Equation of time ſought. Beſides, the place of the imaginary Sun is as far diſtant in the Equator from the point of the Vernal Equinox, as the mean place of the Sun in the Ecliptic is from the ſame: And therefore the foreſaid arc of the Equator, the meaſure of the Equation of the time, is equal to the difference between the Right aſcenſion of the true Sun and the diſtance of the mean place of the Sun from the Vernal Equinox. But it is commonly known, that the difference of any

two Quantities *A* and *B* is equal to the difference between *A* and any aſſumed quantity *X*, and to the difference between this *X* and *B* together. Aſſuming therefore a third arc, namely the diſtance of the true place of the Sun from the Vernal Equinox, the Arc meaſuring the Equation of Time will be the ſum of the two arcs, one of which is the difference between the Right aſcenſion of the Sun and its true place in the Ecliptic, the other the difference between the true and mean motion of the Sun. But ſometimes it happens, that the difference of the two Quantities is equal to the ſum of them taken ſimply; namely when the one is negative and the other poſitive: Therefore an Arc to be converted into Time, to produce the Equation of Time, is always the ſum or difference of two arcs, one of which is the difference of the true Motion of the Sun and its Right aſcenſion, and the other is the difference of the true and mean Motion of the Sun.

But becauſe this Arc conſiſts of two parts, we will conſider them ſeparately. And firſt let us abſtract from the unequal motion of the Sun under the Ecliptic; that is, let us conſider it a little while as equal; and therefore the motion of the imaginary Sun in the Equator will be equal to the motion of the true Sun in the Ecliptic, or the true and imaginary Sun will be equally diſtant from both Equinoxes. And becauſe in the firſt Quadrant of the Ecliptic, or from the beginning of ♈ to the beginning of ♋, the diſtance of the Sun from the beginning of ♈ is greater than its Right Aſcenſion; the imaginary Sun will be farther diſtant from the beginning of ♈ toward the Eaſt, than is the Right Aſcenſion of the true Sun, and therefore the true Sun arrives at the Meridian ſooner than the imaginary, becauſe more to the Weſt; that is, the apparent time precedes the

the mean: hereupon when the Sun is in the firſt Quadrant of the Ecliptic, that part of the Equation of Time, which is the difference of the true Motion of the Sun and its Right Aſcenſion, turn'd into Time is to be ſubtracted from the apparent, to make the mean or equable Time. But when the Sun is in the ſecond Quadrant of the Ecliptic, its Right Aſcenſion is greater than the diſtance of the Sun from the beginning of ♈, (becauſe the complement of the Right Aſcenſion to a Semicircle is leſs than the diſtance of the Sun from the beginning of ♎;) and therefore the imaginary Sun (which, by ſuppoſition, is as far diſtant in the Equator from the beginning of ♈, as the true Sun in the Ecliptic is diſtant from the ſame) is leſs diſtant from the beginning of ♈ than the true Sun; and conſequently arrives at the Meridian ſooner, and therefore the mean Noon precedes the apparent; that is, the mean Time reckons 12 Hours, when the apparent does not yet reckon ſo many, tho' it be the ſame moment, but differently called. And therefore in this caſe, when the Sun is in the ſecond Quadrant of the Ecliptic, this part of the Equation of Time (which ariſes from the obliquity of the Way of the Sun to the Equator, and is the difference between the Sun's motion and its Right Aſcention turned into Time) is to be added to the apparent, to make the mean time, or to be ſubtracted from the mean to make the apparent. But if the Sun be in the third Quadrant of the Ecliptic, the ſame is to be done as if it were in the firſt; for here the diſtance of the Sun from the beginning of ♈ is greater than its Right Aſcenſion, as in the firſt: And the Sun being in the fourth Quarter, the ſame holds as to this part of the Equation of Time as if it were in the ſecond, and for the ſame reaſons.

Let us next conſider the Inequality of natural Days ariſing from the unequal motion of the Sun in the Ecliptic, or the Equation of time ariſing from thence, laying aſide a little while the Obliquity of the Ecliptic; that is, conſidering the Sun as moved after the like manner, and with the ſame celerity in the Equator as it is found really to move in the Ecliptic, and that theſe two Suns the true and imaginary, left the Apogæum together. And ſince the imaginary one moves with the mean Motion, it muſt go before the true one, while it goes forward from the Apogæum to the Perigæum; that is, the imaginary one is more to the Eaſt than the true one: The true one therefore arrives at the Meridian ſooner; therefore the apparent Noon precedes the mean. And the difference between the places of the true and imaginary Sun, or between the true and mean Motion of the Sun, converted into Time, is the Equation of the time ariſing from this inequality of the Sun's motion: This Equation therefore, while the Sun goes from the Apogæum towards the Perigæum, or (in the language of an Aſtronomer,) in the ſix firſt Signs of the Anomaly, is to be ſubtracted from the apparent Time to produce the mean, or added to the mean to produce the apparent out of it. In the Perigæum the true Sun overtakes the imaginary, and from thence to the Apogæum is more to the Eaſt: The imaginary one therefore in all this time, namely in the ſix laſt Signs of the Anomaly, arrives at the Meridian ſooner; that is, the mean Noon precedes the apparent Noon, by as much as the arc between the imaginary and true Sun ſpends in paſſing by the Meridian; that is, as much as that arc, namely the difference between the mean and true motion of the Sun converted into time makes. This part of the Equation of time therefore

fore in the present case is to be added to the apparent time to make the mean, or to be subtracted from the mean to make the apparent.

And when both these causes of the Equation of Time hold, (as they really do) both these things are to be done; that is, to equate the Time you must consider the Sun two ways, *viz.* both in what Quadrant of the Ecliptic it is in, that one part of the Equation may be added to or subtracted from the apparent, as was said before; and in what Semicircle of Anomaly, that the part depending upon that, may be added or subtracted according as the nature of it requires. And on this account it is, that the Equation of natural Days (which, from what has been said above, absolutely speaking, is the Difference of the mean place and the right Ascension of the Sun's true place turned into time) is made up of two parts, And because the vernal Equinox and Apogæum of the Sun, the heads or beginnings of these parts, don't always keep the same position to one another, but the former being carried towards the West leaves the latter, (as was shewn in *B.* 1.) comes to pass, that the (absolute and only) Equation of Time made up of these is not perpetual, but useless in a few Years, the Sun not possessing the same degree of the Anomaly, or the same situation in respect of the Apsides, when it returns to the same point of the Ecliptic.

Ptolemy expresly demonstrates the above mention'd Equation of Time, *Chap.* 10. *B.* 3. *of Great Construct.* notwithstanding *Vendeline* and others have said that it is all useless, and use the apparent and mean Time promiscuously. *Kepler* on the contrary suspected a third cause of equating the time, arising from the diurnal Motion of the Earth's not being intirely equable, but sometimes quicker sometimes slower, according as it is more or less distant

from the Sun, the original of its motion; and calls the part of the Equation depending upon it, the *Physical* Part, leaving the name of the *Demonstrative* to the other described above, which is made up of two parts. *Tycho* alſo had a ſuſpicion of the acceleration or retardation of the Firſt Motion, and was for ballancing the other part of the Equation depending upon the Sun's Anomaly, by means of it; for he only made uſe of the other ariſing from the Obliquity of the Ecliptic, which he calls the *Empyric*, becauſe it would ſuffice for the finding by Calculation the Eclipſes he had obſerved. But *Street* thought this Inequality of the Firſt Motion, to be ſufficient for the making uſe of the part neglected by *Tycho*, under a contrary Title, as he has put it in his *Aſtronomia Carolina*.

PROPOSITION XVIII.

To correct the Place of the Sun, ſeen from the Earth, found by Prop. 16.

If the given Time is the mean, the Place found needs no correction; for it is hitherto the true. But if otherwiſe, let the apparent given Time be converted into the mean (by the preced. *Prop.*) by taking the Place of the Sun found above, *Prop.* 16. as accurate: (For that Place of the Sun is really accurate enough for the determining the Equation of Time, ſince it continues very nearly the ſame, whether the Place of the Sun be that which is found above, or the true and moſt correct:) And to this mean, again (by the ſaid 16th *Prop.*) find the Sun's place, and the place thus found will be the correct and accurate one. For the mean Time only is fit for Aſtronomic uſes, on which account it is call'd the *Aſtronomic Times*; this therfore ſhould be made uſe of: But to determine the mean when the apparent is given, the Sun's place is neceſſary; that therefore was firſt to befound, by

by uſing the apparent Time,as if it were the mean, to find the Equation of the Time. But this once found, and the apparent Time reduced to the Aſtronomic, ſeek for that Time (which is the ſame with the moment propoſed, but expreſſed by a number fitted for an entrance into the Tables) the Sun's place (by the ſame *Prop.* 16.) and the Diſtance of the Earth from the Sun, which is evidently the true and correct one.

The converſe of this Problem may be conſtructed much after the ſame manner, and the Time may by found, when the Sun ſeen from the Earth will be in a given Place.

SECTION III.

Concerning the determining the Orbits of the reſt of the Primary Planets, and compoſing the Theory of them ſeen both from the Sun and the Earth.

PROPOSITION XIX.

TO determine by obſervation the Poſition of the Line of the Nodes of a Planetary Orbit, and the Diſtance of the Planet itſelf, when in the Node, from the Sun. [*Fig.*20.]

In the two Figures fitted to the two caſes of a ſuperior and an inferior Planet, let *S* repreſent the Sun; *Tt* the Orbit of the Earth, whoſe Focus is *S*; *Nn* the Line of the Nodes of the Planet propoſed. And firſt let the Earth be in *T*, and from thence let the Planet be obſerved when in the Ecliptic, and therefore in its Node, ſuppoſe *P*: For the Planet will never be ſeen in the Ecliptic from the Earth, being in the Plane of the Ecliptic, unleſs when it is alſo in the Plane of

the Ecliptic. After an intire revolution, let the Planet be obſerv'd in the ſame Node, when the Earth arrives at *t*. Let the right lines *ST*, *Pt*, be imagined to be drawn, as alſo *St*, *PT* interſecting one another in the point *X*. In the Triangle *STX* there are given the angles *TSt* by the Theory of the Earth and Time between the obſervations, and *STP* the obſerved Elongation of the Planet from the Sun, and the Side *ST* the Diſtance of the Earth from the Sun in the former obſervation. Therefore *SX* is found; and ſo *Xt* ſince *St* the Diſtance of the Earth from the Sun in the ſecond obſervation is alſo given: Again, in the Triangle *tXP* the angle *tXP* equal to the angle *TXS* in the former is given, and the angle *StP* the elongation of the Planet from the Sun in the ſecond obſervation, and the ſide *tX* lately found; and conſequently the ſide *Pt* will be found. Laſtly, in the Triangle *StP*, the ſides *tS*, *tP* being given, together with the angle contained *PtS*, *SP* the Diſtance of a Planet in a Node from the Sun will be found, and the angle *tSP* that the Centric Place (or that ſeen from the Sun) contains with *St* given by poſition, namely, the Place of the Earth ſeen from the Sun at the time of the ſecond obſervation; and the oppoſition to this is the Centric Place of the other Node; that is, the Poſition of the Line of the Nodes *Nn*.

By this means the Motion of the Nodes of the Orbs of the Primary Planets will be determined, by comparing the Place of the Node of each Planet found out by the obſervations of the Ancients, with the Place of the ſame determined by the obſervations of the Moderns.

After the like manner, if a Planet be obſerved twice in any other point of its Orbit from the Earth, the Place of the Planet ſeen from the Sun and its Diſtance from the Sun are determined.

PRO-

Plate 5. Book 3.

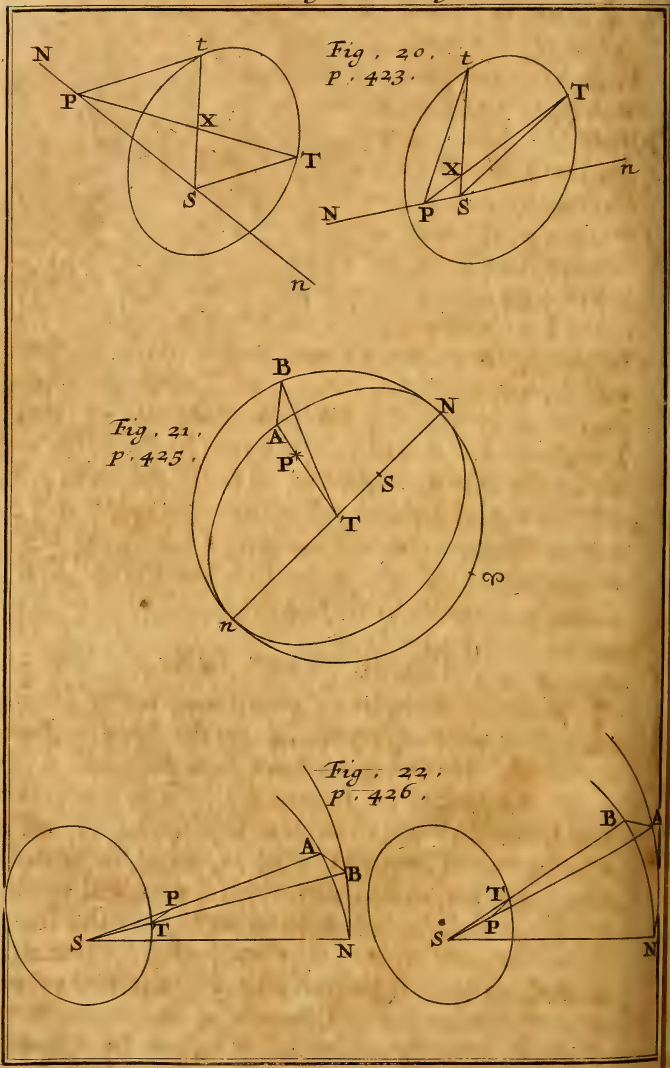


Fig. 20. p. 423.

Fig. 21. p. 425.

Fig. 22. p. 426.

PROPOSITION XX.

TO *define by observation the Inclination of any Planetary Orbit to the Plane of the Ecliptic.*

Let *S* represent the Sun; the Circle *NBn* [*Fig.* 21.] the Ecliptic among the Fix'd Stars; and *NAn* the Path of the proposed Planet, seen from the Sun among them: And the right line *NSn* will be the Line of the Nodes known by position, (by the preced. *Prop.*) Whenever the Earth arrives at the said Line of the Nodes, which it does twice a Year, the Time of which (by *Prop.* 18.) is known; let the Geocentric Place *A* of the Planet *P* be observed (by *Prop.* 27. *B.* 2,) whose Latitude is *AB* an Arc perpendicular to the Ecliptic, and Longitude ♈ *B.* Since the Longitude of the Sun ♈ *N* is known, the difference also of these Longitudes *NB* will be known. In the Spherical Triangle *ANB*, right angled at *B*, the Sides *AB*, *NB* are given; and therefore the angle *ANB*, the measure of the Inclination of the Plane *NAn* of the Orbit of the Planet to the Plane of the Ecliptic *NBn*, will be known.

PROPOSITION XXI.

FRom *the observation of a Planet in Opposition or Conjunction with the Sun, to get away its Second Inequality and find its Distance from the Sun.*

Besides the true Inequality in the motions of the Planets visible to an Eye in the Sun, placed in the common Focus of all the Orbits, there is another Optical one, arising from the annual motion of the Earth, relating to us living on the Earth. The former, which is proper to the Planets, is called by a name proper enough, the *First Inequality*; but the other is secondary, as it were to the Planet, and therefore justly called by Astronomers the *Second Inequality*: And he is said to get

it

it away, who ſhews that Place of a Planet, which an Obſerver would ſee from the Sun. Let *S* (in *Fig.* 22.) repreſent the Sun, *T* the Earth, *P* the Planet in its Orbit, *NB* the Ecliptic among the Fix'd Stars, *NA* the interſection of the Orbit of the Planet with the Sphere of the Fix'd Stars; *N* the Node: And ſeeing the Sun is in the Plane of the Orbit of each Planet, the right line *SN* will be the Line of the Nodes. Let the Earth *T* be ſeen from the Sun among the Fix'd Stars at *B*, and the Planet *P* at *A*. And becauſe the Planet is either in Oppoſition to the Sun, (as in *Fig.* 1ſt) or in Conjunction with it (as in the 2d;) the arc of the great Circle connecting the points *A* and *B*, is the arc of the Circle of Latitude, and conſequently perpendicular to the Ecliptic. And in the Spherical right-angled Triangle *A B N* there are given the angle *A NB* the meaſure of the Inclination of the Plane of the Orbit of the Planet to the Ecliptic, (found by the preced *Prop.*) and the ſide *BN*, namely the inclination of the right line *TS* given (by the Theory of the Earth) by poſition to *SN* the Line of the Nodes found by poſition (by *Prop.* 19.) Therefore there will be known the Side *AB*, the Centric Latitude of the Planet, and *AN* the Diſtance of the Planet in its Orbit, ſeen from the Sun, from the Node *N*, which is to get away the Second Inequality of the Planet. Moreover in the rectilineal Triangle *PST* there are given *ST* from the Theory of the Earth, and the angle *PTS* the Latitude of the Planet by obſervation, or its complement to a Semicircle, and the angle *PST* the Centric Latitude of a Planet found above; wherefore *PS* and *PT* the Diſtances of the Planet from the Sun and Earth will be known.

PRO-

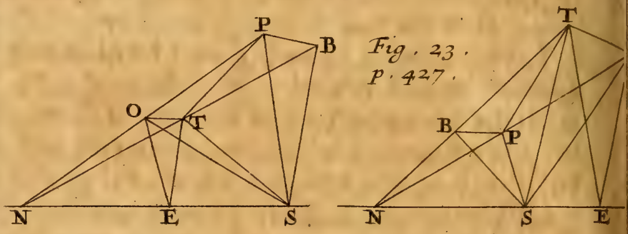


Fig. 23. p. 427.

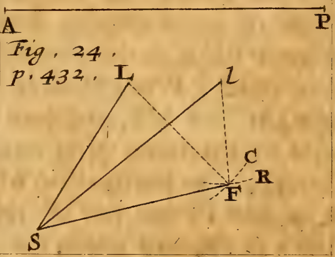


Fig. 24. p. 432.

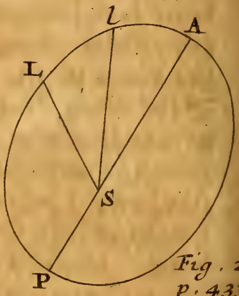


Fig. 25 p. 433.

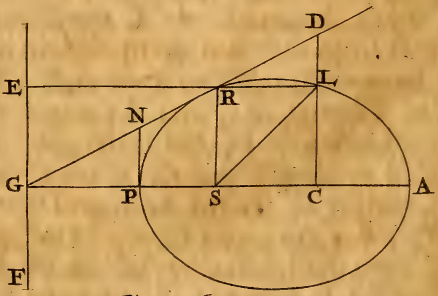


Fig. 26. p. 434.

PROPOSITION XXII.

BY any single observation of a Planet whatever, to clear it of its Second Inequality, and find its Distance from the Sun.

Let the Planet *P* (in *Fig.* 23.) be observed, the Line of whose Nodes is *NS*, from the Earth *T*, with the apparent Latitude *PTB*. Let the Plane of the apparent Latitude be produced, till it cut the Plane of the Orbit of the Planet in *PN*, and the Plane of the Ecliptic in the right line *BTN*. From P let fall *PB* perpendicular to the right line *NB*, and from *T* erect *TO* perpendicular to the same *NB*, which (by *Prop.* 38. *El.* 11.) will be perpendicular to the Plane of the Ecliptic, because the Plane of the Latitude *PNB* is perpendicular to the Plane of the Ecliptic. From *T* let fall *TE* perpendicular to the right line *NS*, and join *OE*, which is perpendicular to the same *NS*; and the Angle *OET* will be equal to the Inclination of the Planes of the Orbit of the Planet and of the Ecliptic. In the Triangle *NST*, are given the Side *ST*, and angle *TSN*, by the Theory of the Earth and the known place of the Node, and by observation the angle *NTS*, the Elongation of the Planet from the Sun computed in the Ecliptic, or its complement to two right; therefore the Sides *TN* and *NS*, and the angle *TNS* will become known. In the Triangle *TEN* right angled at *E*, *NT* and *TNE* are given; and consequently *TE*. In the Triangle *OTE* right angled at *T*, there are given *TE* the Side found just now, and *TEO* the Inclination of the Planes of the Orbit of the Planet and of the Ecliptic, known by *Prop.* 20. of this Book; therefore *OT* will become known. In the Triangle *OTN* right angled at *T*, there are given *OT* and *TN* found before; and therefore the angle *ONT* also. In the first angle

PNT,

PNT, there are given *NT*, and the angle *TNP*, alſo *PTN* the apparent Latitude of the Planet known by obſervation, or its complement to two right; therefore *NP* will be known. In the Triangle *NPB* rightangled at *B*, the ſide *NP* and angle *PNB* lately found being given, *PB* and *NB* will be found. In the Triangle *BNS*, *NB* and *NS* and the angle *BNS* being given, *NSB* the Centric Longitude of the Planet computed from its Node, and the ſide *SB* will be known. Then in the Triangle *PBS* rightangled at *B*, *PB*, *BS* being known, *PS* the Diſtance of the Planet from the Sun will be known, and the angle *PSB* its Centric Latitude. Laſtly, in the Triangle *PNS* all the ſides being given, the angle *NSP* is known, the Centric Diſtance of the Planet in its Orbit, computed from the Line of the Nodes given by poſition (by *Prop.* 19.) in its Orbit. The Second Inequality of the Planet therefore is got rid of, and its Place ſeen from the Sun is determined: And beſides the Diſtance of the Planet from the Sun is found in the parts of the mean Diſtance of the Earth from the Sun, which is taken for the meaſure.

PROPOSITION XXIII.

To define the Periodic time of any Planet about the Sun.

Let the Planet propoſed be obſerved in the Node (as was done in *Prop.*19.) and again, when it returns next to the ſame Node: The Time between theſe obſervations is very nearly the Periodic Time of the Planet ſought. For ſince the Lines of the Nodes are (by *Prop.* 43. *B.* 1.) almoſt at reſt, when a Planet has deſcribed its whole Orbit, it returns to the ſame Node, *&* *vice verſa*.

The

The Period of a Planet is determined not only by the obſervation of it twice in the ſame Node, but its Periodic time will become evident, if it be obſerved twice in any other point of its Orbit, cleared of the Second Inequality. But if the Centric Place of the Planet be computed from the Equinoxes, regard muſt be had to the Preceſſion of the Equinoctial points made in the mean while. In defining the Periodic times of a Planet, it will be proper to make uſe of that advice given, in determinining exactly the Quantity of a Tropical Year; namely to take the moſt diſtant Obſervations that can be had, that the error may be inſenſible, the diviſion being made by a great number of revolutions. And that an error of an intire revolution may not be committed, the knowledge of a Period near the truth, and expreſſed by a round number will ſuffice, which may be had, if in one revolution of a Planet two or more Centric Places be ſought in Oppoſition or Conjunction with the Sun, (namely when it has no Second Inequality) and the Periodic time be taken in that ratio to the Time elapſed between any two Obſervations, as the diſtance between the centric Places is to an intire Circle. If the Planet ſhould not happen to be obſerved in the Node itſelf, or in the Oppoſition with the Sun itſelf, (which generally happens;) then that moment is determined by obſervations made before and afterwards. If the period of a Planet be taken for that ſpace of Time, wherein the Centric place of a Planet becomes the ſame again in the Mundane ſpace or between the Fix'd Stars as it was before; tho' accurately ſpeaking, there is no ſuch Period at all, ſuppoſing the Nodes move; that is, that the planes of the Orbits themſelves change; notwithſtanding, if this Period be different from the Period defined above,

it

it may be defined (confidered in regard of Longitude) by the help of the obfervations made by the preced. *Prop.* when the Centric Place of the Planet is near the fame Fix'dStars.

PROPOSITION XXIV.

TO *find the length of the greater Axe of the Orbit of any Primary Planet.*

We have hitherto defined the fituation of the planes of the Orbits of the Planets to one another: We now come to defcribe the Orbits themfelves in the planes thus defined. Which we fhall do feveral ways, that this or that may be ufed according as the obfervations are, and the one may correct the other; and that the Orbits determined by all or feveral of thefe ways tending to one and the fame thing, may be efteemed more accurate. And firft we fhall fhew how their greater Axes may be determined. The greater Axe of the Orbit of any primary Planet, is to be taken to the greater Axe of the Orbit of the Earth in the fubfefquiplicate ratio of the Periodic time of the propofed Planet, (found by the preced. *Prop.*) to the Periodic time of the Earth about the Sun, determined by *Prop.* 13. For then the Cubes of the greater Axes of the Orbit of the Planet and of the Earth, will be as the Squares of the Periodic times of thofe Bodies about the Sun, which has been demonftrated (in *Prop.* 40. *B.* 1.) to hold in Bodies revolving about the fame Center, and acted upon by a Centripetal Force, which is reciprocally proportional to the fquare of the diftance from that Center: And that this is the condition of the Primary Planets revolving about the Sun, has been fhewn in *Prop.* 42. *B.* 1.

PRO-

PROPOSITION XXV.

THree Centric Places of a Planet and its Periodic Time being given, to deſcribe its Orbit, and find the moment of Time when the Planet was in the Aphelion.

After the ſame manner as in *Prop.* 14. three Places of the Earth ſeen from the Sun (that is, Centric places of the Earth) being given,together with the Periodic time, the Species of the Orbit of the Earth, and the Poſition of the Line of the Apſides is determined ; the ſame may be done by the help of the ſame like *Data* in the Planet propoſed, and the Orbit thus deſcribed, may be corrected by the method pointed at in *Prop.* 15. And as the Orbit of the Earth is deſcribed in the plane of the Ecliptic, determined before (in *Prop.* 19. *B.* 2,) ſo the plane of the Orbit of the Planet propoſed, wherein its Orbit is to be deſcribed, is determined (by *Prop.* 19 and 20.) Moreover, the ratio of the greater Axes of the ſame Orbits is found from the Periodic times of the Earth and Planet (by preced. Prop.) The magnitude of the Orbit of the Planet will be had from the aſſumption of the greater Axe of the Earth's Orbit (or its half, the mean Diſtance of the Earth from the Sun :) The Orbit of the Planet therefore is wholly determined. And the Time when the Planet was in the Aphelion, is determin'd after the ſame manner,as the like was done *Prop.* 14. in the Earth.

Becauſe the Species of the Elliptic Orbit of the Planet is only determined from theſe *Data*, (as is evident from *Prop.* 14,) and the magnitude of the Orbit in reſpect of the Orbit of the Earth is concluded by the preced. from the compariſon of the Periodic times of theſe Planets ; the ſame may be inferred other ways ; namely, by making an Orbit

Orbit of a certain Species (determined from three Centric Places) defcribed about the Sun in a Focus, to pafs thro' the very Place of the Planet in the line of the Nodes, found by the 19th *Prop.* of this: For this point given in the Orbit of the Planet, ferves as well in the determining its Magnitude in refpect of the Magnitude of the Orbit of the Earth, as its greater Axis defined above: For the diftance of that point from the Sun is determined in the parts of the lines of the Earth's Orbit; and any one of thefe ways of defcribing the Orbit, may be examined and corrected by the other.

And from hence may the motion of the Apfides of any Primary Planet be defined; namely the fame way as the motion of the Nodes was before.

PROPOSITION XXVI. LEMMA.

THE Focus and length of the greater Axe being given, to defcribe an Ellipfe paffing thro' two given Points.

Let *S* [in *Fig.* 24.] be the Focus of the Figure to be defcribed, *AP* the length of the greater Axe, *L* and *l* the two points thro' which the Ellipfe ought to pafs. On the center *L*, at the diftance equal to the excefs of the right line *AP* above the right line *LS*, let a Circle *CF* be defcribed; and after the like manner upon the Center *l*, with a diftance equal to the excefs of the right line *AP* above *lS*, defcribe a Circle *RF* cutting the former in *F*; and *F* will be one Focus of the Ellipfe which is to be defcribed: Confequently an Ellipfe defcribed upon the Foci *S* and *F*, having its greater Axe equal to the right line *AP*, will be the Ellipfe fought. For fince *LF* is equal to $AP - LS$, $LS + LF$ will be equal to *AP*. After the fame manner *AP* is equal to $lS + lF$: Confequently (by *Prop.* 52. *B.* 3. of *Conics.*)

Conics,) an Ellipſe deſcribed upon the Foci *S* and *F*, having a greater Axe equal to the right line *A P*, will paſs thro' the points *L* and *l*.

'Tis eaſy from ſuch a Geometrical conſtruction as this to invent a method of calculating ſuitable to Aſtronomy. For three points *S*, *L* and *l* being given, that is, the three ſides of the Triangle *S L l* being known, the angles become known. Again in the Triangle *L l F*, all the ſides are given; namely *L l* as before, *L F* the difference of the given right lines *A P* and *L S*, and *l F*, the difference of *A P*, *l S*; therefore the angle *F L l* becomes known: But *S L l* was known before, therefore the angle *S L F* will be given. Therefore in the Triangle *S L F*, *L S*, *L F*, with the angle *S L F* being given, the ſide *S F* the diſtance of the Foci, and the angle *L S F*, between *L S* given by poſition and the greater axe, will be known; this axe therefore is given by poſition; and it is already given by magnitude: From whence all that belongs to the Ellipſe may be drawn.

PROPOSITION XXVII.

Given two Centric Places of a Planet and the Diſtances from the Sun, together with its Periodic Time, to determine the Orbit of the Planet.

Let *S* be the Sun [*Fig.* 25.] let the two Centric places of the Planet given be in the right lines *S L*, *S l* given by poſition, and let the Diſtances of the Planet from the Sun be the right lines *S L*, *S l* given in magnitude. Wherefore *L* and *l* are the points thro' which the Ellipſe is to paſs. Moreover, the periodic Time being known (by *Prop.* 24.) *PA* the greater axe of the Elliptic Orbit is determined. Deſcribe an Ellipſe therefore (by the preced.) whoſe Focus is *S*, having its greater axe of a given length, that may paſs thro' the given points *L* and *l*. The Centric places of the

 Planet

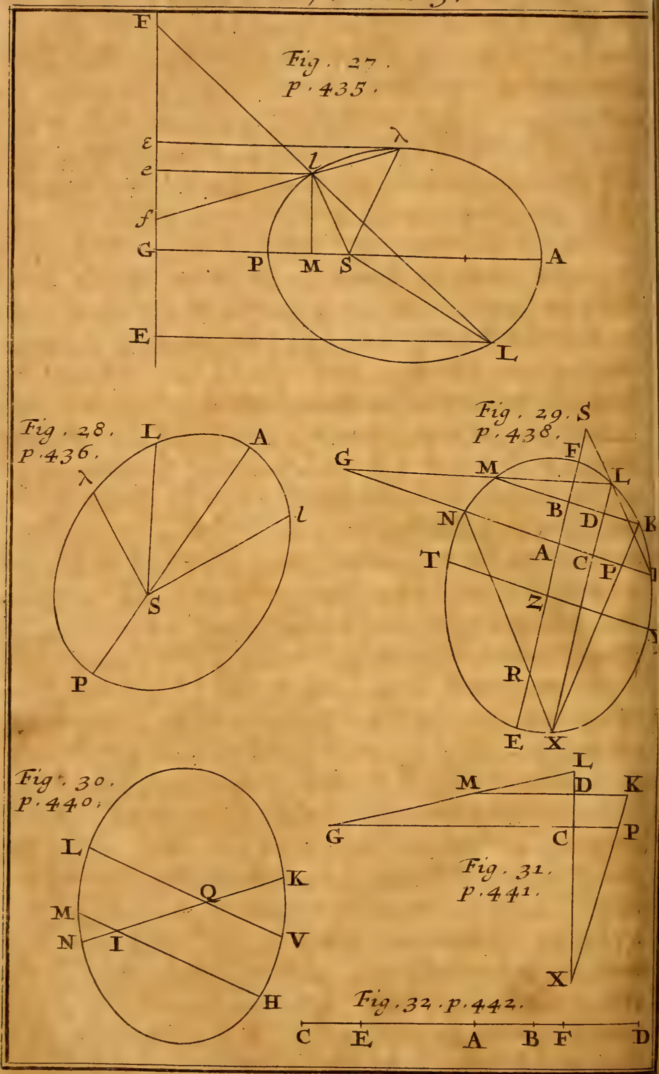
Planet and its Diſtances from the Sun, either in Oppoſition or Conjunction with the Sun are determined as in *Prop.* 21; or from an Obſervation made any where, as in *Prop.* 22. Nevertheleſs that may be aſſum'd, which is determined *Prop.* 19. when the ſituation of the Planet's Orbit in regard of the Ecliptic was defined. But if you have more than two Centric places with their correſponding Diſtances from the Sun, then their Orbit being defined by any two, you muſt ſee whether they agree to it thus defined, and that is to be retained (if they diſagree) which is confirm'd by moſt Obſervations, as is uſually done in moſt Aſtronomical Calculations.

PROPOSITION XXVIII. LEMMA.

TO *the greater Axe* AP *of the Ellipſe* ALP [*Fig.* 26.] *from the Focus* S *let a perpendicular* SR *be erected meeting the Ellipſe in* R, *thro' which let a right line* DR *be drawn touching the Ellipſe, and meeting with the Axe* AP *produced in* G, *and from* G *let* GF *be drawn perpendicular to the Axe. Theſe things being ſuppos'd, if from any point* L *of the Ellipſe two right lines* LS, LE *be drawn, one as* LS *to the Focus aſſumed* S, *the other* LE *perpendicular to* FG; *I ſay, that* LE *is to* LS *in the given ratio of* GP *to* GS.

Thro' the point *L* to the Axe *AP* draw the perpendicular *LC* meeting the Axe in *C* and the Tangent *DR* in *D*, and from *P* draw another line *PN* parallel to it: And *CD* will be equal to the right line *SL* (by *Prop.* 139. *B.* 4. of *Gregory St. Vincent*) and *PN* to *PS*. But *GC* is to *CD* in a given ratio, namely of *GP* to *PN*; and therefore *EL* is to *LS* in the given ratio of *GP* to *PN* or *GP* to *PS*. From hence it follows that *GA* is to *AS* in the ſame ratio of *GP* to *PS*; which is a particular Caſe of this Propoſition, namely when the point *L* taken at pleaſure coincides with

Plate 7. Book 3.



with A. And this is evident from hence, that (by *Prop.* 34. *B.* 1. *of Conics.*) RG being a Tangent, AS is to SP as AG to GP; from whence, by permutation and inversion, GA is to AS as GP to PS. This Proposition holds also in the other Sections of the Cone.

PROPOSITION XXIX. LEMMA.

TO describe an Ellipse about a given Focus, that shall pass thro' three given Points.

Let the Points given be L, l, λ, (in *Fig.* 27.) and the Focus S. Join the right line Ll, and produce it to F, so as that LF may be to lF, as SL to Sl. So let λl joined, be produced to f, so as that λf may be to lf, as $S\lambda$ to Sl. Join the right line Ff, and upon it, from L and S let fall the perpendiculars LE, SG, divide SG in P so as GP may be to PS as LE to LS, and produce it to A, so as that GA may be to AS in the same ratio: An Ellipse described having A and P for its Vertices and S for its Focus; that is, having its greater Axe PA, and its *Latus rectum* quadruple a right line which is to SP as SA to AP (by *Prop.* 54. *B.* 1. *of Conics.*) will pass thro' the Points L, l, and λ.

From l and λ, upon FG let fall the perpendiculars le, $\lambda\varepsilon$. Since by construction LF is to lF as SL to Sl; LE will also be to le (by reason of the Parallels LE, le) as SL to Sl, or LE to SL as le to Sl. But by construction also GP is to PS, and GA to AS, as LE to SL; and therefore as GP is to PS so is le to Sl. After the same manner λf is by Construction to lf (that is, λe to le) as $S\lambda$ to Sl; and therefore $\lambda\varepsilon$ to $S\lambda$ as (le to Sl; that is, as was just now demonstrated, as) LE to SL. Since therefore SL is to LE, also Sl to le, and $S\lambda$ to $\lambda\varepsilon$, as GP to PS, or GA to AS, and this is a property of an Ellipse demonstrated in the preced Lem. 'tis evident that the Points L, l, λ,

 are

are in the Ellipſe, whoſe Focus is S, and extremes of the greater Axe or Vertices, A and P. And from hence it is eaſy to be known, when the Problem is impoſſible, and when inſtead of an Ellipſe, a Parabola or Hyperbola comes forth. If a Calculation familiar to Aſtronomers be requir'd, it may eaſily be drawn from the preceding Conſtruction; for inſtance, after the following manner. Since $S\lambda$ is to Sl, as λf is to lf; then dividedly $S\lambda - Sl$ to Sl as λl to lf: But the three firſt terms of the Proportion are given, from the Points S, l, λ, being given; therefore the fourth lf will be known. And after the like manner, becauſe $SL : Sl :: LF : lF$. By dividing $SL - Sl : Sl :: Ll : lF$, and the three firſt terms being given the fourth term will be known; namely the right line lF. Moreover, the three Sides of the Triangle $Sl\lambda$ being given, the angle $Sl\lambda$ is given, and the three Sides of the Triangle SLl being known, SlL becomes known; therefore there is given both the Angle $Ll\lambda$ the difference or ſum of them, and the Vertical to it Flf. Therefore in the Triangle Flf, the Sides lF, lf, juſt found, being given, together with the angle Flf contained; the angle Ffl will be found, and conſequently its complement to a right fle. But the angle $Sl\lambda$ being known before, its complement to two right Slf will be known; therefore the ſum or difference els of the known Angles Slf, elf, will be given, and conſequently lSG its complement (becauſe of the Parallels el, GS) to two right; that is, the inclination of the greater axe to the right line lS, given by poſition. Beſides, in the Triangle fel rightangled at e, all the angles are given, together with the ſide fl; therefore el, or GM is known, ſuppoſing lM perpendicular to GS. In like manner in the right angled Triangle SMl, the Angles and Side Sl

being

being given, SM is given; and GS is the ſum or difference of GM, MS. Again, becauſe by the preced. Lem. $el.\,lS :: GP.\,PS$, by compounding $el + lS.\,lS :: GS.\,SP$, and the three firſt terms of this Proportion being given, the fourth SP will be known: In like manner, becauſe $el : lS :: GA : AS$, by dividing $el - lS : lS :: GS : AS$; from whence AS becomes known: The ſum of PS, SA is the greater Axe. The other Focus of the Ellipſe is as far diſtant from A, as S is from P. The Foci and Vertices A and P being given, the reſt will not be unknown.

PROPOSITION XXX.

THree Centric Places of a Planet, and the Diſtances from the Sun being given, to deſcribe its Elliptic Orbit.

Let the Centric Places of the Planet be given, and its correſponding Diſtances from the Sun; namely the right lines SL, Sl, $S\lambda$, in poſition and magnitude [*Fig.* 28.] ſuppoſing S to be the Sun. By the prec. Prop. let an Ellipſe, whoſe Focus is S, be deſcribed or determined by Calculation, paſſing thro' the Points L, l, λ. This will be the Orbit ſought. The relation this has to the Orbit of the Earth is therefore known, becauſe the magnitude of the Lines SL, Sl, $S\lambda$ is given, in the parts of the mean Diſtance from the Sun, and the ſituation of the Plane of the Orbit in reſpect to the Plane of the Ecliptic, is ſuppoſed to be determined by *Prop.* 19. and 20.

Upon this Propoſition depends the Method that Dr. *Halley* lays down, in the *Philoſop. Tranſact. An.* 1676. N° 128. for determining the Orbits of the Planets. Namely, if any Planet be obſerved thrice from the Earth in the ſame Point of its Orbit (which, when it may happen, is well enough known from its periodic Time nearly known)

 there

there will be had three Places of the Earth, ſeen from the Sun, and three correſponding Diſtances of the Earth from the Sun, expreſſed in the parts of the diſtance of the Planet from the Sun; and therefore the Orbit of the Earth itſelf is determined by the preceding: Which being known, if any Planet be obſerv'd twice from the Earth in the ſame Point of its Orbit, and conſequently (by *Prop.* 19.) its Centric Place and Diſtance from the Sun be determined, and this be repeated in two other Places of the ſame Planet; three Points of the Planet's Orbit will be had, by which together with one of the Foci (namely the Sun's Place) being given, the Orbit it ſelf (by the prec.) is determined. If the Periodic Time be pretty exactly known, the ſame may be done by two Centric Places and Diſtances from the Sun being known, which may be gather'd by *Prop.* 27. from twice two Obſervations of a Planet obſerved in the ſame two points whatever they be: And by the ſame pains the Place of the Nodes and the Inclination of the Orbit to the Ecliptic are determined by the 19th and 20th Propoſitions.

PROPOSITION XXXI. LEMMA.

TO find the conjugate Diameters of an Ellipſe, which paſſes thro' five given Points. [Fig. 29.]

Let the five given Points be *N, M, L, K, H*, that the Perimeter of the Ellipſe paſſes thro'; a Diameter as *FE* of that Ellipſe is to be found, together with its Conjugate *TY*, in magnitude and poſition. Let right lines be ſuppoſed to be drawn connecting the given Points, ſome two of which are either parallel, or there are no two ſuch. This is eaſily ſeen; for ſince the Points *N, M, L, K, H* are given, the right lines that connect them all manner of ways are given, and the angles contained between thoſe right line

If

If therefore it be found that two angles placed alternately, (for inſtance KMH, MHN) are equal, it may be concluded that the two right lines MK, NH, in which the ſaid angles are alternately placed, are parallel.

CASE I. Suppoſe firſt, that there are two of them, as MK, NH parallel to one another. Let them be biſected in B and A, and let BA connecting them, be produced, till it meet the Ellipſe in F and E; and it will be (*by the Elements of Conics*) a Diameter of the Ellipſe, and (by 26. *Dat.*) given by poſition; becauſe paſſing thro' the given points B and A. Draw thro' L, the Line LX, parallel to EF, meeting the Ellipſe in X, the right line MK in D, and NH in C. And let the right lines XK, LM connected, meet the right line HN (if need be produced) in the points P and G. By *Prop.* 17. *B.* 3. *Elem. Conic of Apollonius*, the rectangle XDL, is to the rectangle MDK as ZFq to ZYq. And $XCL : NCH :: ZFq : ZYq$. An therefore $XDL : MDK :: XCL : NCH$. But (as ſhall be ſhown hereafter,) $XDL . MDK :: XCL . GCP$. And therefore (by *Prop.* 9. *Elem.* 5.) the rectangle NCH is equal to the rectangle GCP. But the rectangle NCH is given, becauſe its ſides NC, CH are given; for the whole line NH is given at firſt as well in magnitude as poſition, alſo the right line LC by poſition (by 28. *Dat.*) wherefore (by 25. *Dat.*) the point C will be given: So likewiſe will the right line GC, ſince its extremes G and C are given: Therefore (by 57. *Dat.*) CP is given in magnitude: But it is alſo by poſition; therefore the point P itſelf is given. And the point K is given; therefore the right line KP connecting thoſe points is alſo given by poſition, and conſequently (by 25. *Dat.*) the point X in the Ellipſe, where the lines LC, KD given by poſition meet) is

is alſo given. Join the right lines NX, LH meeting the diameter EF (if need be produced) in R and S: And then (by the above cited *Prop.* 17. *B.* 3. *Elem. of Conic.*) both NCH will be to XCL, and NAH to EAF, as ZYq to ZFq; from whence $NCH. XCL :: NAH. EAF$. And by what is below demonſtrated, $NCH. XCL :: NAH. RAS$; therefore (by *Prop.* 9. *Elem.* 5.) the rectangle RAS is equal to the rectangle EAF. But RAS is given (ſince the three points A, R, and S are given, A as was ſhewn above, R and S by the 25 of the *Data*, namely the concourſe of the right line EF given by poſition with the right lines XN, HL given by poſition;) therefore the rectangle EAF will be given. After the like manner the rectangle EBF may be found; and the points A and B are given, therefore ſo is EF in magnitude; as we ſhall ſhow hereafter. But its poſition was found before; wherefore the diameter EF is given in Poſition and Magnitude. Moreover TY^q is to EF^q as NCH to XCL, as was ſhewn above; that is, (by 1 *Dat.*) in a given ratio, becauſe the rectangles NCH, XCL are given; wherefore the ſubduplicate ratio of the former, namely the ratio of TY to EF (by 24 *Dat.*) is alſo given: But EF is given, as was ſhewn above; therefore by 2 *Dat.* TY will be given in magnitude. But its poſition is given by 28 *Dat.* becauſe paſſing thro' the point Z, which is in the middle of the right line EF given in poſition and magnitude, parallel to the right line NH given in poſition. Therefore the right lines EF, TY are given in Poſition and Magnitude, namely the conjugate diameters of the Ellipſe, paſſing thro' the five given points N, M, L, K, H.

CASE II. But now let us ſuppoſe that the right lines MH, NK, [*fig:* 30.] are not parallel,

lel, nor any other two: Therefore these lines *MH*, *NK* (at least produced) will intersect one another; for instance in the point *L*, which will be given by 25 *Dat.* draw thro' *L* the right line *LV* parallel to *MH*, meeting the right line *NK* in *Q*, which (by 28 and 25 *Dat.*) will also be given. And the ratio of the rectangle *NQK* to the rectangle *LQV* will be given; namely the same (by *Prop.* 17. *B.* 3. of the *Elem. of Con.*) with that of the given rectangle *NIK* to the given *MIH*. But the rectangle *NQK* is given, since its sides *NQ* and *KQ* are given; and therefore the rectangle *LQV* is also given: From whence, since *LQ* is given by 57 *Dat.* the right line *QV* is given in magnitude; but also in position, because a parallel thro' the given point *Q* to the right line *MH* given by position; therefore the point *V* itself by 27 *Dat.* since the magnitude and position of the right line *QV* and the other extremity *Q* are given. Thus therefore we come back to the former Case; for there are given the five points *N*, *M*, *L*, *V*, *H*, or *M*, *L*, *K*, *V*, *H*, thro' which the Ellipse may pass, and yet such as the two right lines *LV*, *MH* may be parallel. By what has been said therefore the conjugate diameters of that Ellipse may be found. *Q. E. F.*

Let there be two right lines GP, MK, *parallel to one another, on which let the right line* XCDL [Fig. 31.] *fall, and from any points of it* X *and* L *draw any how the right lines* LMG, XPK *meeting the parallels in* M, G, P, *and* K: *I say, the rectangle* XDL, *is to* MDK *as* XCL *to the rectangle* GCP.

The ratio of the rectangle *XDL* to the rectangle *MDK* (by *Prop.* 23. *Elem.* 6.) is compounded of the ratio of *XD* to *DK*, and of the ratio of *LD* to *DM*; and the ratio of the rectangle *XCL* to the rectangle *GCP* of the ratios of *XC* to *CP* and *LC* to *CG*. But because of the parallels

rallels *MDK*, *GCP*, *XD* is to *DK* :: *XC*. *CP*; and *LD*. *DM* :: *LC*. *CG*. Since therefore the compounding ratios are equal, the compounded ratios will alſo be equal; that is, *XDL* to *MDK*, as *XCL* to *GCP*.

If both the rectangles E A F, E B F *are given, and the points* A *and* B *are given; the points* E *and* F *are alſo given, ſuppoſing* E A B F *a rightline.*

For ſuppoſe the rectangle *BAC* to be equal to the rectangle *EAF*, and the rectangle *ABD* equal to the rectangle *EBF*: And therefore, ſince (by *Hyp.*) *EAF* and *BA* are given, *AC* (by 57 *Dat.*) is alſo given. In like manner, ſince *EBF* and *AB* are given, *AD* will be given. By reaſon of the former equality (by 16. *Elem.* 6.) *FA*. *AB* :: *AC*. *AE*, and by converſion of ratios, *FA*. *BF* :: *AC*. *CE*. Again, by reaſon of the latter equality *EB*. *BD* :: *AB*. *BF*, [*Fig.* 32.] and by compounding *ED* : *BD* :: *FA*. *BF*. From whence (by *Prop.* 11. *El.* 5.) *AC*. *CE* :: *ED*. *BD*: And therefore the rectangle contained under *AC* and *BD* is equal to the rectangle under *CE*, *ED*. But the rectangle under *AC* and *BD* is given, (for both right lines are given) therefore the rectangle under *CE*, *ED* is alſo given. And becauſe this given rectangle is applied to the given rightline *CD*, (compoſed of the parts *CA* and *AD* which are given) and wants of a ſquare Figure, *CE* (by 58 *Dat.*) will be given; and conſequently *AE* (by 4 *Dat.*) and becauſe the rectangle *EAF* was given at firſt, the right line *AF* will alſo be given; that is, the points *E* and *F* will be given.

PROPOSITION XXXII. LEMMA.

TO find the Foci and Vertices of an Ellipſe that is to paſs thro' five given Points, and conſequently to deſcribe the Ellipſe paſſing thro' the five given Points.

Pappus in *Book* 8. *Math. Coll.* propoſes and conſtructs

ſtructs this Prob. and the prec. for the uſe of Architects: Namely, having a portion of the ſuperficies of a right Cylinder given, that has no part preſerved intire in the circumferences of the Baſes, to find the thickneſs of the Cylinder. Theſe the Aſtronomers afterwards converted to their uſe. By the preced. *Lemma* find any two conjugate Diameters of the Ellipſe, that is to paſs thro' the five given Points, and let them be *A B*, *C D* biſecting one another in the Center *E*. Produce *B A* to *H* [*Fig.* 33.] ſo as that the rectangle *E A H* may be equal to the ſquare of *D E*; by which means (by 58 *Dat.*) *E H* is given. Let a perpendicular *K L*, from *K* the middle of *EH*, meet with *M A G*; a right line parallel to *D C*, thro' the point *A* in the point *L*, which conſequently is given on the account of the right lines *M G*, *K L* given by poſition. On the Center *L* thro' *E* or *H*, draw a Circle interſecting the right line *M A G* in *M* and *G*, whoſe diameter *MG* conſequently is given as well in poſition (becauſe 'tis parallel thro' the given point *A* to the right line *C D* given by poſition,) as in magnitude, (ſince it is double to the right line connecting the given points *E* and *L*) the right lines *G E*, *M E* connecting them, are alſo given as well in poſition as magnitude. But their poſition is the ſame with the poſition of the Axes ſought: For ſince the rectangle *E A H* (by 35. *Elem.* 3.) is equal to the rectangle *M A G*, and by conſtruction *E A H* equal to the ſquare of *D E*; *E D*q will be equal to *M A G*, or (by 17 *Prop. Elem.* 6.) *E D* will be a mean proportional between *MA* and *AG*; and conſequently (by *Prop.* 88. *B.* 3. *of Greg. St. Vincent.*) the Axes of the Ellipſe deſcribed upon the conjugate Diameters *A B*, *C D* are in the right lines *E G*, *E M*. And the magnitude of the Axes is thus expeditiouſly determined. Thro' *A*, let fall perpendiculars *A Q*, *A N* to *E G*. *E M*, which therefore are given.

ven. Take in the right line *E G*, *E O* a mean proportional between *E G*, *E Q*; in like manner, in *E M*, *E R* a mean proportional between *E M*, *E N*; the right lines *E O*, *E R* (given by 24 *Dat.*) will be the Semi-axes of the Ellipſe (by *Prop.* 37. *B.* 1. *Conic.*) If in *G E* produced you take *E P* equal to *E O*, and in *M E* produced, *E T* equal to *E R*, the Axes *O P*, *R T* of the Ellipſe will be given in poſition and magnitude, together with with the points Q, *R*, *P*, *T*, which are the principal Vertices of the ſame. Moreover, if the greater Axe *P O* be ſo divided in *S* and *F*, that the rectangle under *P S* and *S O*, as alſo the rectangle under *P F* and *F O* be equal to the Square made upon *R E*, *S* and *F* will be the Foci of the Ellipſe (by *Prop.* 52. *B.* 3. *Elem. Conic.*) But theſe Points thus found (by 58 *Dat.*) are given: and if the Axes and Foci of the Ellipſe are given, the Ellipſe may be deſcribed by *Prop.* 54. *B.* 1. *Conics of Apollonius.*

There is no need of annexing here the form of a Trigonometrical Calculation ſuited to this Problem: For it is well known to Geometers, that whatever is found out by the help of *Euclid*'s *Data*, may alſo eaſily be found by a Calculation, proceeding exactly after the ſame manner, and generally in a Trigonometrical manner, as it happens here: Indeed the inveſtigation of any Problem by the help of *Euclid*'s *Data*, is an univerſal method of determining it, and very uſual with the Ancients.

PROPOSITION XXXIII.

Five Centric places of a Planet and the Diſtances from the Sun being given, to find its Orbit.

In the preceding Propoſitions, while we were determining the Orbits of the Planets, we ſuppoſed nothing, but what was agreeable to Obſervations exactly made as well as to the Phyſical Cauſes of their

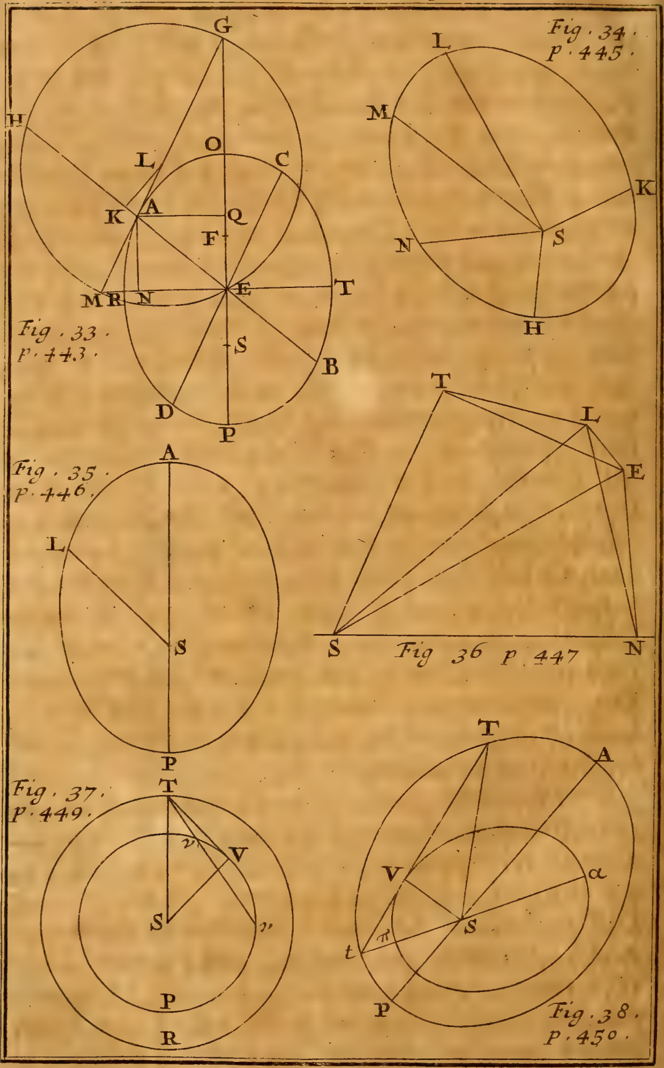


Fig. 33. p. 443.

Fig. 34. p. 445.

Fig. 35. p. 446.

Fig 36 p. 447

Fig. 37. p. 449.

Fig. 38. p. 450.

their Motions demonſtrated in *Book* 1: Namely, that the Orbit of any Planet is an Ellipſe, one of whoſe Foci is the Sun at reſt, and that a Planet deſcribes it after ſuch a manner, as that its Area's or Sectors contained between right lines drawn to the Sun, and the Arcs of the Ellipſe are proportinal to the Times wherein the ſaid Arcs are deſcribed by the Planet. For tho' we have ſuppoſed in *Prop.* 14. with *Ward*, for a little while, that a Planet deſcribes its Orbit ſo, as that its angular motion at the Focus of the Ellipſe in which the Sun is not; yet this was only done, becauſe in the Orbit of the Earth, which is not very excentric, an approximation is made to the equal deſcription of Areas by a radius drawn to the Sun, as is not to be diſregarded, as was ſhown in *Prop.* 7. And we corrected the Orbit of the Earth defined according to that Hypotheſis, as much as we pleaſed in the following 15th Propoſition. But in this Propoſition we ſhall not ſo much as ſuppoſe that the Sun is placed in the Focus of each Orbit; but ſhall deſcribe the Orbit by the help of five given Centric Places, and as many Diſtances from the Sun, independently upon it: And by this means the truth of the preceding Prop. will appear clearer, ſince the Sun will be found to be placed in the common Focus of the Orbits thus deſcribed.

The Centric places of the Planet being given, the right lines, wherein it is ſeen from the Sun, are alſo given by poſition: Let them be SN, SM, SL, SK and SH; [*Fig.* 34.] and the correſponding Diſtances from the Sun being given, the points N, M, L, K and H (by 27 *Dat.*) are given. Thro' theſe points (by the help of the preceding Lemma) deſcribe an Ellipſe, and it will be the Orbit of the Planet ſought. And by the ſame Lemma the Vertices will be found, as alſo the Foci, Poſition

of

of the greater axe and the other things that are required in the Orbit of the Planet. *Q. E. F.*

PROPOSITION XXXIV.

TO *determine the Centric Place and Distance from the Sun, of any Planet proposed, at any given Time.*

The plane of the Planets Orbit, the position of the Line of the Apsides and the Species of the Planet's Orbit being determined, (by the preced. *Prop.*) that is, the Orbit it self being described in its proper Situation about the Sun, suppose *ALP* [*Fig.* 35.] and the Moment being known (by *Prop.* 25.) when the Planet was some time or other in the Apsis also the Periodic Time of the Planet (by *Prop.* 23.) the Time since it was last in the Apsis, (*v. g. A*) will be known, or the ratio of the said Time to the whole Periodic Time will be known; that is, by the Law of Motion of the Planets, demonstrated in *B.* 1. supposing the Place of the Planet *L*, the ratio of the area *ALS* to the whole Ellipse *ALPA* is known. Find therefore the Angle *ASL* by *Prop.* 3, 4, 6, or 7. Therefore the right line *SL* is given by Position, since it is inclined in a given angle to *PA* the Line of the Apsides given in position: As also in magnitude, by the same Propositions, (since this is given in the parts of the Aphelion distance of the Sun *SA*, which is defined above in the parts of the Mean distances of the Earth from the Sun, which is looked upon as the common measure of these lines;) that is, the Place of the Planet proposed seen from the Sun, and its Distance from the same.

And *vice versa* the Time may be found when a Planet shall be in a given Heliocentric Place; by finding the mean Anomaly (by the preced. *Prop.*) from the coequated Anomaly given, to the former of which the Time is proportional.

PRO-

PROPOSITION XXXV.

AT a given Time to find the Geocentric Place of a Planet, as to Longitude and Latitude, and the Distance of the Planet from the Earth.

Find (by *Prop.* 18.) the Heliocentric Place of the Earth in its proper Orbit, at the Time given, and (by the preceding) the Heliocentric Place also of the Planet proposed in its Orbit, with the respective Distances from the Sun.

Let *S* (in *Fig.* 36.) represent the Sun, *T* the place of the Earth in its Orbit, *L* the place of the Planet in its Orbit. Let the right line *SN* be the common intersection of the Planes of these Orbits, which is extended thro' the Sun, (because the Sun is in both Planes) and whose position is given (by *Prop.* 19.) Besides, the place of the Earth *T* being given, the right line *ST* and the angle *TSN* in the Plane of the Ecliptic, are given; and the Place *L* of the Planet being given, the right line *SL* and the angle *LSN*, in the plane of the Orbit of the Planet are given. Draw thro' *L* and *S*, and through *L* and *T*, the Planes of the Circles of Latitude perpendicular to the plane of the Ecliptic, whose common intersection the right line *LE* (by *Prop.* 19. *Elem.* 11.) is perpendicular to the plane of the Ecliptic. Draw in the plane of the Planet's Orbit the right line *LN*, perpendicular to *SN*, and the line *EN* (by the converse of *Prop.* 11. *Elem.* 11.) will also be perpendicular to the same *SN*.

In the Triangle *SLN* rightangled at *N*, *SL* and *NSL* are given; from whence the Side *LN* will be given. In the Triangle *LEN* rightangled at *E*, there are given the Side *LN* lately found, and the angle *LNE*, which (by *Def.* 6. *Elem.* 11) is equal to the Inclination of the Orbit of the Planet to the Ecliptic, found by

Prop.

Prop. 20. *LE* and *EN* are therefore given. Besides in the Triangle *LES*, rightangled at *E*, the Sides *LE* and *LS* are given; therefore the Side *SE*, the *Curtate Distance of the Planet from the Sun* is given. In the Triangle *ESN*, besides the rightangle at *N*, there are given *ES*, and *EN* found before; therefore the angle *ESN* will be given, the Heliocentric Longitude of the Planet computed in the Ecliptic from the Node *N*: But the Place of the Node in the Ecliptic is given; wherefore the Heliocentric Place of the Planet in the Ecliptic will also be known. Again, in the Triangle *TSE*, *TS* is given at first, and *SE* was found just now; the angle *TSE* is also given (equal to the sum or difference of the given angles *TSN*, *ESN*, because the three right lines *ST*, *SE*, *SN*, are in the Plane of the Ecliptic, which is also called *the Angle of Commutation*: And therefore all the rest is given, namely, the angle *TES*, call'd, *The Parallax of the Orbit*, and *STE the Elongation of the Planet from the Sun*; that is, the position of the right line *TE*, or the Geocentric Place of the Planet in the Ecliptic, and *TE* the Magnitude of the same. Lastly, in the Triangle *TEL*, rightangled at *E*, the Sides *TE* and *EL* are given; therefore *TL* the Distance of the Planet from the Earth, and the angle *LTE* the Geocentric Latitude of the Planet will be known.

SECTIO-

SECTION IV.

Of the greateſt Elongation of the Planets from the Sun, their Direction, Station, and Retrogradation.

PROPOSITION XXXVI.

TO determine the greateſt Elongation from the Sun of a given Planet, ſeen from a ſuperior one alſo given, and its Time.

Let *S* be the Sun, *T R* the Orbit of a more remote Planet, [*Fig.* 37.] *VP* of a nearer. Let the radius *SV* of the nearer Planet be drawn any how, and perpendicular to it *VT*, meeting the more diſtant Orbit in *T*, which conſequently (by 16. *El.* 3.) will touch the Circular Orbit *VP* in *V*. The inferior Planet will be ſeen by the ſuperior in *T*, to be at its greateſt Elongation from the Sun in *V*: For a right line drawn from any other point (as *v*) of the Orbit *PV* to *T*, will contain a leſſer Angle with *TS*. Therefore in the Triangle *TVS*, rightangled at *V*, the ratio of the Sides *ST*, *SV* (namely the Diſtances of the given Planets from the Sun) is given; therefore the other angles *TSV*, *STV* are known, the latter of which is the greateſt Elongation of the Planet from the Sun, the other *TSV* is the angle of Commutation or Heliocentric Diſtance of the Planets. Whenever therefore the angle of Commutation, found juſt now, happens, it at the ſame time happens that the inferior Planet, ſeen from the ſuperior, is at its greateſt Elongation from the Sun.

We have hitherto ſuppoſed the Orbit of both Planets to be Circular; but if due regard be had

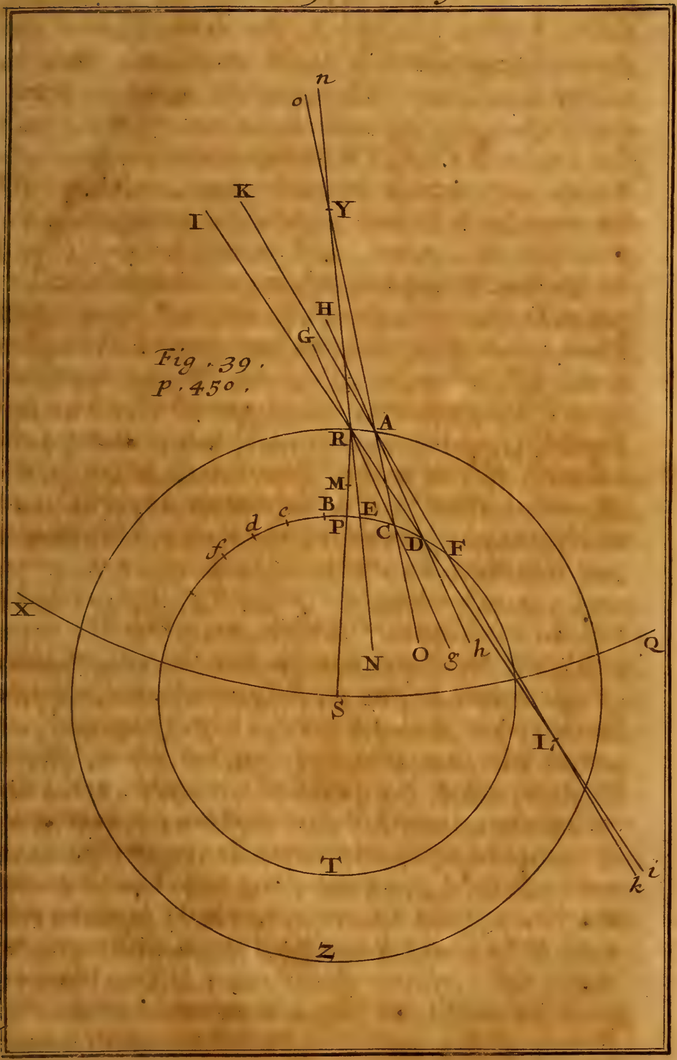
to their Elliptic Figure, the Problem is ſolvable much after the ſame manner. For the angle *SVT* is given, that the right line *VT* [*Fig.* 38.] touching the Orbit, makes with the radius *SV*, inclined in an aſſumed Angle to the Line of the Apſides; beſides *SV* is given in magnitude, ſince its Inclination to the Diſtance of the Aphelion *S*α already given in magnitude is known; and becauſe the angles *A S* α, α *S V* are known, alſo the angle *V S T*, (very nearly equal to that, which would be were both Orbits circular;) the angle *A S T* will alſo be known, and conſequently *S T* will be known in magnitude; from whence what remains is done as before. In this caſe the greateſt Elongation changes according to the different Diſtance of the point *V* (where it is celebrated) from the Aphelion of its proper Orbit: For it is greater (*cæteris paribus*) in α the Aphelion of the inferior, leſs in π the Perihelion, a mean in the mean Longitude; it is alſo various, the place *V* of the inferior remaining the ſame, according as the ſuperior is ſituated in *T* or *t*.

PROPOSITION XXXVII.

ANY Primary Planet ſeen from any other Primary, appears in ſome part of its Orbit, Stationary, in another Direct, and laſtly in another Retrograde.

Let *S* be the Sun, [*Fig.* 39.] about which two Planets revolve from *D* thro' *C* towards *B*, the Orbit of the nearer *PT*; of the more diſtant *R Z*; the arcs deſcribed by them in equal ſpaces of time *A R*, *P B*. Draw *S R* any how meeting the nearer Orbit in *P*. Between *S R* and *SP* let *S M* be a Geometric Mean. Then (by *Prop.* 27. *B.* 1.) the Velocity of the more diſtant Planet is to the Velocity of the nearer, as *S P* to *S M*: But the Spaces deſcribed by Bodies in Motion, in equal

Plate 9. Book 3.



equal Times are as their Velocities; and therefore *AR* is to *PB*, as *SP* to *SM*; wherefore *AR* is leſs than *PB*. And it is plain that it is poſſible for two parallel right lines *HADb*, *GRCg* to be drawn thro' *A* and *R*, ſo as that the arc of the nearer Orbit intercepted by them, is equal to the arc *PB*. For parallel right lines may be ſo drawn, that as much as *CD* (equal to *PB*) is greater than *AR*, ſo much it may be more oblique, (and the arc of the interior Orbit is given, which is as oblique to the parallels thro' *A* and *R* as you pleaſe,) that the arc *CD* compenſating the greater length by its greater obliquity, may touch both of the parallel right lines in their extremities. Which being ſuppoſed, if the Planets be found in ſuch a ſituation in regard of one another as *A* and *D* are here; that is, if the angle of Commutation at the Sun be equal to the angle *ASD*, (which, it is evident, it will be twice neceſſarily between the two next Conjunctions of the Planets propoſed ſeen from the Sun,) the one ſeen from the other all that ſpace of time, wou'd be refer'd to the ſame Fix'd Stars; that is, it would appear Stationary. For the ſuperior at *A* is ſeen from the inferior at *D*, in the right line *DAH*, and while the ſuperior is going to *R*, the inferior is going to *C*, (ſince by ſuppoſition *DC* is to *AR* as the Velocity of the Planet, which is nearer to the Sun, is to the Velocity of the more diſtant:) That therefore ſeen from this, in the right line *CR* will be referred to the ſame place among the Fix'd Stars, to which it was before referred, ſince the diſtance between the Parallels *DA*, *CR* vaniſhes in compariſon with the diſtance of the Fix'd Stars. And, for the ſame reaſon, the inferior Planet deſcribing the arc *DC* viewed from the ſuperior, while it deſcribes the arc *AR* in the ſame

time, will alſo appear Stationary. In like manner, if the Planet next the Sun be on the other ſide of the point *P*, (in which being viewed from the Sun, it ſeems in conjunction with the more diſtant Planet; that is, in which the Sun and ſuperior Planet viewed from the inferior, are ſeen in oppoſition, and where the inferior ſeen from the ſuperior appears in conjunction with the Sun,) and as far diſtant from it, namely where the viſual rays of any ſenſible time are again parallel to one another, for inſtance, at *cd*, the one ſeen from the other will again appear Stationary.

I ſay further, that before the Planet, which is nearer to the Sun, arrives at the beforemention'd ſituation in reſpect of the ſuperior, that is, the ſuperior being in *R*, before the inferior arrive at *D*; the one ſeen from the other will appear to move *in conſequentia*. For let the arc *DF* be taken equal to *PB* or *DC*, and join the right lines *FA*, *DR*, and produce them both ways in *KAFL* and *IRDL*; they will not be parallel, but diverge towards the parts beyond *RA* (in reſpect of the Sun, but on this ſide they will converge and concur, for inſtance, in the point *L*, from whence they will again diverge. For ſince (by ſuppoſition) *DA*, *CR* are parallel, and *DF* equal to *CD*, but more oblique to *RD* than *CD* is to *RC*; 'tis evident that *DR*, *FA* diverge on the other ſide of *RA*, and converge on this ſide of *RA*; *viz.* either behind *DF*, if it be in that part of the nearer Orbit that is turned towards *R* (as in the caſe of this Figure) or between *RA* and *DF*, if in that part which is turned from it. If now the inferior being in *F*, the ſuperior be in *A*, the latter viewed from the former will be ſeen in *K*: But the inferior being arrived at *D*, the ſuperior is got to *R* (by ſuppoſition) and is ſeen from it at *I*.

But

But to come from the ſituation of *K* to the ſituation of *I* is to move *in conſequentia*, or to be Direct; for any of the Planets ſeen from the Sun, moves *in conſequentia* from *A* to *R*, or from *D* to *C*. Nay, even the inferior ſeen from the ſuperior at the ſame time ſeems Direct: For the inferior in *F* will be ſeen from the ſuperior *A* at *k*; and afterwards the inferior at *D* will be ſeen from the ſuperior in *R* at *i*: But to change place from *k* to *i*, is alſo to tend *in conſequentia*; for the Sun ſeen from any Planet ſeems to move *in conſequentia*; namely from *S* towards *Q*. In like manner, after the Planet, which is nearer to the Sun, has paſſed the ſecond Station at *d*, it becomes direct again, and continues ſo till the inferior arrive again to a ſituation in reſpect of the ſuperior, (as is *D* in regard of *A*;) which it will do after it has compleated an intire revolution from the former place *D*, and ſo much of another beſides, as is requiſite for it to recover the ſaid ſituation again, in regard of the more diſtant Planet, that had in the mean time gone forwards.

I ſay thirdly, that the one will appear Retrograde to the other, after that the Planet that is nearer to the Sun has got beyond the above deſcribed boundary *D C* of the firſt Station, and has not arrived at the ſecond *c d*. Let *C E* be taken equal to *PB*, or *D C*; draw the right lines *C A*, *E R*, and let them be produced both ways into *O C A Y o*, and *N E R Y n*, which are not parallel, but diverge towards the parts *C E*, and converge towards *A R*, and being produced meet, for inſtance in *Y*, becauſe by ſuppoſition *D A*, *C R* are parallel, and *C E* is equal to *D C*, but leſs oblique towards *R E* than *C D* is towards *R C*. If now while the ſuperior is in *A*, the inferior be in *C*, this will be ſeen from that in *O*, and that *vice verſa* will be

ſeen from this in *o*: But when the ſuperior is got to *R*, and the inferior is found at *E*, this will be ſeen from that at *N*, but that will be ſeen from this at *n*. But 'tis evident, that to ſeem to move from O to *N*, or from *o* to *n*, is to move contrary to the order of the Signs; ſince from *S* to *Q*, or from *A* to *R* is according to the order of the Signs; namely that wherein the Primary ſeen from the Sun, or *vice verſa*, the Sun ſeen from the Primary ſeems to tend. And this apparent Retrogradation of the Superior ſeen from the Inferior, or of the Inferior from the Superior, continues from the celebration of the firſt ſtation at *D C*, to the ſecond at *c d*: But by reaſon of the motion of the more diſtant Planet, this ſecond ſtation is removed a little *in conſequentia*, or beyond the place *c d* here expreſſed. 'Tis therefore evident, that any Primary ſeen from another Primary in ſome part of its Orbit is Direct, and in ſome Retrograde; and laſtly, in ſome Stationary, *Q.E.D.*

COROLLARY. 1.

'Tis evident from the demonſtration of this Propoſition, that a ſuperior Planet ſeen from an inferior appears at the ſame time Direct, Stationary, and Retrograde, that the inferior ſeen from that ſuperior does.

COROLLARY 2.

The more unequal the ratio between the Orbits of two Planets is, the nearer its place in the Orbit of the inferior, in which the inferior is found while the one ſeen from the other appears Stationary, is to that point of it, from which a right line drawn to the more diſtant Planet, touches the Orbit of the inferior; that is, where the inferior ſeen from the ſuperior (by *Prop.* 26.) appears in its greateſt Elongation from the Sun; and the angle greater wherein the Sun and ſuperior

rior Planet, ſeen from the inferior, ſeem to be diſtant from one another. For the greater the ratio of *S R* is to *S P*, the greater will the ſubduplicate of the latter, namely the ratio of *R S* be to *M S*; that is, (by conſtruction) the ratio of *C D* to *R a*. And therefore the line *C D* muſt be more oblique to *R C* or *A D*, that *D A*, *R C* may become parallel; but it is evident that *C D* is ſo much the more oblique as it is nearer to the point of the inferior Orbit, from whence a right line drawn to *R* touches it.

From hence it is, that to make Saturn Stationary there is need of ſo ſmall a receſſion of the Earth from the point of its Orbit, where a right line drawn to Saturn touches it, towards the point where the Earth comes between the Sun and Saturn; to make Jupiter Stationary, a greater receſſion from the point of contact of the Orbit of the Earth, with a right line touching it drawn to Jupiter; to make Mars the greateſt. Therefore Saturn is Stationary, when ſeen from the Earth it is diſtant a little more than a Quadrant from the Sun; that is, between the Quartile and Trine Aſpect; Jupiter will appear Stationary almoſt in the Trine of the Sun; Mars beyond the Trine.

Hence it is alſo that the arc of the Orbit of Mercury intercepted between the point where Mercury is ſeen from the Earth at its greateſt Elongation from the Sun, and the point where its next Station is ſeen, ſubtends a leſs angle at the Sun than the arc of the Orbit of Venus contains between its greateſt Elongation from the Sun and next Station: And therefore ſince it ſubtends both a leſs angle at the Sun, and belongs to a leſs Circle, and beſides is preſented more Obliquely to the Earth, obſerving it, and is more diſtant from it; 'tis evident that the Stations of Mercury

are much nearer to its greateſt Elongations from the Sun than thoſe of Venus.

COROLLARY 3.

And from hence again it follows, that in reſpect of the ſame inferior, in two ſuperior Planets compar'd together; the farther off the Sun the ſuperior Planet is, the longer its Retrogradation continues; for (by the preced. *Corol.*) it begins ſooner and ends later. But the angle of Retrogradation is leſs, becauſe the Orbit of the given inferior Planet ſeen from the more diſtant ſuperior Planet appears under a leſs angle, than if it were ſeen from a nearer: From this *Prop.* 37. 'tis evident, that the angle of Retrogradation (contain'd between two right lines from the ſuperior Planet oppoſite to the Sun, to the places of the inferior Orbit, where the one appears Stationary to the other) is almoſt equal to the angle wherein the Orbit of the inferior is ſeen from the ſuperior, ſince the above mention'd points of the Stations are not far from the points of contact.

Hence it is, that the Retrogradation of Saturn ſeen from the Earth is longer, but made along a leſs arc, than that of Jupiter; and that of Jupiter in like manner to that of Mars.

COROLLARY 4.

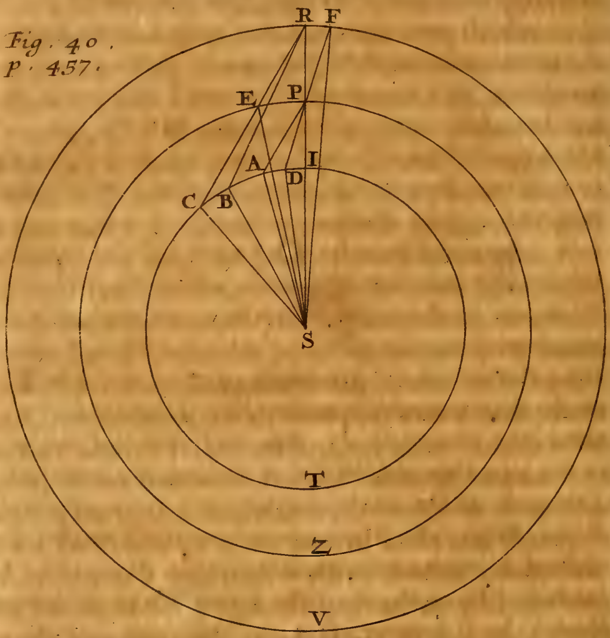
But in reſpect of the ſame ſuperior, in two inferior Planets compared together; the farther off the ſuperior Planet is from the Sun, the longer is its Retrogradation and along a greater arc. The former is evident from the preced. *Corol.* ſince the Retrogradation of two Planets is mutual by *Corol.* 1. The latter is evident from hence, that a Planet that is more diſtant from the Sun, while it appears Retrograde, deſcribes a longer arc ſlower, and therefore ſpends more time.

Hence it is, that Venus ſeen from the Earth, is both longer Retrograde, and along a greater arc than Mercury.

COROL.

Plate 10. Book 3.

Fig. 40. p. 457.



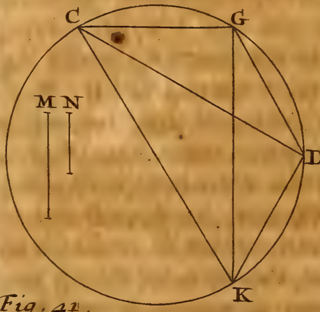


Fig. 41. p. 458

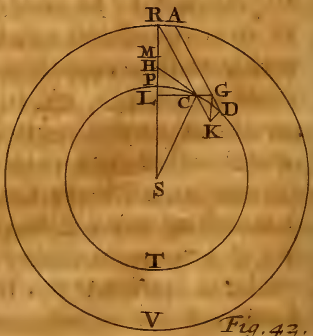


Fig. 42. p. 460.

COROLLARY. 5.

Hence it alſo follows, that when two ſuperior Planets are ſeen from an inferior in Conjunction, if that which is farther off from the Sun be Direct, that which is nearer the Sun will alſo be Direct; if that which is nearer the Sun be Retrograde, that which is farther off the Sun will alſo be Retrograde. Let *S* repreſent the Sun in [*Fig.* 40.] *IT* the Orbit of an inferior Planet; *P Z*, *RV* the Orbits of two ſuperior Planets. Draw *S T* any how interſecting the Orbits of the ſuperior in *S* and *R*. Let *B* be the Place in the Orbit of the inferior, where the inferior Planet is when the more diſtant one in *R* is ſeen from thence to be Stationary; and let *A* be the Place in the ſame, where the inferior is found when the next in *P* ſeen from the inferior appears to be Stationary. Join the right line *AP*, *AS*, *B R*, *B S*. From *Corol.* 2. it is evident that the Place *A* is nearer to the point *I* than *B*, and that the angle *PAS* is greater than *RBS*. When the Planet in *R* appears Direct from the inferior, the Place of the inferior ought to be farther diſtant from *I* than the Place *B*, for inſtance in *C*. Suppoſe now that the nearer of the ſuperior Planets ſeen from the inferior is in conjunction with the more diſtant one, and it will be therefore in the point *E*, where the right line *C R* interſects the Orbit *P Z*. By *Prop.* 21. *Elem.* 1. the angle *ECS* is leſs than the angle *RBS*; therefore it is much leſs than *P A S*. Therefore a Planet ſeen in *E* from *C* is Direct; becauſe to make it Retrograde, 'tis requiſite that *ECS* be greater than *P A S*, which is requiſite to make it Stationary. But the inferior being between *B* and *A*, the more diſtant at *R* will ap-appear Retrograde, and that which is nearer to it in conjunction, ſtill Direct; becauſe the Planet that takes the view is not as yet arriv'd to the limit *A*.

But

But if the nearer of the ſuperior Planets appears Retrograde at *P* from the inferior, the inferior will be between *A* and *I*, for inſtance in *D*. If now the more diſtant Planet appear in conjunction with it, that will be in the right line *DS* produced to the Orbit *RV*, namely in *F*: But the angle *FDS* is greater (by *Prop.* 21 *El.* 1.) than *PAS*, which again is greater (by what has been ſhown) than *RBS*. And therefore the more diſtant Planet ſeen in *F* from *D*, appears Retrograde, ſince to make it appear Direct, a leſſer angle at the inferior than *RBS* is requiſite.

And hence it is that when Saturn is ſeen from the Earth to be Direct, Jupiter and Mars can't be in conjunction with him unleſs they alſo be Direct; or Mars can't be in conjunction with Jupiter Direct, unleſs when he himſelf is Direct: But either Jupiter or Mars Direct may be in conjunction with Saturn Retrograde, and Mars Direct may be in conjunction with Jupiter Retrograde. But Jupiter and Saturn can't be in conjunction with Mars Retrograde, unleſs they themſelves be Retrograde; or Saturn with Jupiter Retrograde, unleſs Retrograde.

LEMMA.

Given in the quadrilateral figure CGDK [Fig. 41.] *inſcribed in a Circle, (one of whoſe ſides* CK *is a diameter of the Circle,) the ſide* CG *and the diagonal* CD, *together with the ratio between the oppoſite ſide* KD *and the other diagonal* GK, *(the ſame that is between the given right lines* N *and* M;) *to find the angle* GCD *contained between the given ſide and the given diagonal.*

Becauſe the ſum of the Squares of the right lines *CG* and *KG* is equal to the ſum of the Squares of the right lines *CD* and *KD*, (for each of the ſums are equal to the Square of *CK*;) the difference of the Squares

Squares of the right lines *G K* and *K D* will be equal to the difference of the Squares of the right lines *C D* and *C G*, and consequently given. Moreover, because *G K* is to *D K* as *M* is to *N*, the Square of *G K* will be to the difference of the Squares of *G K* and *D K*, as the Square upon the right line *M* is to the difference of the Squares upon *M* and *N*; being made out of those in the same ratio with them that are similarly made out of these, which are proportional to those: Therefore the Square upon *G K* (by *Dat.* 2.) will be given; the Square also of *C K*, made up of the given Square of *G K* and the Square of *C G*, is also given: Consequently the right lines themselves *C K* and *G K* are given, the right line *D K* is also given, because it has a given ratio to *G K*. But it is a known Theorem among Geometers, demonstrated by *Ptolemy, B.* 1. *of the great Construction, Chap.* 9. that in a quadrilateral *C G D K* inscribed in a Circle, the rectangle contained under *C D* and *G K* is equal to the rectangle under *C G* and *D K*, together with the rectangle under *C K* and *G D*. And therefore the rectangle under *C D* and *G K*, lessen'd by the rectangle under *C G* and *K D* is equal to the rectangle under *C K* and *G D*. But the rectangled Parallelograms that are contained under the given right lines are given, and the difference of the given rectangles is given; and consequently the rectangle under *C K* and *D G* is given, and *C K* is given; therefore *G D* is given. And the three sides in the Triangle *GCD* being given, the angle *G C D* will be found. *Q. E. F.*

PROPOSITION XXXVIII.

TO *find the Angle of Commutation, wherein two given Planets seen from the Sun are distant, and the angle wherein the Sun and either of the Planets seen from*

from the other are distant, when one of the given Planets seen from the other appears Stationary.

Let *S* be the Sun [Fig. 42.] *R V*, *P T* the Orbits of the given Planets; and let *C D*, *R A* be the Arcs described in the same time by the Planets, while they are seen from the other Stationary, which are given by what has been said at *Prop.* 37. Join *S R*, *SC*, and *RC*, *A D*, which (by the preced.) will be parallel. Thro' *C* draw the lines *CG* parallel to *RA*, also thro' *G* the right line *GK* parallel to *R S*, and thro' *D* the right line *DK* parallel to *C S*, which may meet in *K*, join the right line *CK*. The Triangle *GKD* will be equiangular to the Triangle *R S C*, (since each of the sides of that, is parallel to each of this,) and consequently similar. Because the Time of the Station of the Planet is very small, if compared with the periodic Time even of the swiftest, the line *C D* does not sensibly differ from a right line, as also *R A* or *C G*, much less. The former therefore does not differ from *H C* a right line produced touching the Orbit *P T* in *C*, and the latter produced will be the right line *C L* perpendicular to *S R*, as *A R* is to the same. Moreover, a Circle made upon the diameter *C K* passes thro' *D* and *G*, because the angles *C G K*, *C D K* are right; being by construction equal to the right ones *SLC*, *SCH*. *C GDK* therefore is a Quadrilateral inscribed in a Circle, one of whose sides *C K* is the diameter of the Circle, and in which are given the side *CG* and the diagonal *C D*; as also the ratio between the opposite side *D K* and the other diagonal *GK*, namely, the same that there is between the Semidiameters *C S*, *R S* of the given Orbits. And therefore (by the Lemma premised) the angle *GCD* will be found, to which the vertical angle *HCL* is equal; and to this again the angle *RSC* is equal, which is the Angle of Commutation, where-

whereby the two given Planets ſeen from the Sun are diſtant, when the one ſeen from the other appears Stationary. Moreover, in the Triangle *R S C* the two ſides *S R*, *S C*, and the angle *R S C* contained by them being given, the angles *S R C*, *S C R* will be found; the former of which is that angle whereby the inferior is diſtant from the Sun, while it appears Stationary from the ſuperior, the latter is the angle whereby the ſuperior is diſtant from the Sun, while it appears Stationary from the inferior.

This Method of defining the Points of the Stations fit for Calculation and more natural, and as it were depending upon the motion of the Spectator about the Sun, I have demonſtrated; paſſing over the Geometrical Conſtruction of this Problem laid down by *Ptolemy in B.* 12. *of his Almageſt.* accomodated to the Hypotheſis of the Earth's being at reſt, which *Apollonius Pergæus* formerly gave, and which *Copernicus* notwithſtanding ſays, that it no leſs agrees with his Principles of the Motion of the Earth, in *B.* 5. *Ch.* 35. *Revolut.* and which you may ſee in the abovemention'd Authors.

If the plane of two Orbits be not the ſame, the one may be reduc'd to the other, as letting fall from all the points of the Orbit, which is to be reduced, perpendiculars to that other, by which means the circular Orbit becomes Elliptic, the the Elliptic is chang'd into another Ellipſe, or perhaps into a Circle. But a new Orbit is more eaſily drawn (by *Prop.* 33.) thro' five Points, two of which are the ſame with two of the Orbit, to whoſe Plane the other is reduc'd, namely, the common Sections of that with this; the three others are found, by ſeeking the Curtated Diſtances from the Sun, as was done in *Prop.* 35. And if the other Orbit, or both after the reduction become Elliptic, the Problem is conſtructed in them after

after the like manner, and the preceding Calculation applied to this case, after the same manner as was done in *Prop.* 36. in a case not much different. But to find the ratio between *RA* and *CD*, *Prop.* 41. *B.* 1. is to be used instead of *Prop.* 27. of the same Book; which was used in the case where both the Orbits were circular.

SECTION V.

Concerning Tables of the Primary Planets, and the Use of them.

FOR reasons much like those mention'd in *Sect.* 9. *B.* 2. concerning making Tables of the First Motion, Tables are also made for the more expeditious finding the situation of the Planets, both in regard of one another, and in respect to the Fix'd Stars at a given Time. The Art and Method of making them may be learnt from what goes before, but the Order from what follows.

PROPOSITION XXXIX.

TO describe the Tables, by the help of which the Heliocentric and Geocentric Place of any Primary Planet at a given Time is expeditiously determined.

In the first place, we have *a Table of the Equation of Time,* consisting of two Parts, of which we have treated in *Prop.* 17. And because in a complicated Calculation it would be a tedious piece of work to take out the two component parts with their Signs, add or subtract them as occasion requires, they form one Table made out of them mixt together, fitted to the Age the Calculator lives,in since in that Age the situation of the Points of

of the Ecliptic, in regard of the Earth's Aphélion, is not ſenſibly changed.

Tho' this Table of the Equation of Time may be reckoned among the general Tables, ſince Time muſt be uſed to determine the Place of any moveable Body, and that marked out by ſomething moved equably; and all Time is commonly (and moſt aptly) reckoned by the Sun: The motion of this Body which is in its own nature unequal, ought to be reduced to an equality; that is, ſuch a Table as this ought to be uſed: Notwithſtanding, ſince the Equation of Time marked by the Sun depends, on a double account, upon the ſituation of the Earth in regard of the Sun, this Table tho' general in its uſe, and neceſſary in determining the motion of the Fix'd Stars from the preced. Book, as of the Secondary Planets from the following, ought to be reckon'd from its own nature, among the Tables of the Primary Planets.

The Tables are to be made ſeparately for each of the ſix Primaries, namely Saturn, Jupiter, Mars, the Earth, Venus and Mercury. Tho' this, or the inverſe of this, be the true order of the Planets, and the Tables of them (each ſeparately) ought to be diſpoſed after this order naturally; yet with us, who are the Inhabitants of the Earth, 'tis neceſſary that the Tables of the Earth have the firſt place, both becauſe by the help of theſe, the motion of the Earth is found, on which the apparent motion of the Sun (which leads all the other Planets) depends; which is neceſſary in the firſt place to determine the motion of any of the reſt, by reaſon of the Equation of Time depending upon it.

In the Tables of each of the Planets firſt of all the Radix of the Motion of the ſaid Planet is to be ſettled; that is, at any celebrated point of mean

mean Time pitched upon according to the reckoning of the place, for which the Tables are made, you have marked by the help of the preceding Propofitions, the Heliocentric Longitude of the faid Planet, of the Aphelion and afcending Node of the fame; the firft whereof, namely the Diftance of the Planet from the Equinoctial, confifts of two heterogeneous parts; namely the diftance of the Aphelion of the Planet from the Equinox, expreffed by the angle, and the diftance of the Planet from the Aphelion, expreffed by the area of its Orbit.

In the fecond place, the Mean Motions from the Equinox of the Planet it felf, of the Aphelion and Node, for fingle Years, Tens, Hundreds, *&c.* alfo for Months, Days, Hours, and parts of Hours, are difpofed into Tables.

In the third place, a Table is fubjoined, exhibiting the true Anomaly of the Planet to the feveral degrees of the mean Anomaly of the fame. This Table may be made either by *Prop.* 3. or by Approximation by the 4th, 6th, or 7th Propofitions, according to the nature of the Approximation which a Perfon has a mind to ufe.

In the fourth place, you have a Table exhibiting the Diftance of the Planet from the Sun, (found by the fame Propofitions, together with *Prop.* 22.) in parts, whereof the mean Diftance from the Earth contains 100000, for the feveral degrees of the mean Anomaly. Thefe two Tables are advantageoufly reduced into one, by fetting next to each degree of the mean Anomaly, the agreeable true Anomaly and correfponding Diftance. To thefe Diftances, for eafe of Calculation, are annex'd the correfponding Logarithms; or even the Logarithms of the Diftances alone are difpofed into Tables, fince they are fufficient for Calculation.

In

In the fifth place, is put a Table of the Centric Latitude, or Inclination of a Planet to the Plane of the Ecliptic for the ſeveral degrees of the Diſtance from the neareſt Node, which Diſtance from the Node is commonly called by Aſtronomers, *The Argument of the Latitude.*

In the ſixth place, you have a Table of *Reduction*, whereby the Place of a Planet in its proper Orbit is reduced to a Place in the Ecliptic, ſhewing how much more or leſs forward it is in the latter than in the former.

In the ſeventh place, you have a Table ſhewing how much the interval of a Planet from the Sun in its proper Orbit is to be curtated, to reduce it to the plane of the Ecliptic, by letting fall from the Planet a perpendicular upon it. And ſince theſe three (namely the Inclination, Reduction and Curtation) are fitted to the ſeveral degrees of the Argument of Latitude, the three preceding Tables will commodiouſly come into one; as was ſaid before concerning the third and fourth Tables. Theſe three things are found by common Trigonometry, having the Inclination of the Orbit of the Planet to the plane of the Ecliptic, which is done by *Prop.* 20.

In the eighth place, is advantageouſly placed the Angle of Commutation (found by *Prop.* 38.) making in that Planet an apparent Station from the Earth. And becauſe neither the Orbit of the Planet nor of the Earth is circular, neither can the ſame Angle of Commutation ſerve in every caſe, nor the ſame in the two next Stations; the Angles of Commutation producing both the firſt and ſecond Station of the Planet, are diſpoſed into a Table, for four or eight of the principal Anomalies.

In every one of the Inferiors may commodiouſly be added a Table, (made by *Prop.* 36,) coa-

lexing with the former, of the Angles of Commutation in the greateſt Elongation of the Planet from the Sun, as well towards the Eaſt as towards the Weſt, fitted to the ſame Anomalies. To theſe, the Depreſſion of the Sun below the Horizon in the Moments of Heliacal Riſing or Setting is alſo added. And theſe are the common Tables uſually made by Aſtronomers for the ready finding out the Places of the Planets.

PROPOSITION XL.

TO calculate the Heliocentric and Geocentric Place of any Planet, in regard of Longitude and Latitude at any time propoſed, by the help of the above deſcribed Tables, and to define its Paſſions.

Firſt of all, let the Time propoſed be equated, as was ſhewn in *Prop.* 17; let the Place of the Earth ſeen from the Sun, and its Diſtance from the Sun be found for that mean Time, as in the 18th. And ſince by the help of the firſt Table at the aſſum'd Epocha there are given the diſtances of the Planet propoſed, and of its Aphelion and aſcending Node from the Equinox; and by the ſecond, their Mean Motions made between the ſaid Epocha and the Time propoſed, being equated, as was ſaid; the mean Diſtance of the Planet from the Equinox, and the Diſtances of the Aphelion and the Nodes from the ſame will be given. The difference of the two former is the Planets mean Anomaly; to which, the correſpondent true one or angular Heliocentric Diſtance from its Aphelion may be taken, out of the third Table; and out of the fourth, the Diſtance of the ſame from the Sun. The Diſtance of a Planet from the Node, or the Argument of Latitude will likewiſe be given; to which, the correſponding Inclination, or Centric Latitude may be

taken

Plate 11. Book 3.

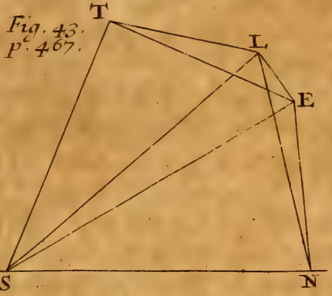


Fig. 43. p. 467.

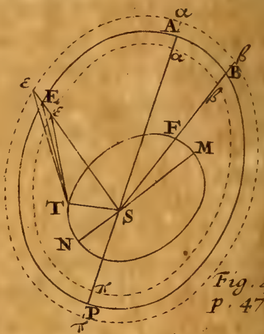


Fig. 44 p. 470.

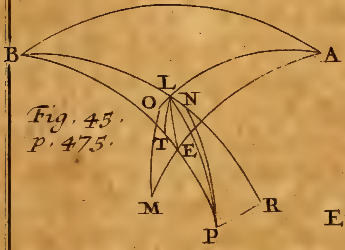


Fig. 45. p. 475.

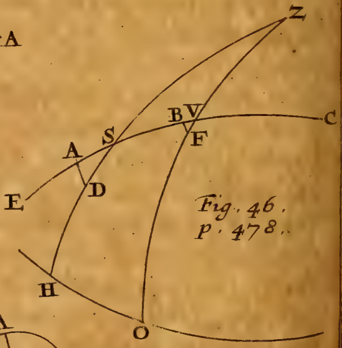


Fig. 46. p. 478.

Fig. 47. p. 481.

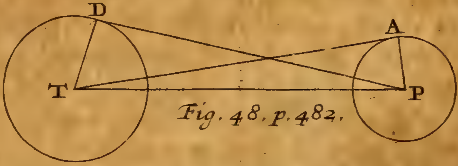


Fig. 48. p. 482.

taken, out of the fifth Table; and out of the ſixth, the Reduction, whereby it is reduced to the Ecliptic; and out of the ſeventh, the Curtation, which being ſubtracted from the Diſtance of the Planet from the Sun in its own Orbit, firſt taken out of the fourth Table, leaves the Curtate Diſtance of the Planet from the Sun.

After this manner may be had the following things in the Diagram of *Prop.* 35; *viz.* the right line *ST* [*Fig.* 43.] in magnitude, namely the Diſtance of the Earth from the Sun; the ſame in poſition, namely the Heliocentric Place of the Earth; the right line *SE* in magnitude, *viz.* the Curtate Diſtance of the Planet from the Sun; and the ſame in poſition, namely the Place of the Planet reduced to the Ecliptic: And conſequently the angle *TSE* is given, contained between right lines given by poſition, which is alſo the angle of Commutation; therefore the angle *STE* the Elongation of the Planet from the Sun ſeen from the Earth will be found. But the Place of the Sun ſeen from the Earth is given; and therefore the Geocentric place of the Planet in the Ecliptic, and *TE* the Diſtance of the Planet from the Earth are given.

Again, becauſe *LE* is perpendicular to the plane of the Ecliptic *TSNE*, the Triangles *SEL*, *TEL* are rightangled at *E*; and therefore the Tangents of the angles *ETL*, *ESL* are in the ſame ratio with *ES*, *ET*: But *ESL* is the Heliocentric Latitude of the Planet firſt taken out of its proper Table, and the ratio of *ES* to *ET* is given; for the right lines themſelves are given, (but the ratio of theſe is the ſame with the ratio of the Sines of the oppoſite angles *STE* of Elongation, and *TSE* of Commutation;) wherefore *ETL* the Geocentric Latitude of the Planet is known.

Moreover, by the angle of Commutation and Anomaly of the Planet found above, in the eighth Table it may be found whether the ſaid Planet be in the firſt or ſecond Station. But if the angle of Commutation be leſs than that which in this Planet makes the Station, the Planet will be Retrograde; if greater Direct; as was ſhewn above. And after the ſame manner, by the ſame Table it may be found whether Venus or Mercury be in their greateſt Elongation from the Sun. And by the Place of the Planet found above, and the known Depreſſion of the Sun requiſite to it, annexed to this Table, it may be determined (by *Prop.* 38. *B.* 2.) whether the Planet is riſing or ſetting Heliacally, or lies hid under the rays of the Sun. And laſtly, by the ſame method, with that whereby the Phaſis of the Moon is delineated, *Prop.* 17. *B.* 1, the Phaſes of Mars, Venus or Mercury, at a given time may be delineated. But Jupiter and Saturn always appear full; as was ſaid in *Prop.* 9. *B.* 1.

In this Propoſition we do not attend to the variety of all caſes, *viz.* whether this Arc be to be ſubtracted from that, or *vice verſa* that from this; nor to the different caſes of Inclination and Latitude Northern and Southern, Elongation Eaſtern and Weſtern, and the like of the reſt; nor how out of two numbers, the intermediate Tabular number ſought is to be found by proportion: For we do not here give rules of calculation to be obſerved by a Calculator literally, but the Geometrical principles of a Calculation.

SCHOLIUM.

Tho' the Tables above deſcribed are natural and commonly made uſe of, yet to find the Places of the Planets more readily, Tables are made after this or the like manner advantageouſly enough, by ſuch as look upon the Aphelia and Nodes

Nodes of the Orbits of the Planets to be at reſt; as *Street* of all the Planets, *Flamſtead* at leaſt of the Apſides of the Earth.

For every Planet's Radixes are to be ſettled, not of the Longitude of a Planet, and of its Aphelion and Node, as before, but of the mean Anomaly of the Planet: And the Motion of the Anomaly for Years and ſums of Years, Months, Days, (or rather for the particular days of the ſeveral Months current,) Hours, &c.

To theſe are added the following things, which always continue the ſame, determining the ſituation of the Orbit of the Planet, and exhibiting the intire Theory expreſſed by numbers; namely 1. The Diſtance computed in the Ecliptic of the Aphelion from ſome one of the Fix'd Stars (for inſtance, the firſt of Aries, or the difference of Longitude of the ſaid Fix'd Star and Aphelion of the Planet, which is called *the Longitude of the Aphelion from a Fix'd Star*. 2. The Longitude of the aſcending Node of a Planet from the ſame Fix'd Star. 3. The Inclination of the Orbit of a Planet to the plane of the Ecliptic. 4. The Magnitudes of the greater Axe of the Orbit of a Planet and of its Excentricity, expreſſed in parts, whereof the mean diſtance from the Earth contains 100000. For from theſe, by the reſpective Propoſitions, the Longitude of the Planet from the firſt Star of Aries, the Inclination or Heliocentric Latitude Northern or Southern, and the Curtate diſtance of a Planet from the Sun are found, for any mean Anomaly of the Planet. Out of theſe artificially annexed to the mean Anomaly, to which they belong, are made Tables; though inſtead of the Curtate Diſtance itſelf, the Logarithm alone will ſuffice for Calculation. For ſince (by *Prop.* 43. *B.* 1.) the mutual ſituation of a Fix'd Star and the Orbits of the Planets remains the ſame, or the Lines of the Nodes

and Apſides are at reſt, (at leaſt they are taken to be ſo by thoſe Authors,) to the ſame point of the ſame Orbit, that is, to the ſame mean Anomaly, theſe three continue the ſame and unvaried in the ſame Planet. Theſe being taken out of the Tables, the reſt is done, as above in the Propoſition. But the Longitude of the firſt Star of Aries at a given Time is found by the Tables of the Fix'd Stars, of which ſee *Prop.* 68. *B.* 2. What remains about the Paſſions of the Planets is to be done after the ſame manner as above.

Such Tables as are more ready for uſe than thoſe mention'd above, are called *Ephemerides of the Celeſtial Motions*, being calculated, from the perpetual Tables deſcribed above, for a certain number of Years, and they ſhew the Places of the ſeveral Planets as to Longitude and Latitude, their Paſſions, and mutual Aſpects, as well when they are viewed from the center of the Sun, as from the center of the Earth, for every day; to which alſo are added the Places of the Moon computed for every day, its Phaſes, the Eclipſes of the Sun and Moon, and other things found in the following Book.

PROPOSITION XLI.

IN the Primary Planets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Elliptic Orbits, which they deſcribe about the Sun.

Let *S* [*Fig.* 44.] repreſent the Sun; *MFTN* the Orbit of the Earth, whoſe Apſides are *M* and *N*, *AEPB* the Orbit of any other Planet, whoſe Apſides are *A* and *P*, which are ſuppoſed to be ſo delineated in the Scheme, as that the Inclination of the lines *MN*, *AP* may be the ſame with that found by what has been ſaid about the Lines of the Apſides of the ſaid Planets in the Heavens; and

and the ratio of the Cubes of them, the ſame with that of the Squares of the Periodic Times of the ſame Planets. I ſay, that the Orbit of the Earth remaining, another Planet can't move in any other Orbit beſides that juſt now deſcribed *A E P B*. For if it can, let the different Orbit of the ſaid Planet be $\alpha \varepsilon \pi \beta$; that is, greater or leſs than the Orbit *A E P B*. Let *F* denote the point in the Orbit of the Earth, wherein the Earth is when it is in conjunction with any other Planet Heliocentrially. The line *S F* drawn (and if need be, produced) will ſhew in the Orbit *A E P B*, the point *B*, wherein the other Planet would then be found if it moved in this Orbit; and in the Orbit $\alpha \varepsilon \pi \beta$, which it is ſuppoſed to deſcribe, the point β, in which it is really found. The Earth removing from *F* and the other Planet from β, after a certain time, let the former be found in *T*, the latter in ε: $S\varepsilon$ joined will meet the Orbit *A E P B* in the point *E*, in which the Planet would have been found, if it had deſcribed the Orbit *A E P B*, ſince the Periodic Time is ſufficiently known and certain. The right lines *E T*, εT being drawn, the angle $ST\varepsilon$ will be the Elongation of the Planet from the Sun at the ſaid time: But this Elongation is not agreeable to obſervation, for 'tis the Elongation *S T E* (drawn from the ſuppoſition, that the Planet deſcribes the Orbit *A E P B*) that does. And therefore ſuppoſing *M F T N* to be the Orbit of the Earth, that other Planet does not deſcribe the Orbit $\alpha \varepsilon \pi \beta$ different from the Orbit *A E P B*. But the Cubes of the greater Axes of the Orbits *A E P B*, *M F T N* are as the Squares of the Periodic Times of the Planets moved in thoſe Orbits. And the ſame is collected of the other Planets compared with the Earth, for the like reaſons: And therefore *ex æquo*, in any two Primary Pla-

nets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Orbits, wherein they are carried.

And generally the agreement of the Places by calculation with thoſe obſerved, argues the true Situation and true Species of the Orbit of any Planet, as well as the true Magnitude. For the Situations of the Earth and the other Planets are ſo variouſly intermingled, that any notable errors committed in the ſettling the Orbit and Theory of any Planet would ſometimes manifeſtly diſcover themſelves, and become very ſenſible: But on the contrary, the Places of the Planets drawn from Tables depending upon the ſaid Theories, ſtrangely agree with their Places by obſervation. The very ſmall difference from obſervation in the mean while is owing to other cauſes; namely the action of the Planets upon one another, the effects whereof in diſturbing the motions of the Planets are afterwards to be conſidered, and their Orbits rectified from thence.

PROPOSITION XLII.

THE Geocentric Place of a Planet at a given Time being given, to find by the Tables the Place of the ſame ſeen from a given Place or Habitation upon the ſurface of the Earth as that time.

If the Habitation given be not in the ſame Terreſtrial Meridian with the Place to which the Tables are fitted, let the given Time be reduced to the reckoning of the Place of the Tables, by the Tables deſcribed *Sect.9.B.2.* To that Time find (by *Prop.36. B.2.*) its Altitude, ſeen from the center of the Earth, above the horizon of the Habitation given, and the Azimuth. Let the Parallax of the Sun, that correſponds with the Altitude thus found, be taken out of the Table made by *Prop. 47. B. 2*; from the Parallax of one Diſtance and of

of one Altitude, or known by ſome Prop. of the 2[d] Book, or by one of the following: And (by *Prop.* 48. *B.* 2. the Parallax of the Planet in the ſame Altitude above the Horizon, will be to the Parallax of the Sun taken above, as the Diſtance of the Sun from the Earth, to the Diſtance of the Planet from the ſame: But theſe Diſtances in the Place of the Planet to be computed by the preced. were known; wherefore the Parallax of the Planet corresponding to that Altitude is found. This therefore is to be ſubtracted from the Geocentric Altitude found above, that it may become the Altitude once corrected in the given Habitation. To the Altitude thus corrected add the correſponding Refraction, from the Table of Refraction made by *Prop.* 66. *B.* 2. of the uſe of which we have ſpoken in *Prop.* 68. of the ſame; and you will have the apparent Altitude of the Planet in the given Habitation, the Azimuth firſt found remaining unchanged. From this apparent Altitude and Azimuth, you may find by *Prop.* 26, and 27. *B.* 2. the Planet's place, which will be the apparent one in the given Habitation. The Time propoſed reckoned in the Meridian to which the Tables are fitted, and afterwards reduced to the Meridian of the given Habitation, ſince it is a mean, (ſuch as it ought to be when Tables are uſed) 'tis reduced to apparent Time, by *Prop.* 17. Therefore the apparent Place of the Planet, at the given Time now made the apparent, is found.

SECTI-

SECTION VI.

Of the Magnitude of the Orbits of the Planets.

IN what precedes, we have conſidered the Planets themſelves, as ſo many points moving in their Orbits; that is, we have abſtracted from their Magnitude. And in this caſe it is the ſame, as to the Phænomena, what Magnitude the Orbit of any of them is ſuppoſed to be of, provided all the reſt have that ratio and poſition to this, as they have in nature. Therefore we have every where ſuppoſed the common meaſure of all the Orbits and their parts to be the greater Axe of the Orbit, that the Earth (the Habitation of the Spectator) deſcribes about the Sun, or half of it, the mean Diſtance of the Earth from the Sun, without being concerned about the Magnitude of this apparent Diſtance. But now when the Magnitude of the other Planets comes to be compared with the Magnitude of the Earth, the aforeſaid Diſtances muſt be compared with the Diameter of the Earth; which may be done by the help of the Parallax being known, as was ſhewn in *Book* 2. And tho' the Parallax of the Planet might be found by one of the methods of finding the Parallax delivered in *Book* 2, if you could obſerve it as accurately as you pleaſe; yet notwithſtanding, becauſe there are ſeveral things that are too obvious which are an hindrance to it, I ſhall propoſe two methods by which this obſervation, ſo ſlippery as it is, may be made without any danger of a notable error. In the firſt it ſhall be done by two obſervations made together in two Places of the Earth; in the ſecond, by two made in the ſame Place.

PRO-

PROPOSITION XLIII.

BY the Conjunction of two Planets, observed in two given places of the Earth, to find the Parallax of both.

Let the true Places of the two Planets observed in *Corporal* Conjuction (that is, where the one seems to touch or cover the other) be *L* and *E*. Imagine the Circle *A B* [*Fig.* 45.] to be drawn thro' the vertices *A* and *B* of the Places on the Earth. Draw the vertical Circles *A E M, A L O, B E P, B L N.* Let the Planets be observed from one of the given Places (namely that whose vertex is *A*) in O and *M*. And since the Places are given where the observations are made, (by *Prop.* 32. *B* 2.) the difference of the Meridians will be given; and so the names of the Hours, whereby any the same instant of time is expressed in the said two Places: And therefore the same moment of Time may be pitched upon to make the observation in the given Places. Therefore at the same instant that the Planets in *O* and *M* are observed from the Place whose vertex is *A*, let them be observed from the Place whose whose vertex is *B*, for instance, in *N* and *P*; Draw the great Circles *L E, O M, N P.* Observe also (either by a Micrometer, or what is instead of it) Threads stretched various ways in the Focus of the Telescope, or by the help of an Icoscope throwing the Image upon a Plane, or by any other method to be left to the industry of the Artist) the magnitudes of the Arcs *NP, O M*, (which in a corporal Conjunction, by reason of their smallness, will be right lines) and of the Angles *P N B, N P B, A O M,* and *A M O.* Now, by the Theory of these Planets, there is given the ratio of their Distances from the Center of the Earth, from whence, and their Altitudes (obser-

ved

ved exactly enough for this purpoſe) there are alſo given (by *Prop.* 49. *B.* 2.) the ratios between the Sines of the Arcs *L N*, *E P*, *L O*, *E M*, and conſequently (by 2. *Dat.*) one being given all are known. From the point *P* let the Arc *P R* be let fall perpendicular upon the Azimuth *B N* produced: And in the rightangled ſpherical Triangle *P N R*, the ſide *P N* and angle *P N R* being given, you may find the ſides *N R*, *R P*, and the angle *R P N*; therefore the angle *R P E* is known. Suppoſe (after the manner of Analyſts) the Sine of the Parallax *L N* in the computation to be the Root: And by the given Sines of the Arcs *L N*, *NR*, you may find (by what *Ptolemy* delivers in *Cap.* 9. *B.* 1. of the *Almageſt*) the Sine of the Arc *LR*, their ſum. Then, in the ſpherical Triangle *L P R* rightangled at *R*, by the given Sines of the ſides *L R*, *R P*, you may find the Sines of the ſide *L P* and Angle *LPR*. Therefore, by the given Sines of the angles *L P R*, and *E P R* (by what is demonſtrated by *Ptolemy* in the place quoted above) you may find the Sine of their difference; namely of the angle *L P E*. From the point *L* let fall the Arc *L T* perpendicular to the Azimuth *B P*. Therefore in the ſpherical Triangle *P L T* rightangled at *T*, by the given Sines of the ſide *L P* and angle *LPT*, you may find the Sines of the ſides *L T*, *T P*: Therefore the Sine of the arc *E T*, namely the difference of the Arcs *E P*, *T P* is given. And laſtly, in the ſpherical Triangle *E T L* rightangled at *T*, by the given Sines of the ſides *LT*, *ET*, you may find the Sine of the ſide *EL* Juſt after the ſame manner you may find the Sine of the ſide *EL*, from the Sines of the Arcs *LO*, *EM* given, expreſſed by the before ſuppoſed Root. Therefore you have given an Equation between the Sine of the Arc *E L* found firſt, and the ſame Sine found

ſecondly,

ſecondly, the reſolution whereof will give the value of the Root, or ſine of the Arc LN; from whence all the reſt that is ſought will become known. But if ſuch Places upon the Earth be choſen, as have the Planets paſſing the common Azimuth of the Places, and that inſtant of Time wherein one of them is in the ſaid common Azimuth; this conſtruction will become that particular one in *Prop.* 87. *Opt. Prom.* of *James Gregory.*

LEMMA.

THE difference of two Quantities and the ratio between them being given, to find the Quantities themſelves.

Let a quantity be taken, that is to the known difference, as the the greater term of the given ratio, is to the difference of the terms; this quantity will be the greater of the quantities ſought: For in four proportional quantities, the products of the two former are in the ſame ratio with the ſimilar products of the two latter. If the greater be leſſen'd by the given difference, what remains will be the leſs.

PROPOSITION XLIV.

BY two Obſervations of the ſame Corporal Conjunction of two Planets, made in the ſame Place, to determine the Parallax of both Planets.

Let the apparent motion of the two Planets reduced to the Ecliptic, made mutually by one another (or to one another) in a given time, be obſerved near the Corporal Conjunction: And by the Theory of both Planets, known by the preceding, let their motion from one another mutually be determined, computed in the Ecliptic, viewed from the Center of the Earth, made in the ſame time; which may be done accurately enough, if the time be not too great. From theſe

arcs

arcs therefore given, namely, the true and apparent motion of the Planets from one another, made in the ſame time, the Parallax of both is to be determined.

Let *HO* [*Fig.* 46.] repreſent the Horizon; *Z* the Zenith, and *EC* the Ecliptic: And let *SV* be the arc of the Ecliptic, wherein the Planets (viewed from the Center of the Earth) move mutually from one another in a given time, known from the Theory; and *AB* the arc of the Ecliptic, whereby they ſeem to move from one another in the ſame time, known by Obſervation; therefore their difference is given. Thro' *A* and *B* imagine Circles of Latitude *A D*, *B F* drawn, meeting the Verticals *Z H*, *Z O*, drawn thro' *S* and *V*, in *D* and *F*; and *S A* is the Parallax of the Longitude of the nearer Planet from the more remote, or the exceſs of the greater Parallax of Longitude above the leſs in this ſituation; and *VB* the exceſs of the greater Parallax above the leſs in the other. The ratio of the arc *A S* to the arc *B V* is compounded of the ratios of *A S* to *S D*, *S D* to *VF*, and *VF* to *BV*; but the ratio of *AS* to *SD* is known, becauſe in the rectilineal Triangle *A S D*, rightangled at *A*, the Triangle *A S D* is given, found by *Prop.* 36. *B.* 2; the ratio of *SD* to *VF* is found, by *Prop.* 47. *B.* 2; and the ratio of *V F* to *B V* is found as before, in the ratio of *AS* to *SD*; wherefore the ratio of *AS* to *BV* compounded of them is known: But the difference of the arcs *AS* and *BV* is alſo known, namely the ſame with the given difference of the arcs *AB* and *SV*; and therefore (by Lem. preced.) the arcs themſelves *AS* and *BV* will be given. And therefore (by Dat. 2.) *SD* and *VF* will be given; that is, the exceſs of the greater Parallax above the leſs in a given Altitude: For the Altitude *HD* or *OF* may be obſerved exact enough

enough for that purpoſe. And from thence the exceſs of the greater Horizontal Parallax above the leſs (by *Prop.* 47. and 63. *B.* 2.) will alſo be given: But (by *Prop.* 35.) the ratio between the Diſtances of the given Planets from the Earth at the time of Obſervation is given, (for the Diſtance of both from the Earth is expreſſed in the parts of the mean Diſtance of the Earth from the Sun;) and therefore the ratio of the Horizontal Parallaxes of the ſame Planets is alſo given, becauſe (by *Prop.* 48. *B.* 2.) the reciprocal of the former: And therefore (by Lem. preced.) the Horizontal Parallaxes of the Planets themſelves are alſo given.

And from hence the Parallaxes of all the Planets are determined. For the Parallax of one of the Planets being given, the Parallax of all of them (by *Prop.* 49. *B.* 2.) are given; becauſe (by the preceding Propoſition) at a given time the ratio between their Diſtances from the center of the Earth is given.

SCHOLIUM.

Tho' any two Planets in any Conjunction (after the manner ſhewn in the preced. Propoſition) may be uſed to determine the Parallax of both, nay any Planet (Mars, for inſtance, in his Achronical ſituation, and neareſt to the Earth) in conjunction with a Fix'd Star, whoſe Parallax is nothing, may ſuffice to determine its Parallax, (at that time the greateſt,) one of the inferior Planets paſſing over the diſc of the Sun, is fitteſt for that purpoſe; eſpecially Venus, whoſe Parallax is very ſenſible, becauſe ſhe is then nearer; and the Theory, ſo far as it is neceſſary for this purpoſe, is accurately enough known.

PROPOSITION XLV.

TO *determine the greater Axes of the Orbits of the Planets, Excentricities, &c. in known meaſures.*

For any time (by one of the two preceding, or any other method ſhewn in *B.* 2.) find the Parallax of any Planet, correſponding with any known apparent Altitude; and from thence the ratio of the Diſtance of the Planet from the center of the Earth to the Semidiameter of the Earth will be given; namely the ſame (by *Prop.* 46. *B.*2.) with that which is between the ſine of the apparent Diſtance from the vertex and the ſine of the Parallax. But (by *Prop.* 35.) the ratio between the Diſtance of the Planet from the center of the Earth and the Diſtance of the ſame from the Sun at that known time; and (by the preced. Prop.) for the ſame Time the ratio of the diſtance of the Planet from the Sun to the greater Axe of the Orbit, and its Excentricity, *&c.* is given: And therefore, *ex æquo*, and by (*Dat.*2.) the ratio of thoſe right lines to the Diameter of the Earth will be given, that is, all theſe Diameters of the Orbits will be given, expreſſed by Diameters of the Earth. And ſince, by the *Schol.* of *Prop.* 17. *B.* 2. the Diameter of the Earth is found in known and familiar meaſures, the aforeſaid Diameters of the Orbits will be given in the ſame.

By this means the Diſtances of the Sun and Primary Planets from the Sun at any time propoſed, expreſſed by Semidiameters of the Earth or any other known meaſures, become known. For thoſe Diſtances at that time expreſſed in the parts of the mean Diſtance of the Earth from the Sun are known, (by *Prop.* 34. and 35,) and (by this) the ſaid mean Diſtance of the Earth from the Sun is known in known meaſures.

SECTION VII.
Of the Magnitude and Denſity of the Sun and Primary Planets.

PROPOSITION XLVI.

TO *define the Magnitude of the Sun and Primary Planets.*

'Tis very proper in this place, having laid down the Theory of the Planets, to treat alſo of their Magnitude and Denſity, upon which conjunctly the Quantity of Matter in each depends; eſpecially ſince ſome things in their Motion and the dimenſion of their Orbits come to be rectified by this means.

Let ſome one of them be *ABC*,[*Fig.* 47.] whoſe Center is *P*, and let O be the Eye of the Obſerver. Join the right line O *P*, along which imagine a plane drawn cutting the ſpherical Body of the Sun or Planet in its greateſt Circle *BAC*. From O draw right lines O *A*, O *B* touching it, and join the right line *P A*. The angle *A* O *B* is the ſame with the apparent Diameter of the ſaid Body at O, and is biſected by the right line O *P*. Let this be obſerved by the methods well known to Aſtronomers, by which means, in the Triangle *A* O *P*, all the angles will be known: For (by *Prop.*16. *El.* 3.) *A* is a right angle, and *A* O *P* is half the obſerv'd apparent Diameter; hence the ratio of the ſides O *P* and *A P* is known: But (by *Prop.* 45.) the ratio between the Semidiameter of the Earth, and *OP* the Diſtance of the Planet is given; therefore *ex æquo*, the ratio between the Semidiameters and the Earth and Planet is given; therefore the triplicate of this is alſo given, namely the ratio between the Earth and the Planet. But the Magnitude of the Earth is given by *Schol. Prop.* 17. *B.* 2; and therefore the Magnitude of the Planet (by 2 *Dat.*) is given. *Q. E. F.*

If thro' *T* and *P* the centers of the Earth and Planet [*Fig.* 48.] a Plane be drawn cutting them both, the common ſections of this plane with thoſe Bodies will be great Circles in them. But if you draw the Tangents *TA*, *PD*, and join *TP*, *TD*, *AP*; theſe two laſt do almoſt coincide with arcs deſcribed upon the Centers *T* and *P* and at the diſtance *TP*; and conſequently they are in the ſame ratio with the angles *TPD*, *PTA* that they ſubtend, the latter whereof is the apparent Semidiameter of the Planet, the former its Horizontal Parallax: And therefore reciprocally the Diameter of the Planet is to the Diameter of the Earth, as the apparent Semidiameter of the Planet to its Horizontal Parallax. But the three latter terms of this Proportion are given; and therefore the firſt, namely the Diameter of the Planet.

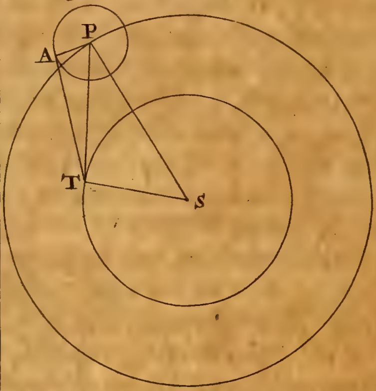
SCHOLIUM.

By the aforeſaid method, the Magnitudes of the Sun and all the Planets, excepting the Earth, may be accurately compar'd together. For their Diameters are in a ratio compounded of the ratio of the Diſtances, and of the ratio of the Sines of the ſemi-angles under which they appear, that is, almoſt of the apparent Diameters: But the ratio of the Diſtances found by *Prop.* 35. is the ſame with the true by *Prop.* 41; and the ratio of the apparent Diameters is certain by obſervation. But the compariſon of the Magnitudes of thoſe Bodies with the Magnitude of the Earth depends upon the Parallax of ſome Planet. If this be not accurately given, the ſaid compariſon can't be made accurately. We have already given the beſt methods of finding it in their proper places: But the laſt, which you have in *Prop.* 44. ſeems moſt fit for uſe, if it be applied to Venus ſeen in the diſc of the Sun; which has not been yet done, nor can

be

Plate 12. Book 3.

Fig. 49. p. 483.



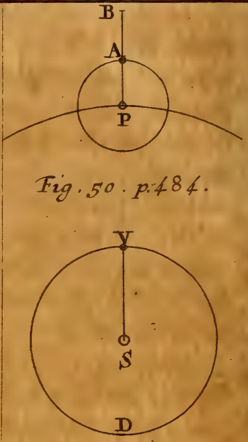


Fig. 50. p. 484.

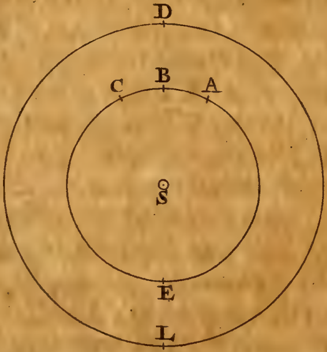


Fig. 51. p. 488.

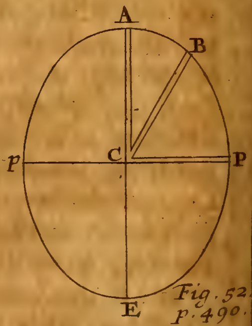


Fig. 52. p. 490.

be done till the year 1761, (which will be the ſecond time of Venus being to be ſeen by us in the Sun.) If in the mean while the Diameter of the Earth ſeen from the Sun be made a mean betwee the Diameters of the five others ſeen from the ſame, (which 'tis very probable is not far from the truth, and beſides agrees very nearly with the Parallax of the Sun determined by an obſervation of Mercury ſeen in the Sun, and other obſervations made other ways,) the Magnitudes of all the Planets and Sun will come out near enough to their true Magnitudes, and ſuch as are to be taken for the true, 'till ſomething more certain be ſettled about the Parallax of the Sun by obſervations.

PROPOSITION XLVII. LEMMA.

TO find the ratio of the Diſtance of a given Secondary from its Primary, to the Diſtance of the given Primary from the Sun.

Let *S* repreſent the Sun; [*Fig.* 49.] *T* the Earth; *P* any Planet, whoſe Satellite is *A*. At *T* obſerve its greateſt Elongation from *P*, namely the angle *ATP*. Therefore in the Triangle *PAT* right-angled at *A*, the angle *T* being given, the ratio of the ſide *AP* to *PT* will be given: And by *Prop.* 35. the ratio of *PT* to *TS* is given; and the ratio of *TS* to the Diſtance of any other Planet from the Sun is alſo known; and therefore the ratio between the Diſtance of the given Satellites from its Primary and the Diſtance of any Primary from the Sun will not be unknown.

But if the Satellite propoſed be our Moon, the Problem can be ſolved no otherwiſe than by finding the Diſtance of the Sun from the Earth, (by one of the methods above, or ſome like it) and the Diſtance of the Moon from the ſame (by the ſame, or ſome other already delivered or hereafter to be delivered in *Book* 4th.)

PROPOSITION XLVIII.

TO find the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in a given Primary, about which a Satellite revolves.

Let *S* be the Sun; [*Fig.* 50.] *P* the Primary Planet given, about which its attendant *A* revolves; and let *V* be any other of the Primaries taken at pleaſure: Make *P B* equal to *S V*, and imagine any Body placed at *B*; and (by *Prop.* 49. *B.* 1.) the Quantity of Matter in *S* to the Quantity of Matter in *P*, will be as the accelerative gravity of the Body *V* towards *S* to the accelerative gravity of the Body *B* towards *P*. And the ratio of the accelerative gravity of the Body *V* towards *S* to the accelerative gravity of the Body *B* towards *P*, is compounded of the ratio of the accelerative gravity of the Body *V* towards *S*, to the accelerative gravity of the Body *A* towards *P*, and of the ratio of the accelerative gravity of the Body *A* towards *P*, to the accelerative gravity of the Body *B* towards the ſame *P*. But (by *Prop.* 26. *B.* 1.) the accelerative gravity of the Body *V* towards *S*, is to the accelerative gravity of the Body *A* towards *P* in a ratio compounded of the ratio of the right line *S V* to the right line *P A*, and of the duplicate ratio of the periodic Time of the Satellites *A* about *P*, to the periodic Time of the Primary *V* about the Sun; and the accelerative gravity of the Body *A* towards the Planet *P* is to the acceleravity gravity of the Body *B* towards the ſame, in the duplicate ratio of the right line *P B* (or its equal *S V*) to the right line *P A*. And therefore the accelerative gravity of the Body *V* towards *S* is to the accelerative gravity of the Body *B* towards *P*; that is (as was ſhewn above) the Quantity of Matter in *S* is to the Quantity of Matter in *P*, in a ratio compounded of the three follow-

following; namely, the ratio of the right line *S V* to the right line *P A*, the duplicate ratio of the periodic Time of the Satellite *A* about the Planet *P*, to the periodic Time of the Primary *V* about the Sun, and the duplicate ratio of the right line *S V* to the right line *P A*. But the first and last compounding ratio make the triplicate ratio of the right line *S V* to *P A*: From whence the Quantity of Matter in *S* is to the Quantity of Matter in *P* in a ratio compounded of the triplicate ratio of *S V* to *P A*, and the duplicate ratio of the periodic Time of the Satellite *A* about *P* to the periodic Time of the Primary *V* about the Sun. But both the compounding ratios are given; namely the ratio of *S V* to *P A*, by the *Lemma* premised, and the periodic Times themselves by observation: Therefore the ratio compounded of them will not be unknown; namely the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in the Primary *P*.

Hence the ratio of the Quantity of Matter in the three Planets that have Satellites may be found; *viz.* by finding the ratio of the Quantity of Matter in every one of them to the Quantity of Matter in the Sun.

SCHOLIUM.

After the same manner you may find the ratio of the accelerative Gravity in the superficies of the Sun to the accelerative Gravity in the superficies of any Planet that has a Satellite. For the ratio of Gravity in the surface of the Sun, to the Gravity in *V*. *viz.* the duplicate of the ratio of the Distance *S V*, to the Semidiameter of the Sun, known from what goes before, and the ratio of Gravity in *V* towards the Sun to the Gravity in *A* towards the Planet, found as above, are had; as also the ratio of Gravity in *A* to the Gravity in the superficies of the Planet *P*, the

duplicate of the known ratio that the Semidiameter of the Planet has to the Diſtance of the Satellite from the Primary. Now it is evident, that the ratio compounded of theſe three known ratios, is the ſame with the ratio of Gravity in the ſuperficies of the Sun to the Gravity in the ſurface of the Planet *P*. And from hence again, *ex æquo*, the ratio of Gravity in the ſuperficies of one Planet to the Gravity in the ſuperficies of the other Planet, if both have Satellites, will be determined.

PROPOSITION XLIX.

TO find the ratio of the Denſity of the Sun to the Denſity of any Primary Planet, about which a Satellite revolves.

If there be two Bodies *A* and *B* of the ſame magnitude, 'tis evident the Quantity of Matter in *A* is to the Quantity of Matter in *B* as the denſity of *A* is to the denſity of *B*: And if they be of the ſame denſity, the quantity of matter in *A* is to the quantity of matter in *B*, as the magnitude of the Body *A* to the magnitude of the Body *B*: And therefore, tho' neither the magnitudes nor denſities are equal, the ratio of the quantity of matter in *A* to the quantity of matter in *B*, is compounded of the direct ratio of the denſities and the direct ratio of the magnitudes of them. Wherefore the ratio of the denſity of the Body *A* to the denſity of the Body *B*, is compounded of the ratio of the quantity of matter in *A* to the quantity of matter in *B*, and the ratio of the magnitude of the Body *B* to the magnitude of the Body *A*; that is, (in the preſent caſe) the Denſity of the Sun is to the Denſity of a Planet in a ratio compounded of the ratio of the Quantity of matter in the Sun to the Quantity of matter in the ſaid Planet, and the ratio of the Magnitude of a

Planet

Planet to the Magnitude of the Sun. But (by prec. *Prop.*) the former of the compounding ratios is given, if the Planet have a Satellite, and (by *Prop.* 46.) the latter is given; therefore the ratio compounded of them is given, namely of the Denſity of the Sun to the Denſity of the Planet.

COROLLARY.

From hence, *ex æquo*, the ratio of the Denſities of three of the Primary Planets is had, namely Saturn, Jupiter and the Earth. The Denſities of the three others may be analogically derived from hence. No doubt the Divine Being, the Wiſe Creator of the World, has placed the Planets at different diſtances from the Sun, that each of them may enjoy a greater or leſs degree of the Sun's heat, according to the degree of its Denſity: For denſer Matter requires a greater heat to perform its natural operations withal. But without doubt, ſome reſpect is had to the Magnitude of the Planet, ſince of like Bodies the leſſer, *cæteris paribus*, are heated ſtronger and more intimately; becauſe in reſpect of their Bulk, they have a greater ſuperficies, and conſequently receive more rays.

PROPOSITION L.

TO correct the greater Axes of the Orbits of the Planets found before.

In Propoſition 24, the greater Axes of the Orbits are taken in the ſubſeſquiplicate ratio of the periodic Time; that is, the Maſſes of the Planets are abſtracted from, as ought to be done before they are determined, and the Planets are looked upon as points revolving in Ellipſes about the center of the Sun at reſt and placed in the Focus. But becauſe the actions of the Sun and Planet are mutual, and a Planet is made by them to deſcribe an Ellipſe (by *Prop.* 58. *B* I.) whoſe

Focus is the common center of Gravity of its ſelf and the Sun, ſimilar to that which it would deſcribe (by *Prop.* 51. *B.* 1.) attracted by the ſame Forces, about the Sun at reſt; and (by *Prop.* 54. *B.* 1.) the greater Axe of the Ellipſe that any Planet deſcribes about the Sun, revolving at the ſame time about the common center of Gravity, is ſhown to be to the greater Axe of the Ellipſe, that the ſame Planet would deſcribe about the Sun at reſt in the ſame periodic Time, in the ſubtriplicate ratio of the ſum of the Maſſes of the Sun and Planet to the Maſs of the Sun; therefore the greater Axe of the Orbit of each Planet (found by *Prop.* 24.) is to be increaſed in the ſaid ſubtriplicate ratio of the ſums of the Maſſes of the Sun and Planet to the Maſs of the Sun, that it may be correct. But (by *Prop.* 48.) the ratio between the Maſſes of the Sun and Planets are given, and therefore the ratio is given, wherein the greater Axes (found by *Prop.* 24.) of the Orbits are to be increaſed, that they may be correct.

PROPOSITION LI.

TO eſtimate the Errors of the motion of the Planets about the Sun, ariſing from their Action upon one another.

If *S* repreſent the Sun, [*Fig.* 51.] *ABCE* and *DL* the Orbits of two of the Planets; 'tis evident that while the nearer to the Sun is in *A*, and the more remote in *D*, (namely when the Planets are tending towards an Heliocentric Conjunction,) the nearer to the Sun will be a little accelerated, and the more diſtant retarded by the Attraction upon one another; and when they are arrived after the Conjunction to ſuch a ſituation as *C* and *D* is, that the more diſtant one is accelerated, and the nearer is retarded; and that near the Heliocentric Conjunction the Diſtance of the more remote

Pla-

Planet from the Sun is leſſened, of the nearer is increaſed. And ſince the Maſſes of the Sun and both the Planets (by *Prop.* 48.) are given, and the mutual Diſtances from the ſuperiors are alſo given, and that (by the preced. *Prop.*) correctly; the ratio of the Attraction of each towards the Sun to the Attraction towards the other will be given, and conſequently the ratio of the Error ariſing from the mutual Attraction to the effect of that Force, whereby each is retained in its proper Orbit about the Sun.

But if all the Error is to be thrown upon the Planet that is more remote from the Sun, the Orbit of the nearer remaining, this will be done almoſt (by *Prop.* 58. *B.* 1.) by placing the Focus of the exterior Orbit in the common Center of Gravity of the Sun and of the interior.

Theſe Errors are moſt ſenſible in Jupiter and Saturn, becauſe of the vaſt Maſſes of thoſe Planets.

SCHOLIUM.

There are alſo ſome inequalities ariſing from the mutual Action of the Primary and their Satellites: For (by *Prop.* 63. *B.* 1.) 'tis not the Center of the Primary, but the Center of Gravity of the Primary and the Satellites, that deſcribes the Elliptic Orbit about the Sun. But becauſe theſe things depend upon the Quantities of Matter in theſe Secondaries, the method of determining which properly belongs to the following Book, we ſhall at preſent ſay nothing of them.

SECTION VIII.

Of the Figure of the Sun and Planets.

PROPOSITION LII.

To determine the Figure of the Earth; that is, the ratio that its Axe has to the Diameters that are perpendicular to them.

In

In the first Book, at *Prop.* 31, we shewed in general, from the proper causes, that the Sun and Planets are of a Figure depressed towards the Poles, but elevated towards the Circle, that lies exactly between the Poles, approaching that of an oblate Spheroid. Now we come to determine this Figure by observations. And first the Figure of the Earth is to be defined.

Let the Figure *APEp*, [*Fig.* 52.] whose Center is C, represent the Earth; *Pp* its Axis, *AE* any Diameter of the Equator, perpendicular to the Axe. Let the lengths of Pendulums oscillating in a Cycloid, in a given space of time, in the Places *A* and *P*; and *AC*, *PC* will be reciprocally as the said lengths of the Pendulums, and therefore in a known ratio. Let the same be done in any intermediate Place *B*, whose Latitude (*viz.* the angle *ACB*) is known, (by *Prop.* 18. *B.* 2;) and *AC*, *BC* will be reciprocally as the lengths of the Pendulums oscillating in an equal space of Time in those Places *A* and *B*, and therefore in a given ratio.

For imagine a Canal filled with a Fluid and bent from *A* to C, and from thence to *P*. From the rest of the Fluid 'tis evident that the Fluid in the Leg of the Canal *AC* is in equilibrio with the Fluid in the Leg *PC*, the centrifugal Force arising from the motion about the Earth's Axis, and the greater distance from the Earth's Center raising and retaining the Fluid in the Leg *CA*, at a greater height than in the Leg *CP*. Wherefore any portion of the Fluid in *CA*, is in equilibrio (or equally heavy) with the like similarly posited portion of the Fluid in the Leg *CP*, (which consequently is also true of any homogeneous Bodies, tho' they be not Fluid;) and the upper points *A* and *P* are similarly posited in the Legs: And therefore homogeneous Bodies placed in *A* and *P*

which

which are as *AC*, *PC* are equiponderant towards the Center of the Earth. But the Gravity of a Body placed in *A*, which is as *PC*, is to the Gravity of another homogeneous Body standing there, that is, as *AC*, as *PC* to *AC*, (namely the weights of homogeneous Bodies placed near one another, are as the Bodies themselves;) and therefore the Gravities of homogeneous equal Bodies placed in *A* and *P* are as *PC* and *AC*; that is, reciprocally as the distances from the Center. Just after the same manner the Gravity of the Body in *B* may be shewn to be to the Gravity of an equal and homogeneous Body in *P*, as *CP* to *CB*; for the Fluid will continue at rest in the Canal *BCP*, as in the former *ACP*: Wherefore *ex equo*, the absolute Gravities of equal and homogeneous Bodies placed any where on the Surface of the Earth are reciprocally as the Distances from the Center. But the accelerative Gravity of a Body is as its absolute Gravity applied to its Mass. And therefore the accelerative Gravities of Bodies placed on the Surface of the Earth are as the distances of the Places from the Center reciprocally. Moreover, because the oscillation of a pendulous Body placed at *A*, and moved in a Cycloid, is supposed (by observation) to be performed in the same space of time with the oscillation of another at *P*; and (by *Prop.* 25. *Part* 2. of *Hugenius's Horologium Oscillatorium*) the time of a free descent along a given axe of a Cycloid has a given ratio to the time of Oscillation in that Cycloid; and (by *Prop.* 6. and 7. *Part* 3. of the said Book) the axis of the Cycloid, which a heavy Pendulum describes, has a given ratio to the length of the String suspending the weight; and consequently the time of a free descent along the given axis of the Cycloid, has also a given ratio to the time of a free descent along the

length

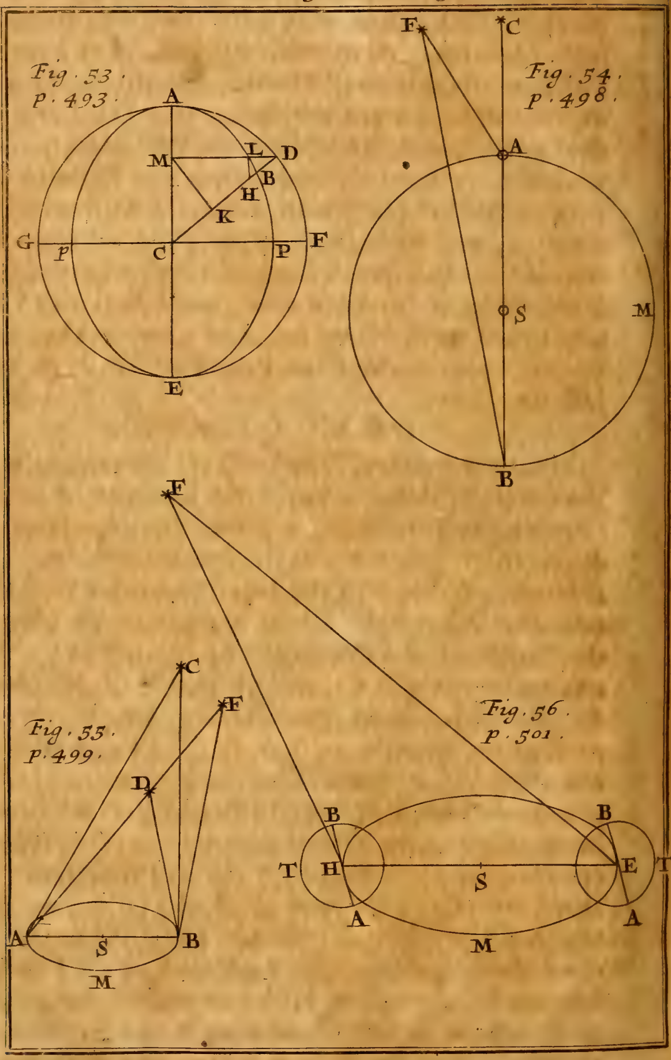
length of the ſtring, namely, the ſubduplicate of the former: The time of a free deſcent along the length of a Pendulum placed at *A*, will be equal to the time of a free deſcent along the length of a Pendulum placed at *P*. But the ſpaces at *A* and *P*, run in equal times by a falling Body, are as the accelerative Gravities in the ſaid Places *A* and *P*, *viz.* the effects are as their cauſes; and the ſaid accelerative Gravities have already been ſhewn to be reciprocally as the diſtances from the Center; and therefore the length of a Pendulum oſcillating at *A*, is to the length of a Pendulum oſcillating at *P* in an equal ſpace of time as *CP* to *CA*. And after the ſame manner may it be ſhewn, that the length of a Pendulum at *B* is to the length of a Pendulumat *P* oſcillating in an equal ſpace of time as *CP* to *CB*: Wherefore, *ex æquo*, univerſally the lengths of Pendulums oſcillating in equal Times are reciprocally as the diſtances of the Places from the Center of the Earth. Since therefore the ratio of *AC* to any *BC* and the inclination of the ſaid *BC* to *AC*, and (among the reſt) the ratio of *AC* to *PC*, are found; the nature of the Figure *APEp* is defined, namely, of the Section of the Earth with the plane of the Meridian, by whoſe rotation about the Axis *Pp*, the Figure of the Earth is generated. *Q. E. F.*

If the Figure of the Earth were to be determined after the manner above deſcribed, there would be no need of meaſuring the length of a Pendulum oſcillating at the Pole *P*, in a given ſpace of time: For by obſerving ſeveral ſuch lengths in different Latitudes, the length of a Pendulum oſcillating at the Pole in the ſaid ſpace of time, may ſafely be concluded upon.

Again, in ſolving this Problem, there will be no need of fitting a Pendulum after ſuch

a

Plate 13. Book 3.



a manner in different places, as to make it perform its oſcillations in the ſame ſpace of time: It will be ſufficient to carry into the other places a Pendulum fitted to one place, ſo as that its ſeveral ſingle oſcillations may anſwer a given ſpace of time, and mark how much a Clock, that has the ſaid Pendulum for its Ballance, is ſwifter or ſlower during one apparent revolution of a Fix'd Star to the ſame place, according as the places, into which it is carried, are nearer the Equator or Pole, than that place is to which the Movement is adjuſted, and from that acceleration or retardation it may be found by calculation, what length is neceſſary to make the ſingle oſcillations to be performed in the ſame ſpace of time, as they are in the place to which the Pendulum was firſt adjuſted.

COROLLARY.

Hence it follows, (the ſame things remaining) if a Circle be deſcribed upon the interval *CA* or *CE* [*Fig.* 53.] and the line *CB* drawn any how be produced to *D*, that *DB* is as the increment of the accelerative Gravity in the receſs from the Equator towards either Pole, or as the increment above the length of the Pendulum oſcillating at the Equator in the given time, agreeable to the place *B*. For it has been ſhewn above, that if *AC* repreſent the gravity or length of a Pendulum at the Pole, *CP* will repreſent the increment of the gravity or length of the ſaid Pendulum oſcillating in an equal time at the Equator; and therefore (producing *Pp* to *F* and *G*) *PF* will repreſent the increment of the gravity or of the length of the ſaid Pendulum at the Pole *P*. In like manner, if *CB* repreſent the gravity or length of the Pendulum at *A*, *AC* (that is, *DC*) will expreſs the gravity or length of the Pendulum at *B*; and conſequently *DB* the exceſs or increment of the gravity

vity or length of the Pendulum at *B*, above the gravity or length of the Pendulum oscillating in the same time at *A*.

If *APEp*, the section of the Earth with the plane of the Meridian be an Ellipse, the increments of the gravity or of the length of the Pendulum in the different places of the Surface of the Earth, above the gravities or length of the Pendulum at the Equator, will be very nearly in the duplicate ratio of the right Sines of the Latitudes of those Places. Thro' *D* draw *DM* parallel to the Axe *Pp*, meeting the Ellipse in *L*, and the right line *AE* in the point *M*, from whence let fall the perpendicular *MK* upon *CD*; take *DH* equal to *PF*, wherefore *CH* is equal to *CP*, and join *LH*: Because *APE* is an Ellipse, (by *Prop.* 31. *B.* 1. *El. Conic.*) CP^q is to ML^q as the rectangle under *AC* and *CE* is to the rectangle under *AM* and *ME*: And, by the same, in the Circle *AFE*, CF^q is to MD^q as the rectangle under *AC* and *CE* to the rectangle under *AM* and *ME*; and therefore CF^q will be to MD^q as CP^q to ML^q. Wherefore *CF* is to *MD* as *CP* to *ML*; and therefore (by *Prop.* 19. *El.* 5.) *PF* is to *LD* as *CP* to *LM*; that is, *HD* is to *LD* as *CH* to *ML*; and therefore (by *Prop.* 2. *El.* 6.) *LH* is parallel to *CM*; and consequently the Triangle *LDH* is similar to the Triangle *MDC*, and similarly posited. But from the right angle *M* of the Triangle *CMD* the right line *MK* is drawn perpendicular to the Hypothenuse *DC*, which also is very nearly done in the Triangle *HLD*, the particle *LB* of the curve of the Ellipse being almost a right line and perpendicular to *CD*, because the Ellipse *APEp* does not differ much from the Circle *AFEG*: Wherefore *CD* is similarly divided in *K*, as *HD* is divided in *B*; that is, *HD* is to *DB* as *CD* to *DK*, or *PF* to *BD* as *CD* to *DK*.

Again,

Again, because of the rightangled Triangle *CMD* divided by the right line *MK* perpendicular to *CD*, (by *Prop.* 8. *Elem.* 6.) *CD*, *DM*, *DK* will be proportional, or the ratio of *CD* to *DK* the duplicate of the ratio of *CD* to *DM*; that is, of the ratio of *CF* to *MD*. But it has already been ſhewn, that *PF* is to *BD* as *CD* to *DK*; and therefore *PF* is to *BD* in the duplicate ratio of *CF* to *MD*: But *CF* is the whole Sine, and *MD* the right Sine of the angle *ACB*, namely of the Latitude of the place *B*; and *PF*, *BD*, are the increments of the gravity or length of the Pendulum oſcillating in the given time, agreeable to the Places *P* and *B*, above the gravity or length of the Pendulum at the Equator: And therefore ſuppoſing *APEp* an Ellipſe, the increment of the gravity or length of the Pendulum at the Pole, above the gravity or length of the Pendulum at the Equator, is to the like increment in any other Place *B* of the ſurface of the Earth, very nearly in the duplicate ratio of the Radius to the right Sine of the Latitude of the ſaid place *B*. And the ſame may be ſhewn in any other place on the ſurface of the Earth: And therefore, *ex æquo*, if *APEp* be an Ellipſe, the increments of the gravity or of the length of the Pendulum in different places of the ſurface of the Earth, above the gravity or length of a Pendulum oſcillating the like Time at the Equator, in the duplicate ratio of the right Sines of the Latitude of thoſe places verynearly.

SCHOLIUM.

All theſe Premiſes are true, ſuppoſing the Earth to conſiſt of uniform and equally denſe matter. For we ſuppoſed the Fluid, wherewith the Canals *ACP*, *BCP* are filled, to be uniform and homogeneous. But if the matter whereof the Earth conſiſts, be denſer at the Center than towards the ſurface, the above mentioned increments of the di-

diſtances from the Center going from the Poles towards the Equator, and the proportional increments of Gravity and of the length of the Pendulum oſcillating in the given Time going from the Equator towards the Poles, will be greater than the Calculus upon the former ſuppoſition makes them; and the Axis of the Earth will be leſs in reſpect of the Diameters perpendicular to the ſame. For if this matter towards the Center, by which the Earth is denſer there, be imagined to be removed, or rather be looked upon ſeparately, that the remaining matter may be every where equally denſe; from what precedes 'tis evident, that the accelerative Gravity in different places of the ſurface of this equally denſe Earth, is reciprocally proportional to the diſtance from the Center. But the ſaid accelerative Gravity is ſtill increaſed, by reaſon of that redundant matter about the Center; and that (as is ſhewn in *Book* 1.) nearly in the duplicate reciprocal ratio of the Diſtance from the Center; Since therefore in the caſe of the uniformly denſe Earth its ſuperficies is elevated towards the Equator and depreſſed towards the Poles; and the Gravity at the Equator leſs than at the Pole in the ratio of the diſtance of the Pole from the Center to the ſemidiameter of the Equator; and further the gravity at the Equator to the aforeſaid redundant matter about the Center ſhould be leſs than at the Pole, in a duplicate ratio of the diſtance of the Pole from the Center to the ſemidiameter of the Equator, which is leſs than the former ſimple ratio, ſince that ratio is a ratio of leſſer inequality; and therefore from both cauſes together, or in the caſe of the Earth's being denſer toward the Center, the Gravity at the Center compounded of the two former, is leſs than the Gravity at the Pole in a ratio leſs than that is which the diſtance

diſtance of the Pole from the Center has to the ſemidiameter of the Equator. And therefore from this leſſer Gravity at the Equator in reſpect of that which holds at the Poles, the Earth will be more elevated at the Equator, than what it would be from what goes before. And the ſame holds in the length of the Pendulum oſcillating in a given ſpace of time; becauſe it is ſhewn above to be proportional to the accelerative Gravity.

PROPOSITION LIII.

TO determine the ratio that the Axis of the Sun, or of a given Planet, has to the Diameters perpendicular to it.

Find by *Prop.* 26. *B.* 1. the ratio of the centrifugal Force in the Circle that is in the middle between the Poles of the Sun or of the given Planet to the centrifugal Force in the Equator of the Earth: But the ratio between the centrifugal Force in the Equator of the Earth to the Gravity on the Surface of the Earth is given, *viz.* the ſame with the ratio found above, that the exceſs of the Diameter of the Earth along the Equator has to the Axis of the ſame; and by *Schol. Prop.* 48. the ratio of the Gravity on the ſurface of the Earth to the Gravity on the ſurface of the Sun or of the given Planet is given: And therefore the ratio compounded of theſe will be given, namely the ratio that the centrifugal Force in the middle Circle, between the Poles of the Sun or of the given Planet, has to the Gravity on the ſurface of the ſame. And this is the ſame with that which the exceſs of the Diameter of the Equator of the Sun or of the given Planet above the Axe of the ſame to the ſaid Axe, ſince (by preced. *Prop.*) that exceſs ariſes only from that centrifugal Force, and conſequently is proportional to it: There-

 fore

fore (by *Dat.* 6.) the ratio between the Axes of the Sun or of a given Planet and its Diameter perpendicular to the Axe will be given.

SECTION IX.

Of the Diſtances of the Fix'd Stars.

PROPOSITION LIV.

TO *determine the Diſtance of a Fix'd Star by Obſervation.*

Having determined the Diſtance and Magnitude of the Sun and Primary Planets, what remains is only to diſcourſe a little of the Diſtance of the Fix'd Stars. And tho' the Diſtance of the Fix'd Stars be ſo immenſely great in compariſon of the Diameter of the Earth, that the latter intirely vaniſhes and becomes inſenſible; and therefore all the ways laid down in this and the preceding Book for finding the Parallax of any Phænomenon, and conſequently the Diſtance, are uſeleſs in this matter: Yet the Diameter of the Orbit that the Earth deſcribes in the ſpace of a Year about the Sun, may ſeem big enough to be compared even with the Diſtance of the Fix'd Stars: and ſince we, who are carried on the Earth, poſſeſs ſometimes this, ſometimes that extremity of the Diameter of the ſaid Orbit, there ought to ariſe a very ſenſible Parallax of the Fix'd Stars from ſo ſenſible a removal of the Obſerver. The Obſervation may be made very commodiouſly after the following or ſome ſuch like method.

Let *S* [*Fig.* 54.] repreſent the Sun, *AMB* the *Orbis Magnus.* Chuſe two Stars C and *F* near

one another, ſituated not far from the Ecliptic. Obſerve their Diſtance, when one, as C, is in oppoſition to the Sun, *viz.* the angle *FAC*. Obſerve their Diſtance again, when C is in conjunction with the Sun, namely the angle *FBC*. Therefore in the Triangle *FAB* all the angles being given, the ratio of the ſides is given, namely of *FA* the Diſtance of the Fix'd Star from *A* to *AB* the Diameter of the *Orbis Magnus* known by *Prop.* 45; therefore *AF* is determined. After the like manner the Diſtance of a Fix'd Star may be found, if the ſecond obſervation of the apparent Diſtance of the Fix'd Star be made, when C is not in conjunction with the Sun, (which perhaps would be difficult, by reaſon of the Sun's brightneſs,) but in any other known ſituation of the Earth, as when it is in *M*: For the right line *AM*, and its ratio to the known one *AB*, is known by the above cited *Prop.* 45.

Having once determined the Diſtance of a Fix'd Star, its Magnitude will be defined by the method laid down in *Prop.* 46, provided the apparent Diameter of the Fix'd Star be ſenſible.

SCHOLIUM.

Becauſe the Obſervations deſcribed above, or others like them neceſſary to determine the Diſtance of the Fix'd Stars, are ſo ſlippery and require ſo much nicety, that Artiſts dare not promiſe they ſhall obtain it by thoſe means; yet they think they have done ſomething very conſiderable, if they can ſhow by obſervation that there is a Parallax of the Fix'd Stars in reſpect to the *Orbis Magnus*: For by this means they could put the Motion of the Earth out of all doubt, which all acknowledge to be a matter worth their while to do. But this may be done pretty well by the following method.

The other things remaining, let the two Fix'd Stars *F* and C (*fig.* 55.) be ſuppoſed not to be near

theEcliptic. Let a Plane drawn thro' them and the Sun cut the Orbit of the Earth in *A* and *B*, which Points will be given exactly enough for the purpose, by what has been said in the preced. Book. Let the angles *CAF*, *CBF* (or the apparent Distances of the Fix'd Stars *F* and *C*, seen from the Earth, when she was in *A* and *B*,) be observed, which will be remarkably enough different from one another, if *AB* be sensible in regard of *AC*, and the Stars *C* and *F* very unequally distant from the Sun, which seems probable to hold in Stars of a different Magnitude. Perhaps if instead of the two Stars *C* and *F*, two, such as *C* and *D*, be observed, *D* will be seen from *A* to be on this side of *C*, from *B* on that side, (which may happen in so great a multitude of Fix'd Stars) or it may be, these two Stars will appear to coincide at one time of the Year, and to be sensibly distant from one another at another time of the Year: In which case the Annual motion of the Earth about the Sun will be evident by the bare use of a Telescope, without any Instrument necessary to take the distance of the Stars,.

The above mentioned method of observing, besides that it is capable of receiving all the improvement, that any Instrnments have or will receive, is hardly any ways chargable, since there needs only a Telescope with a Micrometer, there is no need of erecting an Observtaory nor of rectifying a Perpendicular, all which are very uncertain, since it is not absolutely certain that either Walls tho' the firmest, or Rocks or Mountains themselves always retain the same situation. To this head we may refer perhaps what that celebrated Astronomer Mr. *Cassini* has discovered about the Fix'd Stars; namely that the First of Aries sometimes appears split into two equal ones, and distant by the length of theDiameter of either from one ano-

ther,

ther, which he has also observed of the preceding Head of Gemini; and some others, as some of the Pleiades; and the middle in Orion's Belt, appear to be three, nay, four sometimes.

PROPOSITION LV.

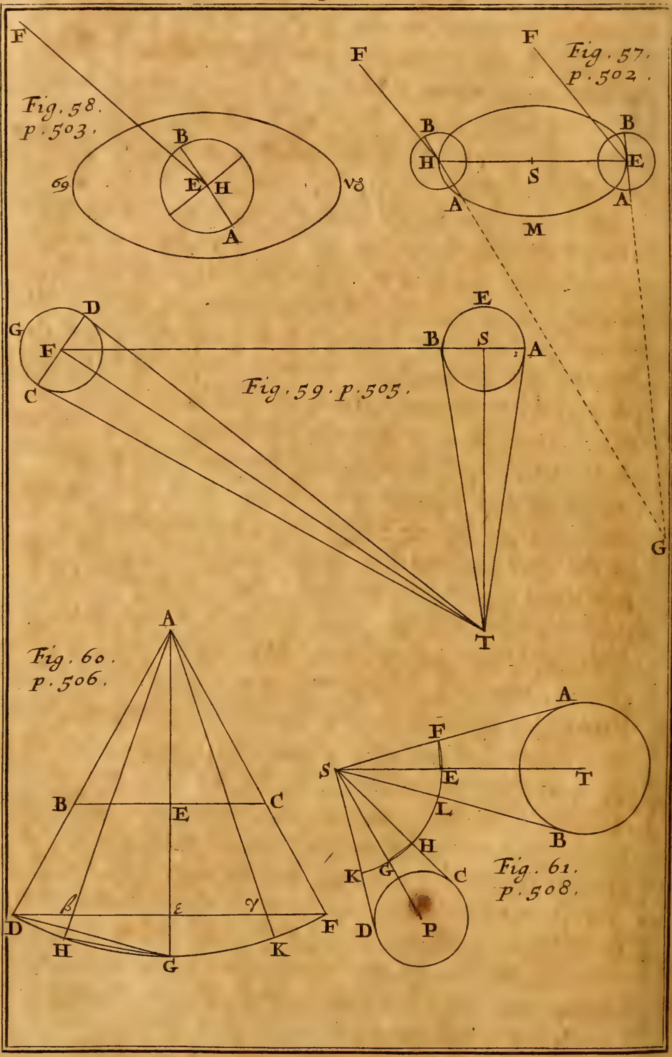
TO *observe the Parallax of the* Orbis Magnus, *by the access and recess of a Fix'd Star to or from the Pole of the Equator made in different Seasons of the Year.* [Fig. 56.]

Let *S* represent the Sun; *HME* the Orbit of the Earth, whose points *E* and *H* are the places of the Earth in the Solstices. Further let *F* be a Fix'd Star near the Pole of the World, that is, let *AB* the Axis of the Earth *ATB* contain with a a right line connecting the Earth and the Fix'd Star an angle *BHF* or *BEF* pretty small; and let this Star be not far from a Solsticial Colure, and therefore in conjunction almost with the Sun in the other Solstice, or let the Plane *EFH* be perpendicular to *HME* the plane of the Ecliptic. 'Tis evident that the angle *BHF* the distance of the Fix'd Star from the Pole of the World, when the Star is in opposition to the Sun, is less than the angle *BEF* the distance of the same from the Pole, when the Fix'd Star is in conjunction with the Sun. And conversely, if the angle *BHF* be found less by observation than *BEF*, this inequality must be thrown upon the Parallax of the Annual Orbit. And the Distance of the Fix'd Star is gained after this manner: The difference of the angles *BEF*, *BHF* is equal to the angle *F*; and in the Triangle *FEH* the other angle *FEH* is given, namely the Latitude of the Fix'd Star at the Time, when the Earth is at *E*: From whence the ratio between *HE* the Diameter of the *Orbis Magnus*, and *HF* the Distance of the Fix'd Star will be known.

SCHOLIUM.

This method of finding by Obſervation the Parallax of the *Orbis Magnus*, that moſt induſtri us Obſerver Mr. *John Flamſteed* makes uſe of, and ſo anſwers the objection of ſuch, as with *Ricciolus*, deny the motion of the Earth about the Sun, becauſe it is confi. med by the obſervation of a Parallax of the *Orbis Magnus*. For he found the Diſtance of the Pole Star from the North Po e to be greater about the Summer Solſtice than about the Winter one, by about 40 or 45 Seconds, and that by a continued ſeries of Obſervations for ſeven Years, allowing for the correction of the Inſtrument, and the change of the Star's place on the account of the receſſion of the Equinoctial points: as Mr. *Flamſteed* learnedly diſcourſes in a Letter to Dr. *Wallis*, dated *Decemb.* 20, 1698 publiſhed in 3 Vol. of *Wallis's Mathemat. Works.* But this method of determining the Parallax of a Fix'd Star, ſuppoſes the Axis of the Earth *A B* [*Fig.* 57.] to be exactly parallel to itſelf, when the Earth is in the points *E* and *H* of its Orbit, when the Obſervations are made. And tho' that ſmall Nutation of the Axis which ariſes from the diminution of the Inclination of the Ecliptic and Equator at the Solſtices, and the increaſe of it at the Equinoxes (by *Corol.* 20. *Prop.* 66. *B.* 1. *of* Newton's *Math. Princip. Nat. Philoſ.*) does not affect or hinder this obſervation, as Mr. *Flamſteed* obſerves in the ſaid Epiſtle; yet another Nutation ariſing from another cauſe may produce all this diverſity in the diſtance of the Pole-Star from the Pole: Namely, if the denſity of the Southern Hemiſphere of the Earth be a little greater than that of the Northern, (either becauſe of the leſſer Heat there and greater Cold here, or becauſe of the inequalities of the Continents of the Earth placed about the Poles, or ſome other cauſe unknown to us,) for then ſince in the Winter Solſtice

Plate 14. Book 3.



the South Pole *A* is inclined toward to the Sun, and at the ſame time is nearer to it than the North Pole *B*, when in the time of the Summer Solſtice the latter is inclined towards the Sun, *H A* will be more inclined to the plane of the Ecliptic in the Winter time than *E A* in the Summer time; on which account *B H F* will be leſs than *B E F*, tho' the right lines *H F*, *E F* ſhould be parallel. Since therefore all that can be made out of Mr. *Flamſteed's* obſervation is, that the angle *B H F* is leſs than the angle *B E F*; and this may ariſe from a double cauſe, namely from the concourſe of the right lines *H F*, *E F* towards *F*, if the Axis of the Earth in one obſervation be parallel to the ſame in the other (which is ſuppoſed to be the caſe by Mr. *Flamſtead*, and expreſſed by the *Fig.* 56.) or from the concourſe of the lines right *H A*, *E A* towards *G* the quite contrary way to *F*, right lines *H F*, *E F* being ſuppoſed to be parallel, as in the Figure 57; from this obſervation the Parallax of the Fix'd Stars is not evinced, becauſe the obſervation may ſtand intire though the right lines *E F*, *H F* remain parallel; that is, tho' the Parallax of the *Orbis Magnus* be nothing.

Nay, this obſervation does not immediately prove even the Annual Motion of the Earth: For tho' the Earth be in the middle (revolving about its Axe, as in the *Semi-Tychonic* Syſtem, [*Fig.* 58.] and ſo make the apparent Diurnal Revolution of the Celeſtial Bodies;) the Sun in the Southern Signs may then attract the Southern Hemiſphere of the Earth turned to it, being nearer and perhaps denſer, ſo as to make the angle *BHF*, the diſtance of the Fix'd Star from the Pole, leſs than *BEF*, when the Sun in the Northern Signs being more diſtant, attracts the Northern Hemiſphere leſs, which alſo perhaps is leſs denſe.

The ſame likewiſe may be ſaid of the different

Distance of a Fix'd Star from the Zenith, which is the way Dr. *Hook* formerly sought after the Parallax of the Fix'd Stars. For if the direction of the Axis of the Earth be changed, the direction of any other right line in the Earth (whose position to the Axe is given) will likewise be changed: And the motion of the Earth about the Sun, can't be certainly concluded, from the various inclination of a right line connecting the Fix'd Star and the Earth to that right line in the Earth. It were therefore to be wished, that our exact Observers would for the future manage their observations for proving the Annual Motion of the Earth after this manner, according to the prec. *Prop.* or its *Schol.* For so at last the Parallax of the Fix'd Stars in regard of the *Orbis Magnus* will with any certainty be found (if it be sensible,) since the change of those angles can't arise from any other cause, but the change of the Earth's place or of the situation of the Fix'd Stars among themselves; but the identity of the situation of the Fix'd Stars in regard of one another is taken as a principle by all such as endeavor to find their Parallax. In the mean while the reasons laid down in the following Proposition incline me to believe with *Copernicus*, that the Diameter of the *Orbis Magnus* in regard of the Fix'd Stars (even the nearest) is insensible.

PROPOSITION LVI.

THE Distance of the Sun from the Earth in respect of the Distance of a Fix'd Star (even the nearest) from the Earth, vanishes almost and is insensible.

Since the Sun and Fix'd Stars are the only great Bodies of the Universe that have any native Light, they are justly esteemed by Philosophers to be of the same kind, and designed for the same uses; and 'tis the effect of a Man's temper, that sets a greater value upon his own things than

he

he ought, that makes him judge the Sun to be the biggeſt of them all. Let them therefore be ſuppoſed equal, ſince that the one exceeds the other has no ground or foundation to ſtand upon.

Let *T* be the Earth, [*Fig.* 59.] *A E B* the Sun whoſe Center is *S*; *CGD* a Fix'd Star equal to the Sun, whoſe Center is *F*. Since the angle *BTS*, that the ſemidiameter of the Sun ſeen from the Earth appears under, exceeds 16 Minutes, then will *B S* or *C F* equal to it, be almoſt the two hundredth part of *ST*; or 200 times *C F* will be equal to *TS*. In the Triangle *FST* rightangled at *S*, the Sine of the angle *S F T* is to the ſide *S T* as the radius to the ſide *C T*, equal to *FT*: From whence (by *Prop.* 11. *Elem.* 5.) the Sine of *S FT* is to *ST* as the Sine of the angle *C T F* to *C F*. And ſince, in theſe ſmall angles, the angles themſelves are as their Sines, the angle *S FT* will be to the angle *C T F* as the right line *ST* to *C F*; and therefore 200 *CTF* is equal to *S FT*, or 100 *CTD* to *TFS*. But that moſt acute Obſerver *Chriſtian Hugens* (in *his Syſtem of Saturn*) affirms, that he, aſſiſted by a Teleſcope, (by the help of which the diameter of an object was magnified an hundred times) could not obſerve any magnitude in the Diameters of the Fix'd Stars tho' the brighteſt, but that they appeared like a point; that is, the angle *DTC* made an hundred times bigger, was ſtill inſenſible; or an hundred times *CTD* (that is, *S FT*) could not be taken notice of by Senſe. Therefore the Semidiameter of the *Orbis Magnus S T* ſeen from the neareſt Fix'd Star (ſuch as we are to ſuppoſe the brighteſt to be) would appear like the ſmalleſt Point; that is, *TS* in reſpect of *S F* or *TF* vaniſhes and is inſenſible.

This ſmallneſs of the apparent Diameters of the Fix'd Stars is the cauſe of their Twinkling: For the ſmalleſt opake corpuſcles, which are always

moving

moving in the Atmoſphere, intirely cover any Fix'd Star, and then immediately uncover it, that is, make it Twinkle.

SCHOLIUM.

Becauſe $S\,T$ compared with $S\,F$ is inſenſible, 'tis no matter in what point of the *Orbis Magnus* the Fix'd Stars be obſerved; for the whole *Orbis Magnus* is but as a point in reſpect of the Diſtance of the Fix'd Stars: And ſince Saturn the outmoſt of the Planets, is not above ten times farther off from the Sun than the Earth, there will be immenſe Spaces between this outmoſt Planet and the neareſt Fix'd Stars to the Sun, into which the very excentric Orbits of the Comets may run, of which we have treated in *Prop.* 35. *B.* 1. And 'tis on the account of this immenſe Diſtance, that the Fix'd Stars have no ſenſible effects upon our Solar Syſtem, and don't diſturb the Planets contain'd in it.

Since therefore the Diſtance of the Fix'd Stars can't be determined by obſervations made in different places of the Earth's Orbit, it remains that we ſhow how to meaſure it by ſome Optical Methods: Two of which we ſhall propoſe, one by Mr. *James Gregory* publiſh'd in the year 1668, the other by Mr. *Hugens* in his *Coſmo-Theoria.*

PROPOSITION LVII. LEMMA.

THE illuminations of the ſame Sphere, in different diſtances from the lucid Body, are reciprocally as the Squares of the diſtances.

Tho' this Propoſition be an eaſy conſequence of *Prop.* 48. *B.* 1. yet I ſhall take the pains to demonſtrate it here. Let *A* [*Fig.*60.] be any point of a lucid Body taken at pleaſure; and let *B C* be the diameter of the Sphere that is illuminated, directly oppoſed to the Rays, at a ſmaller diſtance from the lucid Body; $\beta\gamma$ the ſame diameter at a greater diſtance:

diſtance; that is, let AE, a ray reaching to the Center E be perpendicular to BC, and $A\epsilon$ to $\beta\gamma$: Join AB, AC, and being produced let them meet $\beta\gamma$, alſo produced, in D and F. On the Center A, at the diſtance AD or AF, deſcribe an arc of a Circle DF, which let $A\beta$, $A\epsilon$ and $A\gamma$ produced, meet in H, G and K; and let this Figure be imagined to revolve about the axe AG. 'Tis evident that all the Rays falling upon the Sphere, whoſe diameter is BC, upon the removal of it, will be ſpread upon the Sphere whoſe diameter is DF; And therefore the illumination of the Sphere, whoſe diameter is BC, is to the illumination of the Sphere, whoſe diameter is $\beta\gamma$, as all the rays, contained within the Sector of the Sphere generated by the rotation of ADG, to all the rays contained within the Sector of a Sphere generated by the rotation of AHG; that is, as the Surface of a Sphere generated by the rotation of the arc DG, to the Surface of a Sphere generated by the rotation of the arc GH; that is, (by *Prop.* 49. *B.* 1. *of Archimed. of the Sph. and Cylind.*) as the Circle whoſe radius is DG, to a Circle whoſe radius is HG, or as the Square of DG to the Square of HG. But in ſmall angles, ſuch as we ſuppoſe DAG and HAG to be, DG is to HG very nearly as DE to $\beta\epsilon$: And therefore the illumination of a Sphere, whoſe Center is in E, is to the illumination of the ſame or an equal Sphere, whoſe Center is in ϵ, in the duplicate ratio of $D\epsilon$ to $\beta\epsilon$, or $D\epsilon$ to BE, or $A\epsilon$ to AE; that is, reciprocally as the Square of the diſtances. And ſince the ſame, for the ſame reaſons, is true of any other point of a lucid Body, as well as of A, what was propoſed becomes evident.

And after the like manner, the illuminations of the ſame Sphere made by equal lucid Bodies at unequal diſtances, are reciprocally as the Squares of the diſtances from thoſe lucid Bodies.

PRO-

PROPOSITION LVIII. LEMMA.

ALL the Rays of the Sun falling upon one Planet are to all the Rays of the Sun falling upon another Planet, as the Squares of the Chords of the Horizontal Parallaxes of the Sun ſeen from thoſe Planets. [Fig.61.]

Let *S* repreſent any radiating point of the Sun; and the Spheres *AB*, *CD*, whoſe centers are *T* and *P*, any two Planets. Join *ST*, *SP*, let a plane drawn thro' them cut the Spheres in the Circles *AB*, *CD*; and from *S* draw Tangents to the ſaid Circles. Upon the Center *S*, at any diſtance, draw an arc of a Circle meeting the right lines *SA*, *ST*, *SB*, *SC*, *SP* and *SD* in *F*, *E*, *L*, *H*, *G* and *K*; join the right lines *EF*, *GH*. Let the Figure *AFSET* be imagined to revolve about its Axis *ST*, and *CHSGP* about the Axis *SP*. All the Rays flowing from the point *S* that fall upon the Planet *T* (*viz.* all within the Cone *ASB*) will be to all the Rays falling upon the Planet *P* (namely all in the Cone *CSD*) as the Superficies of a Sphere deſcribed by the rotation of the arc *EF*, to the Superficies of a Sphere deſcribed by the rotation of the arc *GH*; that is, (by *Prop.* 49. *B.* 1. *of Archimedes of the Sphær. and Cyl.*) as a Circle deſcribed upon the radius *EF* to a Circle deſcribed upon the radius *GH*, or as the Squares of the right lines *EF*, *GH*: But *EF* and *GH* are Chords of the angles *AST*, *CSP* to the radius *SE*, and the ſaid angles are the horizontal Parallaxes of the Sun *S*, ſeen from the Planets *T* and *P*; and therefore all the Rays flowing from the point *S* and falling upon the Planet *T*, are to all falling upon *P*, as the Square of the Chord of the Sun's Parallax ſeen from *T*, to the Square of the Chord of the Sun's Parallax ſeen from *P*. And ſince the ſame is in like manner true of Rays flowing from any other point of the Sun, the Propoſition is evident.

Instead of the Chords the Arcs themselves, or the Angles proportional to them, and subtended by them, may be taken, as in the preced. Prop.

SCHOLIUM.

After the like manner, all the rays of a lucid and every way radiating Sphere, are to all its rays that fall upon a given Sphere and illuminate it, in the duplicate ratio of the Diameter (or Chord of the Semicircle) to the Chord of the horizontal Parallax of the lucid Body, seen from the illuminated Body; or as the Square of the Diameter to the Square of the Chord of the said horizontal Parallax. And consequently (by taking the halves of the antecedents,) the half of all the Rays of the lucid Sphere, or all the Rays flowing from a radiating Hemisphere (supposing the other Hemisphere opake) are to all its Rays falling upon the Sphere to be illluminated, to which it appears full, as the half of the Square of the diameter (that is, the Square inscribed in a Circle) to the Square of the Chord of the Parallax of the lucid Body, seen from the Sphere to be illuminated.

PROPOSITION LIX. LEMMA.

THE *ratio of the illumination of the Earth by the Sun to its illumination by a Planet shining at the full, is compounded of the duplicate ratio of the Chords of the Parallaxes of the Sun seen from the Earth and that Planet, and the ratio of a Square inscribed in a Circle to the Square of the Chord of the horizontal Parallax of the said Planet seen from the Earth.*

The Rays, with which the Planet illuminates the Earth, are such as flowing originally from the Sun are reflected back again upon the Earth. We suppose therefore that each Planet reflects every way all the Rays of the Sun that fall upon it; that is (for greater ease) we suppose a Planet to emit Rays every way from that Hemisphere of it

that is turned towards the Sun, *viz.* ſo many as fall upon it from the Sun. 'Tis evident that the illumination of the Earth by the Sun is to the illumination of the ſame by a Planet ſhining in the full, as all the Rays of the Sun falling upon the Earth to all the Rays flowing from the Planet and falling upon it: But the ratio of the Sun's Rays falling upon the Earth to the Planet's Rays falling upon the ſame, is compounded of the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, (that is, all the Rays flowing from the Planet's Hemiſphere,) and the ratio of all the Rays flowing from the Planet's Hemiſphere to all thoſe of them that fall upon the Earth. And the former of theſe compounding ratios, or the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, is equal (by the preced. *Prop.*) to the duplicate ratio of the horizontal Parallax of the Sun ſeen from the Earth to the horizontal Parallax of the Sun ſeen from that Planet; but the latter, namely the ratio of all the Rays flowing from the Planet's Hemiſphere illuminating the Earth to all of them that fall upon the Earth and inlighten it, is equal (by the preced. *Schol.*) to the ratio that a Square inſcribed in a Circle has to the Square of the Chord of the horizontal Parallax of the ſaid Planet ſeen from the Earth. And therefore the ratio of the illumination of the Earth by the Sun to the illumination of the ſame by a Planet ſhining in the full, is compounded of the duplicate ratio of the Chords of the horizontal Parallaxes of the Sun ſeen from the Earth and that Planet, and of the ratio of the Square inſcribed in a Circle to the Square of the Chord of the horizontal Parallax of the ſaid Planet ſeen from the Earth. *Q. E. D.*

PRO-

PROPOSITION LX.

TO *determine the Distance of a Fix'd Star.*

Chuse a Fix'd Star of nearly the same brightness with some Planet, when it is in the Full; in which case the Earth is equally illuminated by these two Stars. But the mutual Distances of the Sun, Earth, and Planet at the time of the abovementioned Observation, and the Magnitudes of these three Bodies (by *Prop.* 46.) being known, the horizontal Parallaxes of the Sun and Planet seen from the Earth (by *Prop.* 48. *B.* 2.) become known, together with the angle the said Planet appears under from the Sun, whose half is the horizontal Parallax of the Sun seen from that Planet. And consequently (by the preced. *Prop.*) the ratio of the illumination of the Earth by the Sun to the illumination of the same by the said Planet becomes known; that is, the ratio between the illumination of the Earth by the Sun and by the Fix'd Star : But (by *Prop.* 57.) the illumination of the Earth by the Sun is to the illumination of the same by a Fix'd Star (equal to the Sun and equally lucid) in the duplicate ratio of the Distance of the Fix'd Star from the Earth to the Distance of the Sun from the same; therefore the ratio between the Distances of the Fix'd Star and the Sun from the Earth is known; and consequently the Distance it self of a Fix'd Star from the Earth is known.

SCHOLIUM.

For the same purpose you may commodiously take Jupiter in its Achronical situation, and Sirius, which (if any other) may be looked upon as equal to the Sun. But since Jupiter in the said situation exceeds Sirius in brightness, and besides the Rays of the Sun falling upon Jupiter are not reflected towards

towards the Earth without being weakened, but are most of them stifled near the surface of Jupiter, the Distance of Sirius determined after this manner will be less than the true Distance.

Proposition LXI.

To measure the Distance of a Fix'd Star another way.

Tho' the diameter of the brightest Fix'd Star can't by any Optical Instrument yet invented, be made so big as to become sensible, and such as may be compared with the Sun's Diameter; however, the Sun's Diameter may be lessen'd so much, as that its Light thus lessen'd may (in appearance) be no greater than the light of Sirius twinkling in the Night; which will be done if its Rays be let into a Tube thro' a hole small enough; or if its image be lessened as much by means of a Lens, or partly by one partly by the other method: For the diameter of the image of the Sun, tho' insensible (if looked for inconsiderately) becomes known by calculation founded upon Optics and Dioptrics, having the Diameter of the hole, the Lens and situation of the Eye given. But the Diameter of an image of the Sun, as bright as Sirius to an Eye given by position, is equal to the Diameter of Sirius; therefore the apparent Diameter of Sirius, before insensible, becomes known by calculation: But the Sun and Sirius are supposed equal; therefore their Diameters are equal: And therefore the Distance of Sirius will be to the Distance of the Sun, as the Sine of the apparent Diameter of the Sun to the Sine of the apparent Diameter of Sirius, or (by reason of the smallness of these angles) as the apparent Diameter of the Sun to the apparent Diameter of Sirius; therefore the Distance of Sirius will be known. *Q. E. I.*

The End of the First Volume.